The Origin of the Moon and the Single-Impact Hypothesis, II

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This is the second paper devoted to the numerical study of planetary collisions as a possible scenario for forming the Moon. We present a series of nine simulations of a collision between the protoearth and an impactor of various sizes. The mass ratio between the protoearth and the impactor ranged from 0.1 to 0.25. We were able to model both planets with iron cores, having modified our smoothed particle hydrodynamics code to allow the inclusion of up to 10 different material types. Two different relative velocities at infinity for the impactor were considered: \( v_\infty = 0 \text{ km/sec} \) and \( v_\infty = 10 \text{ km/sec} \). We show that for a low-velocity collision and an impactor in the mass range \( 6.5 \times 10^{26} M_{\text{impactor}} \leq 8.2 \times 10^{26} \text{ g} \), more than a lunar mass of iron-poor material is thrown into orbit. For an impactor with a mass within this range, the ejected mass that goes into orbit is for the most part divided comparably into material orbiting inside the Roche limit and into material orbiting outside the Roche limit. This material is either spread out in the form of a disk, or, for a relatively narrow range of masses toward the lower end of the range, clumped into an object of about lunar mass beyond the Roche limit. For impactors more massive than about \( 8.2 \times 10^{26} \text{ g} \) we found that there is too little mass thrown into orbit. For very small mass impactors well over a lunar mass is placed in orbit, but a large amount of it is iron. In the high-velocity range we did not find a possible mass range for the impactor that would lead to the formation of an iron-poor disk massive enough to form the Moon. © 1987 Academic Press, Inc.

1. INTRODUCTION

In Benz, Slattery, and Cameron (1986; hereafter referred to as Paper I) we described a three-dimensional smoothed particle hydrodynamics (SPH) code and gave some results of our initial investiga-

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According to this hypothesis, toward the end of the planetary accumulation process, the protoearth collided with a planetary body having a substantial fraction of the protoearth mass. The collision took place with a substantial impact parameter, so that the combined angular momentum of the two bodies was approximately that now in the Earth–Moon system. At the velocity of the collision (11–15 km/sec) most rocky materials are vaporized. It was therefore hypothesized that pressure gradients in the resulting vapor cloud played a crucial role in accelerating a relatively small fraction of the vapor into orbit about the protoearth, forming a disk (Cameron and Ward 1976, Cameron 1985). The dissipation of this disk was then thought to spread matter radially so that the Moon could collect together gravitationally beyond the Roche lobe (Ward and Cameron 1978, Thompson and Stevenson 1983, Thompson and Stevenson 1987). The scenario has recently been reviewed by Stevenson (1987).

Paper I describes our initial investigations of this problem using the three-dimensional SPH code. In that series of investigations we ran several collision cases with a variety of parameters. Both the target planet and the impactor (taken to have 0.1 of the target’s mass) were wholly composed of granite. The results of these simulations showed that substantial amounts of mass could be injected into orbit about the Earth.

In the present paper these simulations have been made more realistic since we have introduced iron cores into the planetary bodies. We find that the iron cores play a distinct and important role in the mechanics of mass injection into orbit around the Earth. We have explored a range of mass ratio parameters of the impactor relative to the target and have identified the mass ratio interval in which the results are promising for lunar formation.

We discuss in this section the various assumptions made in our model. Most of the assumptions are identical to those made in Paper I, so we limit ourselves to giving a list of these assumptions, without many details. However, we shall emphasize the differences between the assumptions in these simulations and those of Paper I.

As in Paper I, we neglect the shear strength of the material and all heating or cooling mechanisms other than kinetic energy dissipation in shocks and the $PdV$ work. The energy lost by radiation from hot fragments was neglected because of the short time scale of the simulation. We used 3008 particles to simulate the colliding planets; hence the mass resolution is about $2 \times 10^{24}$ g. The time needed for such a particle to radiate a significant fraction of its energy is very much longer than the 24 to 48 hr covered by our simulations. This assumption is therefore consistent with the resolution of the code.

The main difference with respect to our first paper is in the structure of the protoplanets. The smoothed particle hydrodynamics code was modified to allow for the inclusion of up to 10 different material types, allowing us to include planetary iron cores in the simulation. Core formation occurs readily in bodies of the sizes we consider in this paper (Stevenson 1980, 1983), therefore it is reasonable to assume that the impactor would be already differentiated at the time of the collision. Each protoplanet was given a pure iron core of one-third the planetary mass, which was surrounded by a granite mantle (granite was chosen for the same reason as in Paper I, namely, it was the only rock for which we had equation of state data at the time of the calculations). With this modification we expected the model to be able to determine not only what fraction of material would go into orbit but also the iron content of the
material which forms the prelunar accretion disk.

