The Origin of the Moon and the Single-Impact Hypothesis I

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Received November 12, 1985; revised March 3, 1986

Recently the single-impact hypothesis for forming the Moon has gained some favorable attention. We present in this paper a series of three-dimensional numerical simulations of an impact between the protoearth and an object about 0.1 of its mass. For computational convenience both objects were assumed to be composed of granite. We studied the effects on the outcome of the collision of varying the impact parameter, the initial internal energy, and the relative velocity. The results show that if the impact parameter is large enough so that the center of the impactor approximately grazes the limb of the protoearth, the impactor is not completely destroyed; part of it forms a clump in a large elliptical orbit about the Earth. This clump does not collide with the Earth, since the effects, first, of vapor pressure gradients during the impact, and later, of angular momentum transfer due to the rotation of the deformed Earth, have modified the ballistic trajectory. However, since the orbit of the clump comes close to the Earth (within the Roche limit) the clump will be destroyed and spread out to form a disk around the Earth. The amount of angular momentum in the Earth–Moon system thus obtained tends to fall short of the observed amount; this deficiency would be eliminated if the mass of the impactor were somewhat greater than the one assumed here. The scenario for making the Moon from a single-impact event is supported by these simulations. © 1986 Academic Press, Inc.

1. INTRODUCTION

There does not as yet exist a consensus of opinion concerning the origin of the Moon. At the Kona meeting on this subject held in 1984 (Hartmann et al., 1985), the classical theories of lunar origin had a few proponents, but these theories received a significant amount of criticism. Much discussion was focused on newer ideas, in particular, on the single-impact hypothesis for its origin (Hartmann and Davis, 1975; Cameron and Ward, 1976).

This single-impact theory has the advantage of overcoming most of the difficulties with the classical theories. By varying the initial conditions of the impact (relative velocity, impact parameter, and mass ratio) in a reasonable way, the angular momentum of the Earth–Moon system can easily be obtained (Cameron and Ward, 1976; Ward and Cameron, 1978) without having to get rid of a large excess. During the collision there would be a major loss of volatile elements. However, the approximate similarity between the Earth’s mantle and the Moon can be explained only if the impactor is already of roughly the same chemical composition since, as shown by these simulations, the Moon is formed almost exclu-
sively by material coming from the impac-
tor. Also Cameron (1983) suggested that
after such a collision the upper layer of the
very hot terrestrial atmosphere will interact
with the prelunar accretion disk, allowing
the lighter noble gases and most of the xe-
non to escape, thus explaining why our at-
mosphere is much less massive than the at-
mosphere of a comparable planet like Venus.

Preliminary calculations by Cameron
(1985) have shown that the single-impact
theory is plausible. However, these compu-
tations were too crude to allow any quanti-
tative prediction to be made. Therefore
more detailed simulations were needed in
which the physics of the shocks and the
vaporization process could be accurately
treated. In this paper we present a series
of 3-D numerical simulations of the colli-
sion between the Earth and an object of
about \( \frac{1}{10} \) its mass. Different impact veloci-
ties, impact parameters, and initial internal
energies are considered. Particular care
was taken in the choice and test of the
equation of state to model as accurately as
possible the thermodynamics of the mate-
rial during and after the shock.

The next section gives a list of the vari-
ous assumptions we made, and Section 3 is
a presentation of the equation of state we
used. The numerical technique and the
tests of the code are described in Sections 4
and 5. A description of how the initial con-
ditions were set up as well as the units used
in these simulations is given in Section 6.
Finally, in Section 7 we present the results.

2. ASSUMPTIONS

The complexity of a simulation of a colli-
sion between the protoearth and an object
of about \( \frac{1}{10} \) of its mass impacting at 11 km/
sec or more is such that several assump-
tions are needed to keep the problem trac-
table.

The first problem is that we do not know
the structure of the Earth or the impactor at
the time of the collision. We think it likely
that the protoearth and its impactor were
formed rapidly, and hence they were proba-
bly molten and should have iron cores. How-
ever, for simplicity, in this first series of
simulations we have assumed that both
the impactor and the protoearth were made
of the same material (granite) without any
core. By looking at the results one can see
that the Earth's core was not severely im-
pacted in the collision; therefore we do not
expect that much of it would go into orbit.
It is quite a different story for the impac-
tor's core as it is completely destroyed.

More detailed simulations in which both
planets have a core are planned.

We had to assume that both objects were
made of granite because we had no detailed
equation-of-state data closer to the present
Earth mantle composition than that.

Ahrens and O'Keefe (1972) have performed
a series of simulations of an impact be-
tween an aluminium or iron projectile on
various rocks to determine the speed
needed by the projectile to melt or vaporize
these rocks. They found that granite needs
a slightly higher velocity than the other
rocks they considered. We therefore expect
our model to underestimate slightly the
amount of vaporization that would occur as
compared to the amount we would obtain
with a material closer to the Earth's mantle
composition. We expect to get improved
equation-of-state data for other materials
and to rerun part of these simulations later.

