The effect of shape anisotropy on TRM direction

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Summary. A more exact treatment than has previously been given of the effect of magnetic shape anisotropy on TRM direction is presented. This takes into account the variation with temperature of the spontaneous magnetization, weak-field susceptibility and distribution of blocking temperatures. Detailed numerical computations for three examples of basalts with very different magnetic properties show that the effects of shape anisotropy can be estimated very accurately from a simple equation derived for a limiting case if one uses in place of the real TRM susceptibility an equivalent TRM susceptibility 0.68–0.85 times as large. Application of these equations suggests that shape anisotropy has a very small average effect in subaerial basalts, but may in some cases substantially shallow the direction of TRM in submarine basalts. In particular, about 10 degrees of the inclination anomaly observed in DSDP leg 37 basalts could be caused by shape anisotropy. These results assume the absence of significantly large and systematic internal fields due to short-range magnetic interactions. Further work is needed to determine how much the TRM direction in strongly magnetic rocks actually deviates from that of the external field.

Introduction

Almost all palaeomagnetic results depend on the assumption that the primary natural remanent magnetization (NRM) of rocks is acquired parallel to the Earth’s magnetic field. This assumption is seldom questioned for igneous rocks because they generally display little or no preferred orientation of minerals and because the primary NRM is thermoremanent magnetization (TRM), which, in questionable cases, can be directly tested for fidelity by cooling in a known field in the laboratory. This has been done for many igneous rocks in many laboratories, and the direction of TRM usually lies parallel to the external field within the 1 or 2 degree uncertainty in orientation and measurement (e.g. Nagata 1961). Moreover, most palaeomagnetic results are drawn from averages of directions for several specimens from the same cooling unit and, very often, from further averages of directions for several cooling units. Errors in palaeomagnetic field directions due to anisotropy with a randomly varying component would thus be even smaller than the error found by testing single specimens.
One type of anisotropy may be important in some igneous rocks, however, because it can produce systematic deviations of direction within a unit or even within groups of units. This is shape anisotropy, the magnitude of which is larger the less equidimensional the cooling unit and the more strongly magnetic the rock. The qualitative sense of the effect of shape anisotropy is to deflect the TRM away from the shortest dimension of the body. Thus, for instance, in a pile of sub-horizontal sheets of lava we would expect shape anisotropy to cause the primary NRM to be inclined systematically less steeply than the external field which produced it. The crucial question for palaeomagnetists is under what conditions might this systematic effect be significantly large.

Several people have studied the effect of shape anisotropy on the direction of TRM and have developed a variety of approximate formulations based on one or more simplifying assumptions. Strangway (1961) derived relations which give the direction of the internal field within an infinite sheet of magnetic material as a function of direction of external field and induced and remanent susceptibilities. This treatment, however, does not permit quantitative prediction of the deviation of direction of TRM because the internal field actually changes continuously during the acquisition of TRM, with the result that each portion of the TRM spectrum is magnetized differently.

Uyeda et al. (1963) derived a quantitative relationship for the deviation of TRM direction due to shape anisotropy in terms of the shape demagnetizing factors and the susceptibility of induced magnetization. This relationship fits the results of their experiments on discs of magnetite fairly well, but cannot be generally applied because it assumes that the TRM has a single blocking temperature and neglects the effects of partial remanence on the demagnetizing field during the blocking process. This is not acceptable because most strongly magnetic rocks are at least as susceptible, and generally far more susceptible, to the TRM than to induced magnetization. The treatment of Stacey & Banerjee (1974) suffers from the same defect.

Vogt (1969) obtained an approximate formula for the effect of shape anisotropy on the acquisition of TRM by substituting remanent for induced susceptibility into the well-known formula for its effect on induced magnetization and averaging as the remanent susceptibility varies from zero to its room temperature value during cooling from the Curie temperature. This treatment is a distinct improvement over the others because it recognizes that shape anisotropy plays no role in the fixing of the first TRM blocked and has maximum effect on the last TRM blocked. However, it takes no account of the increase in the spontaneous magnetization during cooling, the role of induced magnetization or the actual distribution of blocking temperatures on the acquisition of TRM.