As in Paper I we neglected the possible initial spin of the protoearth. This assumption not only simplifies the computations, but one can also argue that the initial angular momentum of the protoearth is likely to be small compared to the present angular momentum of the Earth–Moon system. As shown by Wetherill (1986), the size of the second largest body in a swarm forming a planet is roughly 10% of the mass of the largest body. Since the formation of the Moon requires a planetesimal of roughly that amount, the impactor would plausibly be this second largest object. The main body has accumulated out of many smaller objects. The probability that the randomly distributed impacts in the accumulation process could impart an angular momentum comparable to that of the Earth–Moon system is extremely small.

We note that the model includes self-gravity in a completely self-consistent and rigorous way. Self-gravity is the dominant force when a body exceeds a size of about 50–100 km. It is responsible for internal compression, increasing internal density and making vaporization of material more difficult. It also turns out that the proper computation of gravitational forces plays a key role in determining how much matter ends in orbit following the collision. Simulations that do not include self-gravity and therefore must terminate some tens of minutes after the impact (Kipp and Melosh 1986) are useful in allowing a comparison of those early moments with results from our code but they cannot carry the simulation far enough to determine the orbiting mass fraction. Furthermore, it turns out in our simulations that this fraction is not determined by initial pressure gradients but mainly by gravitational torques. This makes any prediction of the final result from the initial stages virtually impossible.

We used for both granite and iron the so-called Tillotson (1962) equation of state. This is an analytical expression which includes 10 material-dependent constants. Depending on the density and internal energy, there are three different regions in which the equation of state takes different forms: (a) condensed or cold states, (b) expanded and hot states, and (c) intermediate states. The numerical constants for granite (and their definition) are given in Paper I, and we give here the equivalent coefficients for iron: $a = 0.5$, $b = 1.5$, $u_0 = 9.5 \times 10^{10}$ ergs/g, $p_0 = 7.86$ g/cm$^3$, $A = 1.279 \times 10^{12}$ ergs/cm$^3$, $B = 1.05 \times 10^{12}$ ergs/cm$^3$, $u_* = 1.42 \times 10^{10}$ ergs/g, $u'_* = 8.45 \times 10^{10}$ ergs/g, $\alpha = 5$, and $\beta = 5$.

It has been pointed out to us (Stevenson 1987 and private communication) that the Tillotson equation of state does not represent adequately a two-phase medium since such a medium cannot be described by two thermodynamic variables in the usual way. This means, for example, that for any given internal energy one can have an enormous range of pressures, depending on the liquid mass fraction. Therefore pressure gradients computed from the Tillotson equation of state in any two-phase medium may be erroneous. This is a well-justified criticism.

Jay Melosh has kindly provided us with a new equation of state (the ANEOS equation of state developed by Thompson and Lauson 1984) that includes a two-phase medium treatment in a more correct thermodynamic way. This analytical equation of state includes treatment of the melt liquid–vapor and solid–vapor transitions. Each phase is described in terms of its Helmholtz free energy potential. Separate phases are treated as separate components in a mixture that is always in pressure and temperature equilibrium. Mixed phases can coexist at the same Gibbs potential. Since we obtained this equation of state, we have run several problems using both equations of state, trying to determine differences in the results. Our preliminary indications are that the details of the equation of state do not seem to make a significant difference in
the amount of material put into orbit. The
distribution of the material in orbit (the
fractions inside or outside the Roche limit)
seen to depend slightly on the form of the
equation of state. These results will be
presented in a subsequent publication.