A correct description of a solid should
include the calculation of all types of defor-
mations. During the approach to the im-
pact, even a cold body is expected to un-
dergo strong deformation (Kaula, 1984). As
long as the impact occurs at hypervelocity,
shear strength is not important, at least dur-
ing the impact and for some time afterwards
(the shear strength would be important if
we wished to compute, for example, the
size and shape of a crater, but that is not the
subject of this investigation). We therefore
neglect the shear strength of the material.
This, in turn, means that we do not have a
real solid but rather a liquid even at a low
internal energy. The main effect of neglect-
ing the shear strength is to allow the Earth and all debris rapidly to recover a spherical shape. The crater created on the Earth by the impact fills up with material and the Earth regains its spherical shape since there are no nondiagonal terms in the stresses opposed to gravity. The real protoearth, even though hot enough to be liquid, would not be free of viscous stresses. By letting the Earth quickly become spherical, this assumption causes us to underestimate somewhat the amount of angular momentum transferred to the clumps in orbit by a non-axisymmetric rotating Earth.

Since the impact takes place over a short time (about 1 h) we expect that the heating of the material will be due mainly to shocks; all other heat transport mechanisms have much longer time scales. Therefore the only heating mechanism taken into account was the kinetic energy dissipated in shocks. A final simplification concerns the cooling of the material after the shock. Here again we suppose that the only cooling process is the $PdV$ term. We are neglecting the energy radiated by the hot gas or the rocks because of the same time scale argument as before. This argument is certainly wrong for small fragments, but since we have utilized 2048 mass points, each one represents the equivalent of $3.2 \times 10^{24}$ g. For this size, we expect the assumption to be valid.

3. EQUATION OF STATE

In hypervelocity impacts it is a prime requirement to describe accurately the thermodynamics over a wide range of pressure and density; therefore the equation of state plays a very crucial role. This equation must be able not only to describe material at normal pressures and densities, but also its condition after shock, expansion, or change of phase in cases where the shock energy has been sufficient to melt or vaporize.

One equation of state widely used in impact simulations is due to Tillotson (1962). The general analytical form of this equation of state was derived by generalizing the Mie-Gruneisen equation of state (the Gruneisen coefficient was taken to be a function of volume as well as energy instead of volume alone) and by using the Rankine-Hugoniot relations to describe the compression. If, for a given material, the pressures given by the Tillotson equation of state are compared to those given by tables, it usually turns out that there is good agreement. The region in which the difference is largest is the so-called intermediate region (see below).

Depending on density and internal energy there are three different regions in which the equation of state takes a different form:

(a) For condensed states when $\rho > \rho_0$, or for any cold state when $u < u_s$ ($\rho_0$ and $u_s$ are, respectively, a reference density and specific internal energy for a given material):

$$
\frac{P}{\rho} = \frac{1}{\frac{u}{u_0} + 1} + A\mu + B\mu^2
$$

where $\eta = \rho/\rho_0$ and $\mu = \eta - 1$. The important point is that the pressure may be less than zero if the density is less than $\rho_0$ since $\mu$ is negative. This negative pressure provides tension and assures that a condensed matter object does not expand to infinity like a gas. Therefore no boundary conditions need to be added to the code.

(b) For expanded hot states when $\rho < \rho_0$ and $u > u_s$ the equation of state has the form

$$
P_e = au_0 + \left(\frac{b\mu}{u_0} + 1\right) + A\mu \exp(-\alpha(\eta^{-1} - 1)) \exp(-\beta(\eta^{-1} - 1)^2).
$$

If the density is low enough and the internal energy high enough the second term vanishes and the equation of state goes asymptotically to that of a perfect gas.

(c) A smooth transition between both
states is assured by a transition equation for the intermediate region defined by $u_s < u < u'_s$ and $\rho > \rho_0$. In this region the equation of state has the form

$$P_i = \frac{(P_c(u - u_s) + P_c(u'_s - u))}{(u'_s - u_s)}.$$  

The quantities $a, b, u_0, \rho_0, A, B, u_s, u'_s, \alpha,$ and $\beta$ are constants for a given material and were determined by Tillotson in the following way: The constants $a, b,$ and $A$ are derived a priori to fit special equation-of-state data for a given material. $u_0$ and $B$ are then adjusted to provide the best overall $P, V, u$ surface. The values obtained for granite are as follows: $a = 0.5, b = 1.3, u_0 = 1.6 \times 10^{11}$ erg/g, $\rho_0 = 2.7$ g/cm$^3$, $A = 1.8 \times 10^{11}$ erg/cm$^3$, $B = 1.8 \times 10^{10}$ erg/cm$^3$, $u_s = 3.5 \times 10^{10}$ erg/g, $u'_s = 1.8 \times 10^{11}$ erg/g, $\alpha = 5$, and $\beta = 5$.