Vogt (1969) also recognized the possible importance of shape anisotropy on the direction of TRM of the intensely magnetized marine basalts. He reasoned that the inclinations of TRM would be less than the ambient field and argued that this might explain the shallow inclinations of certain seamounts in the northern hemisphere which conventionally are interpreted in terms of northward drift. This suggestion seems to have been ignored for the most part, perhaps because of the very brief and approximate form of the calculations.

Recently, Harrison & Watkins (1977) have expressed the opinion that the stable inclination of NRM of marine basalt in cores drilled by the Deep-Sea Drilling Project is usually shallower than that of the axial dipole field at the latitude that the rocks were erupted (cf. Lowrie, Løvlie & Opdyke 1973; Lowrie 1974; Heirtzler 1976; Ryall et al. 1977). They show that at least for the results of leg 37 (Ryall et al. 1977), which are the most extensive available for general publication use, the negative inclination anomalies are too large and too frequent to represent probable geomagnetic field behaviour. Thus they suggest that large tectonic rotations (55°) of surficial blocks about axes subparallel to the ridge axis
may have occurred, although they express concern that there is no evidence for such large rotations in the area nearby at the ridge axis, which has been surveyed and studied in great detail.

For these reasons a more exact treatment of the effects of shape anisotropy is undertaken in this paper and applied to three examples of basalts with different rock magnetic properties. This calculation takes into account the effects of the distribution of blocking temperatures and the variation of spontaneous magnetization and induced susceptibility on the anisotropic acquisition of TRM. It verifies Vogt’s surmise that significant shallowing of the inclination could occur for flattened or elongated cooling units composed of rocks as strongly magnetic as some of the basalts from the sea-floor. It appears, however, that at most only one-half of the inclination anomaly found in leg 37 basalts can be attributed to shape anisotropy if the remanent susceptibility of these basalts at the time of eruption was similar to that of young basalts presently at the ridge axis.

Shape anisotropy

**Theory**

The total magnetization $J_T$ of any body gives rise to a demagnetizing field $H$ inside the body. $H$ always has a component which opposes $J_T$, but the exact direction depends on both the magnetization and the shape of the body. Stoner (1945) showed that inside a uniformly magnetized ellipsoidal body the demagnetizing field is constant throughout the body and is given by

$$H = -N \cdot J_T.$$  \hspace{1cm} (1)

$N$ is the shape-demagnetizing tensor, a second-rank symmetric tensor with trace $4\pi$ and with principal axes coinciding with the symmetry axes of the ellipsoid. Two limiting cases of most interest are spherical and planar bodies, for which the principal values of $N$ are $N_1 = N_2 = N_3 = \frac{4}{3} \pi$ and $N_1 = N_2 = 0, N_3 = 4\pi$, respectively. Thus in a sphere of intrinsically isotropic material $H$ is always exactly antiparallel to $J_T$, but in a planar body $H$ is perpendicular to the plane in such a sense that a component of it opposes $J_T$ (unless $J_T$ lies in the plane, in which case $H = 0$). In non-ellipsoidal or non-uniformly magnetized bodies equation (1) is not valid, but it still may be used to describe an approximate, average value of $H$ for the body.

The total field acting inside the body is taken as $F + H$, the sum of the external and demagnetizing fields. This ignores the so-called Lorentz field, which some have argued contributes to the internal field as well (Strangway 1961). The concept of the Lorentz field derives from a simplified model for the interactions between the magnetic grains in which the average interaction field is presumed to be $\frac{1}{2} \pi J_T$. This is a reasonable approximation when the magnetic grains are nearly uniformly distributed but not if, as is usually the case in rocks, the grains are clustered. Thus it seems most straightforward to omit the term for the Lorentz field, keeping in mind that grain interactions would be expected to modify the results obtained for shape anisotropy alone. If the grain interactions in a particular case are well approximated by the Lorentz field, it is easy to take account of them in the following calculations by simply substituting $N - \frac{1}{2} \pi I$ everywhere in place of $N$.