Stevenson (1987) points out also that the
vapor fractions given in Paper I are under-
estimated since we counted as vapor only
those particles which had been fully vapor-
ized and not those in the intermediate
Tillotson region which would presumably
be mixtures of gas and condensate. This is
quite true. Particles partially vaporized are
those that have a specific internal energy
between $u_i$ and $u'_i$ (see Paper I for the exact
definitions). The difficulty with the Tillot-
sion equation of state is to know for any
given particle falling within these bounds
what fraction is liquid and what fraction
gas. We counted as gas only those particles
with energies above $u'_i$. Since many parti-
cles ended in the intermediate energy
range, the vapor fraction may well have
been underestimated by quite a large frac-
tion. This, however, affects only the vapor
fraction we quote in our paper and not the
computation itself. The reader should view
the results reported here as underestimat-
ing the effects of pressure gradients; since
we found quite promising scenarios for
lunar formation, we believe that the effects
of gas pressure gradients can only extend
the number of interesting scenarios, if they
are important at all.

3. NUMERICAL METHOD

As in Paper I we used the so-called
smoothed particle hydrodynamics method
(SPH) (Lucy 1977, Monaghan 1986, and
references in Paper I). The smoothed par-
ticle hydrodynamics method is a free
Lagrangian approach for solving the con-
servation equations of hydrodynamics. A
finite set of spatially extended Lagrangian
particles replaces the continuum of hydro-
dynamic variables. The usual cell mesh is
not needed; SPH therefore does not suffer
from mesh tangling or inaccuracies associ-
ated with the severe distortion of the cell
mesh. Hence SPH is particularly suited for
the simulation of highly distorted flows
such as those occurring during impacts. A
simulation evolves in time by computing
the trajectories of all particles from the
various forces acting among them. These
forces are computed from interactions be-
tween the particles that depend on their
relative positions and velocities.

In Paper I we gave an extensive descrip-
tion of all the numerical equations used in
the code. We have now added the possibil-
ity of having several different types of
materials. We associate with each particle a
number that gives the material type. In the
equation of state subroutine, the material-
dependent coefficients are chosen as a
function of this material type. Since iron
has a higher density than granite and since
all particles have the same mass, the mean
separation between iron particles is smaller
than that between granite particles.

In addition to the test cases mentioned in
Paper I, several more applications have
been run using the code. One of those cases
(Kidman and Benz 1986) consisted of hit-
ting a quarter-inch lead plate with a 20-g
lead sphere at 6.6 km/sec. Experimental
data are available for this case so that direct
comparison between the numerical sim-
ulation and experiment could be done. We
also used the new analytical equation of
state. The numerical results were found to
be in very good agreement with exper-
iment. The code was able to reproduce
accurately the size of the hole in the plate
as well as the shape of the vapor cloud on
the other side of the plate. The velocities of
the leading edge of that vapor cloud agreed
to within a few percent with the exper-
iment. These results confirmed the ability
of the code to compute collision prob-
lems accurately, in particular, the trans-
formation of kinetic energy into inter-
nal energy and then the release of that
energy as kinetic energy of an expanding
cloud.
4. INITIAL CONDITIONS AND UNITS

The following procedure was adopted to obtain a planet with an iron core of one-third its mass. First, hydrostatic equilibrium in the planet was computed by assuming that the planet was all iron and by introducing a damping term in the equation of motion so that the particles would relax to that equilibrium. Once equilibrium was reached, the outer two-thirds of the planet was changed into granite, and the whole system was relaxed again. This procedure avoided mixing of particles at the interface between the mantle and the core. After that the planet was very stable, and even without any dissipation term included we could not see any appreciable motions building up for times longer than those covered by the simulations.

The present series of simulations involves collisions between the protoearth and impactors of various masses. The mass ratio between the two planets was obtained by using different numbers of particles for both the protoearth and impactor. We showed in Paper I that the mass and the angular momentum escaping the system during low-velocity collisions (around 10 km/sec) were of the order of a few percent (this result is also recovered in these simulations—see the escaping mass in Table I). Therefore we decided to start all the simulations with a total angular momentum only slightly higher than the present angular momentum of the Earth-Moon system (see Table I for exact values).

This is a key constraint. This assured that when the calculations with systems having the appropriate angular momentum. The total mass of the system was chosen to be equal to the mass of the Earth-Moon system. With those conditions, once the mass ratio and relative velocity are specified, the impact parameter is automatically determined: the larger the impactor the smaller the impact parameter (at fixed velocity). For the three higher-velocity cases we chose a somewhat larger value to account for the larger mass loss occurring during these collisions.

The simulation is initiated by setting both planets on a collision course. The initial positions correspond to a separation of 10 protoearth radii along the x-axis and the appropriate impact parameter along the y-axis. The relative velocity was computed from the principle of energy conservation so as to match the desired relative velocity at infinity.