Two important comments can be made as a conclusion to this section. The first is that to be vaporized the material must not only be hot, but also of low density. This means that during the collision the maximum amount of vapor is not obtained at the time of maximum shock or maximum internal energy, since this corresponds also to maximum compression, but at a later time when the material has already expanded. The second comment is that, since $a = 0.5$, the equation extrapolates to an ideal gas of $\gamma = 1.5$, which is quite a reasonable value.

4. NUMERICAL TECHNIQUES

The numerical method used to simulate these impacts is the so-called "smoothed particle hydrodynamics" (SPH) method. This method was first introduced in the astronomical context by Lucy (1977), and was shown, especially by Gingold and Monaghan (1979, 1981, 1982, 1983; Monaghan and Gingold, 1983), to give very good results in many different applications. A similar technique was used by Hausman (1981) to simulate colliding isothermal gas clouds. In these simulations, however, it appeared that the gas clouds interpenetrated. This forms the main criticism of the application of the particle method to collision simulations. However, Lattanzio et al. (1985) have shown that by using a somewhat more sophisticated version of the particle method, almost all interpenetration can be avoided. By running comparison simulations between our SPH code and a code that does not allow interpenetration at all, we will show that the particle method is indeed very well suited for the calculations of impacts.

The SPH method is a fully Lagrangian one in which the trajectories of a finite number of mass points are followed in time. Each of these particles is assumed to have its mass spread out in space according to a given distribution called the kernel, which is written in the following equations as $W(r, h)$, where $h$ is a characteristic length scale related roughly to the size of one particle. This formalism allows one to get rid of the usual grid, and therefore no rezoning due to a highly distorted path is necessary. It has been shown by Gingold and Monaghan (1982) (hereafter GM 1982) that this technique can be recovered by using standard interpolation techniques. Since the basic principles of the method have been published several times (see, for example, the references listed above), we will only recall how the two main equations (momentum and energy conservation) are written using this formalism.

4.1. Momentum Equation

Since the whole method turns out at the end to be very similar to the usual $N$-body problem, we will write the momentum equation as the equation of motion of the $i$th particle. Therefore

$$\frac{d\mathbf{v}_i}{dt} = -m \sum_{j=1}^{N} \left( \frac{P_j}{\rho_j^2} + \frac{P_i}{\rho_i^2} \right) \nabla_i W(r_{ij}, h) - G \sum_{j=1}^{N} \frac{M(r_{ij})}{r_{ij}^2} \mathbf{f}_{ij} + \mathbf{f}_{i}^{\text{visc}}$$

where $r_{ij} = |\mathbf{r}_i - \mathbf{r}_j|$ and $\mathbf{f}_{ij} = (\mathbf{r}_i - \mathbf{r}_j)/r_{ij}$.

The first term of the right side represents the pressure gradients. Its somewhat sur-
prising form is derived from the equality
\[
\frac{\nabla P}{\rho} = \nabla \left( \frac{P}{\rho} \right) + \frac{P}{\rho^2} \nabla \rho
\]
proposed by Gingold and Monaghan (GM 1982) to assure exact linear and angular momentum conservation.

The second term is the gravitational force for which we made use of Newton’s law since all particles are spherically symmetric. \(M(rij)\) is the mass of particle \(j\) within a sphere of radius \(r_i\) of particle \(i\) and is given by
\[
M(rij) = 4\pi \int_0^{r_i} r^2 \rho(r) dr = 4\pi \int_0^{r_i} r^2 W(r, h) dr.
\]

Finally, the third term represents the force due to the introduction of an artificial viscosity. Monaghan and Gingold (1983) have shown that the standard von Neumann-Richtmyer viscous pressure and the bulk viscosity both introduce unacceptably large post-shock oscillations and therefore have to be rejected. They also propose a new form of a bulk viscosity which is
\[
F_{\text{bulk}}^i = -\sum_{j=1}^N \Pi_{ij} \nabla_i W(rij, h) \quad \text{if} \quad v_{ij}rij < 0
\]
\[
F_{\text{bulk}}^i = 0 \quad \text{if} \quad v_{ij}rij \geq 0
\]
\[
\Pi_{ij} = -\frac{\alpha \varepsilon c_{ij}}{\rho_{ij}} \frac{v_{ij}rij}{r_{ij}^2 + \varepsilon h^2}
\]
with \(c_{ij} = 0.5(c_i + c_j), \rho_{ij} = 0.5(\rho_i + \rho_j); c_i\) and \(\rho_i\) are, respectively, the sound speed and density at particle \(i\), whereas \(\varepsilon\) and \(\alpha\) are arbitrary numerical constants.