The increase in remanence $dJ$ due to uniform cooling of the body from $T + dT$ to $T$ is assumed to be given by (Coe 1967)

$$dJ = (F + H)K(T)dT + \frac{1}{J_s} \frac{dI_s}{dJ_s} J_s dT.$$  \hspace{1cm} (2)
The first term on the right-side of equation (2) describes the increase of remanence due to the blocking of new domains during cooling according to a function \( K(T) \), characteristic of the rock. The linear dependence on total internal field appears to be a reasonable approximation within the range of fields of most interest (0 to 1 Oe (10^-4 T) (see, e.g. Nagata 1961). The second term describes the increase of remanence arising from the increase during cooling of the spontaneous magnetization \( J_s \) of previously blocked grains. This equation is only valid for rocks in which the remanence is carried by one species. If there were \( n \) species, each with a different \( J_s \), equation (2) would have to be replaced by a system of \( n \) coupled differential equations involving \( n \) species of remanence.

The three component equations corresponding to (2) become uncoupled if we choose coordinate axes to coincide with the principal axes of \( N \), because in this reference frame the component equations corresponding to (1) become

\[
H_i = -N_i J_{Ti} \tag{3}
\]

for \( i = 1, 2, 3 \). Note that the total magnetization \( J_T \) is the sum of the remanent and induced components

\[
J_{Ti} = J_i + I_i \tag{4}
\]

where the induced magnetization \( I \) has components

\[
I_i = X(T)(F_i + H_i) \tag{5}
\]

The intrinsic susceptibility, \( X(T) \), is assumed to be isotropic. Equations (3), (4) and (5) show that

\[
F_i + H_i = \frac{F_i - N_i J_i}{1 + N_i X(T)} \tag{6}
\]

which, when substituted into (2), yields the basic component equations

\[
d J_i = F_i B_i(T) K(T) dT + \left[ \frac{1}{J_s} \frac{d J_s}{dT} - N_i B_i(T) K(T) \right] J_i dT \tag{7}
\]

where

\[
B_i = \frac{1}{1 + N_i X(T)} \tag{8}
\]

Each equation in (7) is an ordinary, first-order, linear differential equation whose solution can be written

\[
J_i(T_0, T_1) = F_i J_{s}(T_1) \int_{T_0}^{T_1} \frac{B_i(T')}{J_s(T')} K(T') R_i(T', T_1) dT' + J_i(T_0) \frac{J_s(T_1)}{J_s(T_0)} - R_i(T_0, T_1) \tag{9a}
\]

where

\[
R_i(T', T_1) = \exp \left\{ -N_i \int_{T'}^{T_1} B_i(T'') K(T'') dT'' \right\}. \tag{9b}
\]

If the measurement of remanence is made at \( T_i \), Uyeda et al. (1963) have noted that there will be a self-induced magnetization due to the internal demagnetizing field which will be indistinguishable from the true remanence. This will have negligible effect on the direction that is measured if the specimen is equidimensional, i.e. has little or no shape anisotropy.
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It is common in palaeomagnetic practice to use equidimensionally shaped specimens, and we will assume that to be the case here. The use of significantly flattened or elongated specimens could produce significant deviations of measured TRM directions over and above that calculated in this paper.

A SIMPLE LIMITING CASE

Before presenting the results of numerical calculations for specific examples of the deviation of TRM direction that is predicted by the exact formula (9a), it is useful to consider a limiting case in which \( J_s(T) \) and \( X(T) \) are assumed to remain constant at their maximum values during blocking of all the TRM. The deviation calculated under these conditions will be an upper bound for the deviation that can actually be caused by shape anisotropy. These approximate calculations also turn out to be more relevant to the actual, more complicated case than one might at first expect.

When \( J_s \) and \( X \) are assumed constant, the basic differential equation (7) becomes separable

\[
(1 + N_i X) \frac{dJ_i}{F_i - N_i J_i} = K(T) dT. \tag{10}
\]

Integrating both sides from zero TRM \( (T = T_c) \) to the final TRM \( J_i(T = T_f) \), (10) becomes

\[

\left( \frac{1 + N_i X}{N_i} \right) \ln \left( \frac{F_i}{F_i - N_i J_i} \right) = \int_{T_c}^{T_f} K(T) dT \tag{11}
\]

if \( N_i \neq 0 \). In the special case that \( N_i = 0 \), integration of (1) yields

\[
k \equiv \frac{J_i}{F_i} = \int_{T_c}^{T_f} K(T) dT. \tag{12}
\]