As in Paper I we chose a system of units in which all relevant numbers used in the computation by the code have values close to unity. This system is the following: mass unit \( M_u = 6.0 \times 10^{27} \) g and distance unit \( R_u = 8.08 \times 10^8 \) cm. All other units can be derived from those two and their values are: time units \( T_u = 1148.9 \) sec or 19.1 min and energy unit \( E_u = 4.95 \times 10^{11} \) ergs/g. Unless otherwise specified, all results and plots are given in these units.

5. RESULTS

Paper I reported results from simulations of a collision between the protoearth and an impactor of 0.1 its mass. Various impact parameters, relative velocities, and values of the initial internal energy were exam-
This paper investigates mainly what happens when the mass of the impactor varies. A limited number of "high"-velocity cases are also considered. A total of nine new simulations were run for various combinations of parameters; the initial characteristics of these simulations can be found in Table I. Also given in this table are the relative variations of total energy and total angular momentum. We found in Paper I that the amount of initial internal energy does not make much difference, therefore we chose not to vary it and took $1 \times 10^{10}$ ergs/g as the starting value for all the simulations. The exact phase structure of the planet (solid or molten) is not really known since it depends on the equation of state zero-temperature minimum energy. However, since the code ignores material strength, we actually model liquid planets even at zero temperature.

5.1. Low-Velocity Collisions

$(V_z = 0 \text{ km/sec})$

It is difficult to define a typical history of a collision since the results are very sensitive to the mass of the impactor. However, the first stages are roughly the same. As the impactor approaches the protoearth it becomes deformed by the tidal field. At the time the impactor hits, it slows down and there is a corresponding dissipation of kinetic energy into heat in the strong shock at the interface. Material is then vaporized and ejected in two high-speed jets, one directed forward and a smaller one directed backward. The intensity of this jetting is of course determined by the intensity of the shock, which is in turn determined in part by the impact parameter. In our simulations, the impact parameter is fixed by the mass of the impactor since all our computations are done at (almost) fixed angular momentum. This means that we get more prominent jetting for a large mass impactor than for a small one. Once this jetting begins and once the impactor starts to be destroyed, the various simulations differ quite strongly from one another. For the low-velocity collisions one can, however, group the results in three categories depending on the mass of the impactor. These three domains are: (1) low-mass impactors ($M_{\text{impactor}}/M_{\text{protoearth}} \leq 0.12$); (2) intermediate-mass impactors ($0.12 < M_{\text{impactor}}/M_{\text{protoearth}} \leq 0.17$); and (3) large-mass impactors ($M_{\text{impactor}}/M_{\text{protoearth}} > 0.17$).

For low-mass impactors the collision is almost a grazing one (owing to the angular momentum constraint), therefore the shock is weak, and the impactor does not get completely destroyed but rather "bounces" and is thrown onto a very eccentric orbit that makes it collide again with the protoearth. The second collision destroys the impactor completely and a great part of it is flung into orbit. The problem, however, is that most of the iron core of the impactor ends in orbit as well, leading to an iron-rich prelunar accretion disk. Large-mass impactors, on the other hand, collide with a relatively small impact parameter (again due to the angular momentum constraint). The shock is very strong and large jets are created. The bulk of the impactor is greatly slowed down and is finally accreted by the protoearth. The fraction left in orbit is iron poor but its mass is only about a lunar mass or less. Finally, in the intermediate region the collision leads to an iron-poor disk containing well over a lunar mass.

We present now some more detailed results of one representative case in each of these three categories in the form of snapshots of the course of the collision during the simulations. We plot velocity vectors of individual particles projected onto the plane of the collision. To avoid overly dense regions we superimposed an invisible grid and plotted only one vector per cell. The velocity vectors are normalized to the maximum velocity in each frame. In the upper line we display the time and the coordinates of the region covered by the plot $(x_{\text{min}}, x_{\text{max}}, y_{\text{min}}, y_{\text{max}})$. The numbers are all given in units defined in the preceding section. To allow identification of iron
particles we plotted a “•” in addition to the velocity vector for those particles.