The symmetric form for \(F_{\text{bulk}}^i\) assures also an exact conservation of linear and angular momentum and gives very good results in the shock tube problem (Monaghan and Gingold, 1983). We show in the next section that this form still allows some diffusion of particles and therefore underestimates the amount of kinetic energy dissipated in shocks. To avoid excessive interpenetration of the two bodies we add (following a suggestion of Frank Harlow) another term that has the property of dragging a particle along with the mean flow defined as some mean velocity of neighboring particles. This term is written
\[
F_{\text{drag}}^i = -K \text{sign}(v_{ij}rij) \sum_{j=0}^N \frac{v_{ij}p_{ij}f_{ij}}{p_{ij}} p_{ij} = W(rij, h) \quad \text{if} \quad \frac{r_{ij}}{h} < 4
\]
\[
p_{ij} = 0 \quad \text{if} \quad \frac{r_{ij}}{h} \geq 4
\]
and \(K\) is an arbitrary numerical constant. This force is not symmetric in \(i\) and \(j\), and therefore does not conserve momentum exactly. However, tests have shown that the variation of both linear and angular momentum is very small. To avoid further loss of angular momentum this term was turned off once the center of mass of the two colliding objects had reached their minimum separation. By doing this in all simulations, the amount of angular momentum loss never exceeds 0.15%. Moreover this loss occurs

![Figure 1](https://example.com/figure1.png)

**Fig. 1.** Variation of total, internal, and kinetic energy for the three codes. Solid lines, SPH (smooth particle hydrodynamics) with “drag term”; dotted lines, SPH without “drag term”; dashed lines, Eulerian code.
FIG. 2. Snapshots of run 1. \( \nu_0 = 0 \) km/sec; \( r_{\text{min}} = 0.77 R_{\text{earth}}; E_{\text{int}} = 10^7 \) erg/g. Velocity vectors are plotted at particle locations. The velocity has been normalized to its maximum value in each frame. Time and coordinates of the four corners of the plotted field are given in the upper line (in units defined in Section 3). For particles in the vapor phase a "●" is plotted.

before the time at which the particles spread out in space. Since this happens after the time of closest approach, the trajectories of the various clumps forming after collision are calculated accurately.

The total "viscous" force therefore becomes

\[
F_{\text{visc}}^i = F_{\text{bulk}}^i + F_{\text{drag}}^i
\]

and this completes the description of the equation of motion.

4.2. Energy Conservation Equation

The variation of the internal energy is given by thermodynamics and is written

\[
\frac{du}{dt} = -P \frac{dV}{dt} + \frac{dQ}{dt},
\]
where $dQ$ is the amount of energy absorbed by the system from its surroundings. By writing this equation for each particle one finds that the change of internal energy of the $i$th particle is given by the sum of the following two terms:

$$
- \left( \frac{dV}{dt} \right)_i = 0.5 \sum_{j=1}^{N} \left( \frac{P_j}{\rho_j^2} + \frac{P_i}{\rho_i^2} \right) v_{ij} \nabla_i W(r_{ij}, h)
$$

$$
\left( \frac{dQ}{dt} \right)_i = 0.5 \sum_{j=1}^{N} \Pi_{ij} v_{ij} \nabla_i W(r_{ij}, h)
$$

These two expressions allow us to compute the change in specific internal energy and therefore assure conservation of total energy.

Up to this point we have written all the equations without specifying any special form for the kernel $W(r, h)$. To complete the above description and to allow numerical computations it is necessary to define this kernel. It can be shown mathematically that there are only a small number of constraints on $W(r, h)$ such as that $f W(r, h) = m$ ($m$ being the mass of one particle), and that $W$ should be continuous, together with its first derivative, to assure the continuity of the forces. In practice, however, it is useful to choose a function for which all derivatives as well as the integral can be obtained analytically. One natural choice is the exponential function which we adopted in these simulations. Gingold and Monaghan adopted a Gaussian function (this type of kernel leads to a erf-type function when integrated and an analytical expression is unobtainable). The detailed effects of choosing a different kernel have not been studied thoroughly; however, theoretically there should not be a significant difference as long as the basic requirements for $W$ are met. In practice, the only way is to test the code against known analytical solutions or against results produced by other independent codes. This last procedure is the one that we have chosen and it is described in the next section.