Because (12) refers to a component direction along which the demagnetizing field is zero, the total internal field and the external field are the same in this direction, and therefore \( k \) is the intrinsic TRM susceptibility of the rock. Substituting \( k \) of (12) for the right-side of (11), we see that in the general case \( (N_i \neq 0) \)

\[
J_i = \frac{F_i}{N_i} \left[ 1 - \exp \left( \frac{-kN_i}{1 + XN_i} \right) \right]. \tag{13}
\]

It is interesting to compare (13) with the approximate result of Vogt (1969), which he obtained in analogy to the well-known effect of shape anisotropy on induced magnetization by using a weighted average of the TRM susceptibility in place of the weak-field susceptibility \( X \). In terms of symbols used in this paper, Vogt’s formula is

\[
J_i = \frac{F_i}{N_i} \ln \left( 1 + kN_i \right). \tag{14}
\]

When \( X \) is small enough that \( XN_i \) can be neglected compared to 1, (13) and (14) are very close to each other; in fact, the Taylor’s series expansions of both expressions only differ in the third and higher power terms of \( kN_i \). Thus for \( kN_i < 0.5 \) the two formulae agree to 3 percent or better, but for increasingly larger values of \( kN_i \) (14) diverges rapidly from (13) on the high side. (14) clearly has the wrong sort of limiting behaviour because \( J_i \) increases without bound as \( kN_i \) becomes very large. The limiting value \( J_i = F_i/N_i \) predicted by (13) is reasonable because that is the value at which the internal field is just zero \( (F_i = -H_i) \).
If the body is an oblate ellipsoid of revolution flattened parallel to the 3-axis \( N_3 > N_1 = N_2 = 2\pi - N_3/2 \), the TRM will deviate from the external field toward the plane of flattening. If instead the body is a prolate ellipsoid of revolution elongated parallel to the 3-axis \( N_1 = N_2 > N_3 \), the TRM will deviate from the external field toward the 3-axis. In these cases we may take \( F_2 = J_2 = 0 \) without loss of generality, so that the angle that the TRM makes with the plane normal to the 3-axis can be obtained from (13). For both \( N_1 \) and \( N_3 \) not equal to zero,

\[
\theta = \tan^{-1} \left( \frac{J_3}{J_1} \right) = \tan^{-1} \left( \frac{N_1 \left[ 1 - \exp \left( -kN_3/1 + XN_3 \right) \right]}{N_3 \left[ 1 - \exp \left( -kN_1/1 + XN_1 \right) \right]} \frac{\tan \theta_F}{1} \right).
\]  

(15)

\( \theta_F \) is the angle that the external field makes with the body; i.e. \( \tan \theta_F = F_3/F_1 \). If either \( N_1 \) or \( N_3 \) is zero, one must use in equation (15) the expression for \( J_1 \) or \( J_3 \) that is given in (13).

Let us consider the extreme case of a planar body \((N_1 = N_2 = 0, N_3 = 4\pi)\) with remanent magnetization much greater than induced \((k \gg X); \) that is, \( Q \equiv k/X \) infinite. The angle of the TRM predicted by (15) is given in Fig. 1 as a function of the intrinsic TRM susceptibility \( k \). The deviation of the TRM from the external field depends on the angle the external field makes with the body, being zero when that angle is 0 or 90°. The maximum deviation is negligible for values of \( k \) less than 0.001, is less than 2° for moderate values of \( k \) up to 0.01, but increases very rapidly as \( k \) increases toward 0.1 and beyond.

An illustration of the effects on the TRM angle of induced susceptibility and shape (for ellipsoids of revolution) when the external field angle is 45° is given in Fig. 2. When the

![Figure 1. Angle of TRM versus susceptibility of TRM according to equation (15) for a planar body of magnetic material with infinite \( Q \). External field makes angles of 0, 15, 30, 45, 60, 75 and 90° with the plane.](image-url)
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Figure 2. Variation of TRM angle according to (15) for ellipsoids of revolution with different Qs and different shapes. Curves A, B, C: $Q = 1, 3, \infty$ and $N_3/N_1 = \infty$; curves D, E: $Q = \infty$ and $N_3/N_1 = 6, 2$; curve F: $0 < Q < \infty$ and $N_3/N_1 = 1$ (spherical shape).