5.1.1. Low-Mass Impactors ($M_{\text{impactor}}/M_{\text{protoearth}} \leq 0.12$) and Tidal Disruption

Selected times for simulation 1 (mass ratio 0.10) are shown in Fig. 1. Notice how large the impact parameter is; the relative motion between the two planets is nearly parallel to their colliding surfaces, so there is no appreciable shock, except in the outermost regions, and there is little mass loss by jetting (Fig. 1a). The granite mantle is partially destroyed but the iron core is only slightly deformed (Fig. 1b). In fact, the major damage to the impactor is due to gravitational torques since the impactor was present for a significant amount of time inside the protoearth’s Roche limit. However, the time spent inside that limit was too short to allow complete destruction of the impactor.

Mizuno and Boss (1985) found a similar result in their study of the tidal disruption of dissipative planetesimals. They showed that if the planetesimal that enters the Roche lobe on a parabolic orbit is dissipative, very little damage is done to it due to the short time spent inside the lobe. The larger the dissipation rate, the smaller is this damage. In our simulations we do not include any extra dissipation in addition to the artificial viscosity needed to treat the shocks, and the impactor still survives the first passage (Fig. 1c).

The impactor is now on a bound orbit and total destruction occurs during the next perigee passage (Figs. 1d and 1e). Tidal disruption of a planetesimal can therefore still occur even if the planetesimal survives its first close encounter. The condition for this to happen is the formation of a binary system in which the orbiting object regularly enters the Roche lobe of the main planet. However, if it is not destroyed in a small number of passages, interaction with other bodies is likely to increase rather than decrease the encounter velocity. From our simulation we can estimate the amount of energy dissipated in the encounter and hence get a rough idea of the maximum relative velocity that would still allow the capture. In this way we find a relative velocity of 2.5 km/sec. That means that the same collision, occurring at a relative velocity at infinity greater than 2.5 km/sec, would not lead to the capture of the impactor. This value of 2.5 km/sec is really an upper bound for tidal capture since both planets still actually hit, increasing the rate of dissipation. We think that these results, together with Boss and Mizuno’s conclusions, rule out tidal disruption as a potential mechanism, not only for the formation of the Moon, but more generally for any significant increase in the rate of accumulation of the planets from planetesimals.

On the first return of the projectile, in the perigee passage (Fig. 1d), the encounter is much more violent because the impact parameter has decreased (owing to the decrease of the relative velocity and thermal increase in the radius of both planets), which allows more time for gravitational torques to dismantle the impactor as well as a stronger collisional shock. Figure 1e shows how the impactor is destroyed and spread out. The long elongated tail is a typical signature of gravitational torques. These torques do not distinguish between iron and rocks, resulting in both core and mantle being spread out into orbit in clumps of various sizes. Some of the pieces will collide with the protoearth again and some will stay in orbit. At the time we stopped the calculation (Fig. 1f) (after slightly more than 2 days of model time) there are two major clumps remaining in orbit.

At this point we computed the orbits of all the particles to see whether they will collide with the protoearth, reenter the Roche limit, or even escape the system. Surprisingly, the escaping fraction is quite large. Since there was not a strong enough shock to cause this ejection, this mass was thrown away by the gravitational torques due to the various clumps in orbit. This mechanism is even stronger than the shock
Fig. 1. Snapshots of run 1 (mass ratio: 0.10). Velocity vectors are plotted at particle locations. The velocity has been normalized to its maximum value in each frame. Time and coordinates of the four corners of the plotted field are given in the upper line (see text for units). For iron particles a "●" is plotted.
acceleration we get in the large-mass impactor regime (see next section). This was also found by Benz and Hills (1986) in their simulations of colliding stars: in the low-velocity range the mass loss is a maximum for grazing collisions and not for head-on ones. We also computed the radius of the circular orbit having the z-component of the angular momentum of the particle, since we expect that the particles would collide with one another to form a disk in the x–y plane. We included in this calculation the particles having a perigee greater than the radius of the protoearth and not having enough energy to escape the system.

Table II gives the result of these calculations. We also give in this table the mass fraction with equivalent circular orbits inside and outside the Roche limit. We determined the iron content in each category as well as the amount of material which originated from the protoearth. Note that in none of our simulations does iron from the core of the protoearth get into orbit; any iron in orbit always comes from the core of the impactor. The most striking feature in this table is that about 0.85 lunar masses of iron went into orbit (0.74 orbiting inside the Roche lobe and 0.11 outside the Roche lobe). This is the main criterion which allows us to rule out small impactors (less than about $5.9 \times 10^{26}$ g) as candidates for the formation of the Moon, since in our opinion a realistic mechanism has not been proposed for the separation of orbiting iron from rocks before accumulation of the material into the Moon.