The exponential kernel we chose was proposed by Wood (1981) and has the analytical form

$$
W(r, h) = \frac{m}{8\pi^3} \exp \left( -\frac{r}{h} \right)
$$

with $m$ the mass of one particle.
5. TESTS

The code with an ideal gas equation of state has already been tested in a wide number of applications (Benz, 1984), so we were sure of the general behavior of the code; nevertheless, with the change of the equation of state, new tests had to be done. The following simulation was selected as a test case to be run by the code: a head-on collision between two identical granite spheres. The same problem was then run using another code widely used at the Los Alamos National Laboratory and the results of both compared. The code used for comparison (LASOIL-3D) is a completely Eulerian code, and therefore totally different and independent from ours. The LASOIL family of codes is an extensively modified version of the OIL family of codes.
written by Computer Code Consultants, Inc. (Johnson, 1967). Major modifications in the LASOIL codes are the extension to 32 materials, new differencing techniques, and material transport routines; modifications were made by N.L. Johnson, K.H. Duerre, and J.B. Payne of Los Alamos National Laboratory. One unique aspect of this code is its ability to prevent completely any interpenetration of the two bodies during the collision. As one may expect such interpenetration in a particle scheme (see, for example, Hausman (1981) and Lattanzio et al. (1985)), comparison simulations between the two codes should not only allow a determination of the amount of interpenetration, but also a quantification of its effects on the results of the collision. Since the amount of interpenetration will be maximized if the colliding objects are not centrally condensed, we turned off gravity in the tests so that the two spheres were perfectly homogeneous.

As a first test we wished to check the code’s ability to compute accurately the thermal dilatation. For testing we first computed an equilibrium state of a granite sphere with zero internal energy. The mean density obtained was 2.71 ± 0.0005 g/cm³. This result is accurate to 0.4% since the density for granite is 2.7 g/cm³. The extremely low value for the dispersion around the mean density further indicates that the sphere which was obtained was very close to being completely homogeneous. After this equilibrium structure at zero internal energy was obtained and found to be accurate, we added internal energy to the system and computed a new equilibrium structure. The specific internal energy added was $1 \times 10^{10}$ erg/g (which is a high internal energy corresponding to almost 7% of the sublimation energy). The sphere expanded, and as a result the mean density dropped to 2.02 ± 0.0007 g/cm³. To check whether this was the correct value, we introduced the equilibrium structure into the Eulerian code and integrated the equation further in time. Since no motions were generated and the density remained at this value, we concluded that the SPH code equilibrium calculation was correct. This, of course, assumes that the Eulerian code does the computation accurately. Nevertheless since these two codes are quite independent and totally different in conception, it is very unlikely that by chance both codes would produce the same error.
The second test we ran was to simulate a collision between two of the above spheres. The relative velocity between them at the impact was 19.88 km/sec, which is considerably higher than all the velocities considered in the rest of this paper. Two simulations were done with the SPH code, one with, and the other without, the drag term introduced in the “viscous” force term. The same problem was then run using the Eulerian code and the results of the three simulations compared. Figure 1 shows the various energies as a function of time for the three simulations. As may be seen, the total energy was almost exactly conserved (less than 0.3% variation). The main result is that without the drag term the SPH code underestimates slightly (10%) the amount of kinetic energy dissipated. This underestimation comes from some particles in one
body diffusing into the other one instead of being stopped, whereas with the drag term this diffusion is removed. By using the drag term the difference between both codes at maximum internal energy is 0.6%. The differences in the later stages of the evolution are due to a problem with the SPH technique. After collision both objects blew up. Particles were spread out in space and therefore do not represent a fluid accurately any more. It is significant that the amount of cooling (difference between maximal and final internal energy) is the same in the two SPH simulations.

It is worth mentioning that this interpenetration problem is at its worst in the case of a collision between two homogeneous bodies, since once a particle diffuses even a little bit it sees a high-density region behind it and consequently it is easier for it to diffuse even more. The results are quite different when the impacting objects have internal density gradients. Tests show that in such a case diffusion is much reduced and the drag term is considerably less important.

From running these comparisons it became evident that the Eulerian code must have many more grid points than the SPH code has particles. In fact, to get the above results the Eulerian code was used in a two dimensional version having a grid of $140 \times 100$, whereas the SPH code used 1216 particles and was fully 3-D. Estimates have led to the conclusion that in 3-D the Eulerian code would need between 40 to 60 times more grid points than the SPH code needs particles to get results of the same precision. A similar conclusion was already reached by Durisen et al. (1985). These authors also pointed out that there is an intrinsic difficulty for SPH to stimulate the behavior of low- and high-density regions at the same time. While this difficulty does not appear during the pre- and immediate post-impact phases, we do not expect the trajectory of the outgoing low-density material to be affected, except by internal pressure gradients in the low-density material which may not have been properly simulated in these calculations. This will hardly have any effect on material that is ejected from the system.

In impact calculations the differences in general between two codes are sometimes large since the calculation deals with highly nonequilibrium states and all differences are expected to grow vigorously. The small
differences found between our code and the Eulerian one are therefore very encouraging and allow us to view the results of our simulations with a good degree of confidence.