5.1.2. Large-Mass Impactors \( (M_{\text{impactor}}/M_{\text{protoearth}} \approx 0.17) \)

Larger-mass impactors lead to quite a different result. Due to our angular momentum constraint the impact parameter is relatively small, and the shock is much stronger than that observed in the previous case. Figure 2 illustrates the history of a collision between an impactor only five times less massive than the protoearth (simulation 6). Figure 2a shows the beginning of the collision. The strength of the shock is evidenced by the large jetting of very hot vaporized rocks in the early phase of the collision (Fig. 2b). In such jets the velocity of the material is often very much larger than the preshock velocity. Indeed,

<table>
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<th>Run</th>
<th>( M_{\text{in}} ) ((M_{\text{Moon}}))</th>
<th>%Fe</th>
<th>( M_{\text{out}} ) ((M_{\text{Moon}}))</th>
<th>%Fe</th>
<th>( M_{\text{Earth}} ) ((M_{\text{Moon}}))</th>
<th>( \bar{R}<em>{\text{circular}} ) ((R</em>{\text{Earth}}))</th>
<th>( M_{\text{escape}} ) ((M_{\text{Moon}}))</th>
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<td>0.27</td>
<td>5.2</td>
<td>5.98</td>
</tr>
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</table>

* The columns give, in order, the number of the run, the mass orbiting the protoearth inside the Roche limit, the iron fraction in that mass, the mass orbiting the protoearth outside the Roche limit, the iron fraction in that mass, the total mass of material from the protoearth in orbit, the mean circular radius of the orbiting material, and finally the mass escaping the system. All masses are given in units of one lunar mass.
Fig. 2. Snapshots of run 6 (mass ratio: 0.25). See caption of Fig. 1 for the meaning of the symbols.
we found that 0.65 lunar masses were able to escape the system despite the collision occurring just at the escape velocity. The shock was too weak to cause significant disruption of the impactor’s core (Fig. 2b). Since the escaping material took a good fraction of the energy and angular momentum with it, the material that is left is quickly accreted by the protoearth. Especially impressive is the way the iron core of the impactor plows its way through the mantle of the protoearth (Figs. 2c and 2d).

At the end of the simulation there is not much material left in orbit (Figs. 2e and 2f). We computed the distribution of this material in the same way as above and obtained the numbers in Table II. The total mass in orbit is equal to 0.15 lunar masses. Clearly the mass ejected into orbit in a collision with a large impactor is not sufficient to allow the formation of a Moon-sized satellite.

5.1.3. Intermediate-Mass Impactors (0.12 < \(M_{\text{impactor}}/M_{\text{protoearth}}< 0.17\))

This is the only mass range that, together with our angular momentum constraints, leads to a suitable prelunar accretion disk or even in one case to a Moon-sized object orbiting the protoearth outside the Roche lobe. Figure 3 is a sequence of snapshots from a collision in which the mass of the impactor was 0.136 times that of the protoearth. The impact parameter is such that the iron core of the impactor grazes the protoearth (Fig. 3a). Due to the strength of the shock as well as to the gravitational torques, the incoming planet is destroyed and spread out the first time it hits the protoearth (Fig. 3b). The iron core, however, is more resistant to the shock than was the granite mantle, and as a result it is not completely destroyed. When it gets sufficiently far away from the protoearth its self-gravity is able to pull it back together (Fig. 3c).

We have by now formed primarily a three-body system: the protoearth, the impactor’s core, and most of the remaining parts of the impactor’s mantle. The latter body is by far the smallest of the three. The iron core is on a close, eccentric orbit about the protoearth. The rocky clump is on a trajectory that takes it farther out and therefore it travels at a somewhat smaller velocity, lagging behind the core. This results in a tremendous gravitational torque acting on the outer material arising from the protoearth and the iron core. Enough angular momentum is transferred in the outer regions to make sure that the rocky material will not collide with the protoearth again. Having thus given away a large part of its angular momentum, the iron core hits the protoearth at the end of its first revolution (Fig. 3d), giving rise to a gigantic splash (Fig. 3e) in which the iron cuts right through the mantle to land on the protoearth’s core. There is a negligible chemical interaction between the iron and the mantle at this time, but the protoearth is heated to a temperature of the order of 8000°K by the released gravitational potential energy. At the end of the calculation (after about 1 day of elapsed time) we are left with a pure granite clump of one lunar mass orbiting safely outside the Roche lobe (in the lower right corner of Fig. 3f), together with some smaller pieces. Slightly more than half a lunar mass is also orbiting inside the Roche limit (see Table II). The system has a total angular momentum equal to the present Earth–Moon system to within half a percent.

In the same mass domain we ran simulations 2 and 4. These two simulations left a suitable amount of mass in orbit but did not show the formation of a clump outside the Roche lobe. Instead, the mass formed a disk-like structure. We conclude that the formation of a Moon-sized orbiting clump is only possible for a rather narrow range of impact parameters (due to the great amount of cpu time needed per simulation we did not try to get the exact value for this mass range). It is not clear whether the direct formation of the Moon (without going through a disk dissipation phase) would
allow sufficient time to get rid of most of the volatile elements. The formation of a prelunar accretion disk inside the Roche limit has the nice feature that it easily allows the loss of volatiles during the century or so (Stevenson 1987) needed to transport mater-
rial out of the Roche lobe. We note that the fraction of protoearth material in orbit is relatively small, somewhere between 8 and 16%. This implies quite strongly that the impactor must have about the right chemical composition to form the Moon (allowing for modifications occurring as a result of the collision).

Figures 4 and 5 give a graphical representation of the mass distribution orbiting about the protoearth in the various low-velocity collisions reported so far. If the requirement for formation of the Moon from one of these collisions is at least one lunar mass in orbit together with at most an extremely small amount of iron, we see from those figures that the mass of the impactor has to be in the range $6.5 \times 10^{26} \text{ g} \leq M_{\text{impactor}} \leq 8.2 \times 10^{26} \text{ g}$.

5.2. HIGH-VELOCITY COLLISIONS
($V_c = 10 \text{ km/sec}$)

Only three simulations were run in this velocity regime since the results indicate that there is no chance of forming a Moon or a suitable prelunar accretion disk at this velocity. The starting conditions were also a little different. Mass loss is more important, so it is difficult to estimate how much angular momentum should be present initially so that the system will end with the appropriate value. Table I gives a list of the initial parameters for these three simulations.

The results are summarized in Table II and plotted in Figs. 6 and 7. We see that a
large impactor mass is needed if enough mass is to be put into orbit. A mass ratio of 0.18 is the only one of the three simulations that leads to more than a lunar mass in orbit. However, the main problem resides in the large amount of iron that is included in that mass: more than 1.3 lunar masses! In fact, a large part of the granite mantle escapes as indicated by the large mass fraction that is able to leave the system. We are forced to rule this case out for the same reason we ruled out the low-mass impactor cases in the low-velocity regime. There is no convincing mechanism that would allow the separation of the rocks and the iron in orbit.

5.3. Disk or Clump?

We have already pointed out that the range of impact parameters leading to the formation of a bound clump of material that retains its integrity (orbiting beyond the Roche lobe) is much less than the range that leads to the formation of an orbiting disk. That does not mean that one should reject this scenario. In fact it would simplify substantially the theory! The evolution of a disk of material orbiting a planet inside its Roche limit was first discussed by Cameron and Ward (1976) and by Ward and Cameron (1978). Recently extensive work on the properties of a prelunar disk was done by Thompson and Stevenson (1987) and Stevenson (1987). They showed that the disk has a cooling time of about $10^2$ years, that due to its two-phase structure such a disk is highly unstable even though hot, and that mass is spread beyond the Roche lobe in a time comparable to the cooling time. This outer mass is hot and may not be immediately unstable to fragmentation.

None of our “successful” simulations led to the formation of a disk of material orbiting only inside the Roche limit. There was always at least a comparable amount of material orbiting beyond the limit. This material was either in the form of a major clump or divided into several small pieces. We computed the angular momentum of all material in orbit and deduced a corresponding mean radius from it. This is the radius of the circular orbit of a single clump having all the mass and the angular momentum of the material in orbit. This mean radius is listed in the seventh column of Table II. All of these mean radii for the cases having more than a lunar mass in orbit are larger than the Roche limit. For high-velocity collisions this mean radius is quite large.