6. INITIAL CONDITIONS AND UNITS

The initial protoearth and the impactor were first modeled by adding to the momentum equation a damping term so that the particles settled down into an equilibrium structure. Once this damping term is suppressed the remaining artificial viscosity provides a sufficient dissipation to ensure continuing stability. This structure was obtained by imposing a constant specific internal energy and by letting the density vary until equilibrium was reached. We used 1848 particles for the protoearth and 200 particles for simulating the impactor. It
should be noted that the mean density and radius of the protoearth thus obtained are not equal to the present values for the Earth. These values depend upon the amount of internal energy assumed for the object. We ran two series of simulations, one with a low internal energy \((1 \times 10^7 \text{ erg/g})\) and one with a large value of this energy \((5 \times 10^{10} \text{ erg/g})\). In fact this latter value corresponds to about 27\% of the vaporization energy. The radius and the mean density we obtained are, respectively, \(R_E = 6564 \text{ km, } \rho_E = 4.37 \text{ g/cm}^3\) for the “hot molten” protoearth and \(R_E = 5085 \text{ km, } \rho_E = 9.12 \text{ g/cm}^3\) for the “cold solid” protoearth. The same applies for the impactor for which we obtained the following values: \(R_i = 3718 \text{ km, } \rho_i = 1.27 \text{ g/cm}^3\) in the “hot molten” case and \(R_i = 2853 \text{ km, } \rho_i = 3.92 \text{ g/cm}^3\) for the “cold solid” impactor. The mass of the protoearth and impactor were taken to be \(5.96 \times 10^{27} \text{ g}\) and \(0.65 \times 10^{27} \text{ g}\).

At the start of each simulation the centers of the two objects were always separated by 10 Earth radii along the x axis and displaced by some arbitrary amount along the y axis. The \(xy\) plane corresponds to the equatorial plane of the Earth. The initial relative velocity was then computed such as to correspond to the given velocity at infinity. This starting velocity was always directed along the x axis. This implies that the total angular momentum is not zero even for a zero velocity at infinity; however, even an infinitely small velocity at infinity could produce the required angular momentum.

All of our runs involved initial angular momenta comparable to that in the present Earth–Moon system. Any likely initial spin of the protoearth will be negligible in comparison.

### Table 1

<table>
<thead>
<tr>
<th>Run</th>
<th>(v_\infty) (km/sec)</th>
<th>(r_{\text{min}}) ((R_E))</th>
<th>(E_{\infty}) (erg/g)</th>
<th>(\Delta E_{\text{tot}})</th>
<th>(\Delta J_{\text{tot}})</th>
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<td>0.77</td>
<td>1.10^7</td>
<td>3.3</td>
<td>0.14</td>
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<td>2</td>
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<td>0.77</td>
<td>1.10^7</td>
<td>6.1</td>
<td>0.12</td>
</tr>
<tr>
<td>3</td>
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<td>0.77</td>
<td>5.10^10</td>
<td>0.5</td>
<td>0.003</td>
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<tr>
<td>4</td>
<td>9.0</td>
<td>0.77</td>
<td>5.10^10</td>
<td>0.4</td>
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<tr>
<td>5</td>
<td>4.5</td>
<td>0.77</td>
<td>5.10^10</td>
<td>0.5</td>
<td>-0.01</td>
</tr>
<tr>
<td>6</td>
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<td>0.22</td>
<td>5.10^10</td>
<td>0.5</td>
<td>0.002</td>
</tr>
<tr>
<td>7</td>
<td>3.0</td>
<td>0.92</td>
<td>5.10^10</td>
<td>0.5</td>
<td>0.02</td>
</tr>
</tbody>
</table>

*Given is the denomination of the run, relative velocity at infinity, distance of closest approach (in Earth radii), initial specific internal energy, maximum variation of total energy, and total angular momentum.*
In numerical simulations it is convenient to adopt units in which all quantities have values close to unity. Therefore in these simulations we adopted as a unit of mass, the total mass, $M_u = 6.61 \times 10^{27}$ g; the unit of distance was taken to be $R_u = 8.08 \times 10^8$ cm. All other units were calculated using these numbers: the unit of time was $T_u = \sqrt{R_u^3/GM_u} = 1093.8$ sec or 18.2 min, and the energy per unit mass was $E_u = R_u^2/T_u^2 = 5.46 \times 10^{11}$ erg/g. Unless otherwise specified, all results and plots are given in these units.

7. RESULTS

A total of 7 simulations were run for various combinations of parameters; the initial characteristics of these simulations are given in Table I ($r_{\text{min}}$ would be the distance of closest approach if the two objects were
point masses). Also given in Table I is the maximum relative variation of the total energy and the total angular momentum. For the simulations leading to the formation of a Moon we give in Table II the main characteristics of the resulting Earth–Moon system.