Thompson and Stevenson (1987) have pointed out that a hot and thick disk may extend quite a long way outside the Roche distance without necessarily becoming gravitationally unstable to breaking up into clumps. Thus the formation of a disk extending outside the Roche lobe in many of our simulations does not mean that gravitational instabilities will immediately break up the outer part of the disk. However, that event must happen sooner or later, when the outer part of the disk has cooled sufficiently. If several clumps are formed at that time, many of them will soon collect into a single body, since a circular distribution of mass points is gravitationally unstable, and a few of the clumps may be expelled from the system in the process. It thus appears that a single body will be formed beyond
the Roche lobe after an indeterminate history in which the material may go through the stages of being part of a disk and of being divided among several other bodies. Hence, whether a single clump is formed directly or after a complicated history, it appears that it will form. It also appears that there will inevitably be a great deal of material in a disk orbiting inside the Roche limit when the outer single clump forms.

The outer orbiting clump will greatly perturb the material orbiting inside the Roche limit at least down to a certain distance. It is likely that the outer part of the disk will not last long once the clump has formed. Some of the perturbations may increase the radial distance of part of the material so that it will either accrete onto the main clump or be ejected from the system. But the perturbations will certainly also raise tides throughout the disk, and these tides should be capable of transferring much of the angular momentum in the disk to the clump, with an accompanying inward flow of the material in the disk (Lin and Papaloizou 1979, Goldreich and Tremaine 1980). Thus it is possible that much or little mass is placed in orbit, since gravitational torques can again play a key role, but the orbiting material is rich in iron.

We therefore propose that one should consider as prime lunar formation candidates those simulations that lead to about a Moon mass of iron-free material in orbit in the general vicinity of the Roche limit. This consideration limits the impactor's mass to the relatively narrow range $6.5 \times 10^{26} \text{g} \leq M_{\text{impactor}} \leq 8.2 \times 10^{26} \text{g}$.

6. CONCLUSIONS

This set of calculations confirms the result found in Paper I that the single-impact hypothesis provides a plausible scenario for forming a prelunar accretion disk. However, the introduction of an iron core has led to a few surprises that were not expected from Paper I, since we found that it has a unique and distinctive role to play in the types of planetary collisions studied here.

- We found that gravitational torques play the major role in putting material into orbit and destroying the low-mass impactors. Pressure gradients, which were originally proposed as the mechanism allowing acceleration of material into orbit, play only a secondary role in these calculations (this is confirmed by preliminary results from calculations done with a better equation of state). The gravitational torques are maximized for a relatively narrow range of impactor masses ($6.5 \times 10^{26} \text{g} \leq M_{\text{impactor}} \leq 8.2 \times 10^{26} \text{g}$), since in this mass range the collision results in the physical separation of the core and the mantle of the impactor, forming a three-body system (protoearth–iron core–mantle) which is very efficient in transferring angular momentum outward. For larger impactors, where this separation does not occur, not enough mass is thrown orbit at low velocities and too much iron is thrown into orbit at higher velocities. In the case of low-mass impactors, enough mass is placed in orbit, since gravitational torques can again play a key role, but the orbiting material is rich in iron.

- We found that the distribution of the material in orbit is not in a disk confined within the Roche limit. In the cases in which a lunar mass or more of material was put into orbit, a majority of it was outside the Roche limit. The formation of the Moon may therefore not depend upon the dissipation and spreading of an accretion disk beyond the Roche lobe. Thus significantly less mass may be needed in orbit than previously thought since a lot of mass need not be dumped on the protoearth to bring the remainder out to the Roche limit.

- Around impactors of mass $7.3 \times 10^{26} \text{g}$ (the exact range of possible mass is not known), the result of the collision is not the formation of a disk but the formation of a single clump of almost exactly a Moon mass in an orbit that remains outside of the
Roche limit, together with a disk inside the Roche limit.

- For high-velocity collisions we found no candidate cases. Either there was not enough mass in orbit or too much iron was mixed with the rocks that were placed into orbit around the protoearth.

- Since only small relative velocity collisions lead to a state from which a Moon might form, we conclude that the impactor must have had an orbit somewhat similar to that of the Earth in order to collide with such a low velocity. This excludes the possibility that the impactor was perturbed by Jupiter and hit the Earth while on a very eccentric orbit. Since we find the Moon to be mostly formed from material originating from the impactor, the impactor itself must have been formed in a nearby region. This is consistent with the empirical evidence of the similarities between the lunar and terrestrial oxygen isotopic anomalies.

REFERENCES