Figures 2–8 show snapshots of the evolution of the system during the collision for the various simulations. We plot the velocity vectors of the individual particles. To avoid overly dense regions we superimposed a grid and plotted only one particle per cell. For particles in the vapor phase we plot a “O” at the particle’s location. The velocity vectors are normalized to the maximum velocity in each frame. In the upper line we display the time and the coordinates of the region covered by the plot: \((x_{\text{min}}, x_{\text{max}}, y_{\text{min}}, y_{\text{max}})\). These numbers are all given in the units defined in Section 6.

The typical history of the collision is as follows. As the impactor approaches the protoearth it becomes severely deformed by the tidal field. At the time it hits the protoearth it has a clearly marked pear shape. It was pointed out by Mizuno and Boss (1985) that in the presence of a very strong dissipative impactor (which would be the case only in a partially molten impactor) the deformation should be small. The real effect on the outcome of the collision of a deformed or not impactor is difficult to estimate; we believe, however, that it is only a second-order effect since the material ending in the disk originates exclusively from the side of the impactor opposite to the protoearth that is not from the deformed region. The impactor is then slowed down and there is a corresponding dissipation of kinetic energy into heat in the strong

<table>
<thead>
<tr>
<th>Run</th>
<th>(J / J_{\text{obs}})</th>
<th>(P_{\text{rot}}) (hr)</th>
<th>(M_{\text{clump}} / M_{\text{Earth}})</th>
<th>(\eta)</th>
<th>Vapor fraction (%)</th>
<th>(\overline{E}_{\text{in}}) (erg/g)</th>
<th>(r_{\text{min}}) (RE)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>0.79</td>
<td>8.6</td>
<td>2.4</td>
<td>0.06</td>
<td>0.5</td>
<td>(1.2 \times 10^{10})</td>
<td>1.3</td>
</tr>
<tr>
<td>3</td>
<td>0.99</td>
<td>12.2</td>
<td>3.2</td>
<td>0.0</td>
<td>0.85</td>
<td>(6.7 \times 10^{10})</td>
<td>1.3</td>
</tr>
<tr>
<td>5</td>
<td>8</td>
<td>11.9</td>
<td>2.9</td>
<td>0.03</td>
<td>1.22</td>
<td>(7.2 \times 10^{10})</td>
<td>b</td>
</tr>
<tr>
<td>7</td>
<td>1.10</td>
<td>15.1</td>
<td>4.6</td>
<td>0.0</td>
<td>0.88</td>
<td>(6.5 \times 10^{10})</td>
<td>1.2</td>
</tr>
</tbody>
</table>

* For those leading to a Moon we list: total angular momentum of the Earth–Moon system normalized to the observed value, rotation period of the Earth, mass of the clump (in Moon masses), mass fraction of Earth material in the clump, maximum vapor fraction, mean specific internal energy, distance of closest approach of the clump (in Earth radii).

* Clump escapes from the system.
shock formed at the interface. At this point we emphasize again that both objects do not interpenetrate as can be clearly seen from the various plots. As time goes on, the impactor starts to be destroyed and spread out in space. Once this occurs, the density of the hot, shocked material at the interface decreases, and as a result part of this matter turns into vapor (to vaporize, material has to be hot and in an expanded state). However, most of this vapor becomes solid again since during subsequent expansion the gas cools, and soon its internal energy is lower than the vaporization energy.

The evolution after the impactor begins to spread out in space depends strongly on the impact parameter and the relative velocity. For small impact parameters like run 6 (Fig. 7) the impactor is completely destroyed and spread out in space. If the rela-
tive velocity is small (zero velocity at infinity) almost all material falls down onto the protoearth. For a higher relative velocity (4.5 km/sec) part of the ejected material escapes the system; however, most of it is still accreted by the protoearth. Small impact parameter collisions are therefore unable to lead to the formation of the Moon.

For a larger impact parameter but a high relative velocity (9 km/sec) as in runs 2 and 4 (Figs. 3 and 5), the impactor is again completely destroyed and spread out in space. However, this case differs from the previous ones in the sense that almost all the material spread out in space leaves the system and is not captured by the protoearth. An intermediate case is run 5 (Fig. 6) with a relative velocity at infinity of 4.5 km/sec. In this case the impactor was not destroyed completely so that shortly after impact part of the material flying away clumped together forming a moon of about 3 times the mass of the Earth's moon. However, the impact velocity was still too high to allow this clump to be bound to the Earth.

Finally, for low relative velocities (runs 1, 3, 7; Figs. 2, 4, 8), there occurs the formation of clumps orbiting around the protoearth. The material was able to clump together for two reasons. First, self-gravity of the moderately destroyed impactor was able to prevent further destruction and attracted several pieces together. Second, on very short length scales, the possibility of having tension (negative pressure) in the equation of state assures that the different parts stick together once they come very close, which is the behavior one would expect from molten material. This effect is expected to be small since all the clumps have a very high specific internal energy, making them belong to the “intermediate region” of the equation of state (see Section 3) for which tension is much reduced. Table II gives a list of various important quantities relative to the Earth-Moon systems formed in these simulations. It is worth noting that the very high specific internal energy (column 7) of these clumps indicates that they are all completely molten. Since even in the case where the initial specific internal energy was low (run 1; Fig. 2) we obtained a molten Moon, it follows that the single impact theory predicts that all the material forming the Moon was at some stage completely molten.

It is interesting to note that the particles found in these clumps all originate from a
FIG. 8. Snapshots of run 7. (v∞ = 9 km/sec; rmin = 0.92Rearth; Eint = 5×10⁶ erg/g). See caption of Fig. 2 for the meaning of the symbols.

well defined region of the impactor. This region corresponds to the upper left part of the impactor in Figs. 2, 4, 8 which is the region the farthest away from the interface at the time of maximum compression. There are no particles in the clump coming from the region facing the protoearth or even the opposite side of the impactor. Since we expect that the presence of an iron core is going to change much in the behavior of what happens to the inside of the impactor we will discuss this in a forthcoming paper.

We also computed the orbits of the various clumps around the Earth. All come very close to the Earth again, well within the Roche limit of 2.8 Earth radii (the distance of closest approach if the two objects were point masses is given in column 8). In one case (run 7; Fig. 8) the clump will most
probably collide again with the Earth. (The collision in this case was not strong enough, due to the larger impact parameter, to push the material far enough from its initial ballistic orbit.) We expect, therefore, that these clumps will be destroyed again and spread out into a disk around the Earth. Part of the material will fall down onto the Earth and part of it will be able to move beyond the Roche limit to form the Moon. Since the mass of the clump is several times the mass of the Moon there is enough mass available. The problem is that the total angular momentum of the Earth-Moon system (Table II, column 2) is too small. For this reason we point out that probably the impactor needs to be more massive than 0.1 of the Earth. Calculations in progress and including iron cores (Benz et al., 1986) show that a mass ratio between 0.12 and 0.20 takes care of this angular momentum problem. This conclusion is supported by the fact that most of the Moon is formed from the impactor; therefore, an impactor with a mass closer to the one of the Earth would also (to the first order) have a chemical composition closer to it (Ringwood, 1984).

In almost all of the simulations, after the main impact much material falls back onto the protoearth. This material is heated at the time it hits the protoearth, and since it does not expand again but is attracted to the protoearth by gravity, most of the simulations end with an protoearth surrounded by a hot gaseous atmosphere. The protoearth is also heated by the shock. The final mean specific internal energy was increased by a factor of about 2000 for the initially cold protoearth and by about 1.3 for the initially molten protoearth. This shows that for a cold protoearth the shock is much stronger than in the case of an already molten protoearth. This is also shown by the higher rotation period of the protoearth (Table II, column 3) obtained when starting with a cold protoearth.

8. CONCLUSIONS

From these simulations we conclude that the single-impact hypothesis provides a plausible scenario for making the Moon provided that:

The relative velocity between the impactor and the protoearth is relatively small (less than about 5 km/sec at infinity). If this condition is not fulfilled the impactor is completely dispersed in space.

The impact parameter is not too small. A
A good approximation is that the center of the impactor should graze the surface of the protoearth. Failure to satisfy this requirement again leads to the complete destruction and spreading out of the impactor and to the accumulation of most of its material onto the protoearth.

The impact parameter is not too large. Otherwise the shock is not strong enough to push the clump away from its initial ballistic orbit and it will collide a second time. Since this collision will be at a low velocity we expect that most of the impactor will be accreted by the protoearth.

The mass ratio of impactor to the protoearth must be greater than $\frac{1}{10}$ to $\frac{1}{10}$ (new simulations including iron cores suggest $0.12$ to $0.2$) in order to have enough angular momentum in the Earth-Moon system.

The results show that once these requirements are satisfied, the formation of a prelunar accretion disk is almost straightforward. After collision part of the impactor clumps together in an elongated orbit around the Earth. The orbit brings the clump back within the Roche limit where it will be destroyed again and spread out into a disk. We do not carry out calculations as far as this, but we expect that viscous evolution of the disk will allow part of the material to move beyond the Roche limit and form the Moon (Ward and Cameron, 1978; Thompson and Stevenson, 1983).

In future work we will investigate what happens when both the Earth and the impactor have an iron core. These simulations may allow us to put more constraints on the initial parameters leading to the formation of the Moon from a single impact.

ACKNOWLEDGMENTS

The authors thank Norman L. Johnson and Francis H. Harlow for many helpful discussions and advice. Part of the work of W. Benz was supported by the Swiss National Science Foundation. The work of A. G. W. Cameron was partly supported by NASA grants NGR 22-007-269 and NAG 9-89.

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