Figure 3. Variation of demagnetizing factor ratio versus dimensional ratio for ellipsoids of revolution. Plus signs are for oblate and minus signs for prolate ellipsoids.

induced susceptibility is comparable to the remanent susceptibility (curve A), the TRM direction departs from the external field by as much as $11^\circ$ more than when the induced susceptibility is negligible (curve C). Likewise, the more flattened the shape for a given $k$, the greater the deviation of TRM direction. The shape that corresponds to a given ratio $N_3/N_1$ can be obtained from Fig. 3; for instance, for $N_3/N_1 = 6$ (curve D in Fig. 2), the
Figure 4. Variation of TRM angle with ratio of demagnetizing factors according to (15) for selected values of the TRM susceptibility, \( k \) and \( Q = \infty \).

dimensional ratio of the corresponding oblate ellipsoid is \( a_1/a_3 = 5 \). The variation of TRM angle with \( N_3/N_1 \) is shown more completely in Fig. 4. For the range of \( k \) values from 0 to 0.2, the TRM angle is almost insensitive to further flattening of oblate ellipsoids (or elongating of prolate ellipsoids) when \( N_3/N_1 > 100 \) (or \(< 0.01 \); that is, further departure of the body from a spherical shape would result in no more than \( 1^\circ \) further deviation of the TRM from the external field direction.

The simple example of greatest interest in palaeomagnetism is a horizontal, thin sheet \((N_1 = N_2 = 0, N_3 = 4\pi)\). This is approximately the case for many lava flows and sills. With this geometry the inclination \( I \) of the TRM will be the same as \( \theta \) in equation (15), and the declination will be unaffected by the shape anisotropy. In Fig. 5 the shallowing in inclination of TRM, \( \Delta I \), is plotted versus inclination of the external field for the case \( Q = \infty \). The deviations are considerable for very strongly magnetic rocks; with \( k = 0.1 \) the maximum \( \Delta I \) is \( 16^\circ \). If \( Q = 1 \) for \( k = 0.1 \) the maximum \( \Delta I \) is \( 30^\circ \) (cf. equation (15)).

As discussed earlier, if the distribution of grains in the rock is uniform, it is easy to estimate how magnetic interactions between grains will modify the results calculated for shape anisotropy by using the concept of the Lorentz field. All that needs to be done is
Figure 5. Deviation of inclination of TRM, $\Delta I$, from the external field inclination, $I_F$, due to shape anisotropy of a horizontal thin sheet for selected values of TRM susceptibility, $k$. $\Delta I$ is deflected toward the horizontal according to (15).

to replace each component of the demagnetizing factor $N_i$ by $N_i - \frac{4}{9}\pi$ and the intrinsic TRM susceptibility $k$ by $k'$, where

$$k' = \frac{1 - \exp\left(\frac{\frac{4}{9}\pi k}{1 + \frac{4}{9}\pi X}\right)}{\frac{4}{9}\pi}$$  \hspace{1cm} (16)

and use the same equations. The latter change is necessary because the definition of intrinsic susceptibility in (12) now applies for $N_i - \frac{4}{9}\pi = 0$; i.e. $k'$ is the TRM susceptibility of a sphere while $k$ is the TRM susceptibility in the plane of a thin sheet. The effect of including such interactions is not great. For the strongly magnetic case considered in the paragraph above ($k = 0.1, Q = \infty$), the maximum shallowing of inclination is about 14.5°, only 1.5° different from the result with no magnetic interactions between grains.

RESULTS OF THE FULL CALCULATION

It must be borne in mind that the values and figures (Figs 1, 2, 4 & 5) referred to above are maximum estimates according to equation (15) for the deviation of TRM from the external field direction due to shape anisotropy. They apply rigorously only to the limiting case in which $I_s$ and $X$ are assumed constant over the entire range of blocking temperatures of the TRM. For exact results one must integrate (9), and thus the variation with respect to temperature of spontaneous magnetization $I_s$, weak-field susceptibility $X$ and blocking-temperature function $K$ must be known. Two cases were investigated in detail: a high Curie point rock similar magnetically to many young, fresh Hawaiian basalts and a low Curie point rock patterned magnetically after an unoxidized pillow basalt sample from the axis of the Indian Ocean Ridge (104-6S, Marshall & Cox 1971, 1972; Grommé et al., in preparation).
The magnetic parameters for these two cases are shown in normalized form in Fig. 6. The units of \( J_0 \) are immaterial since it always occurs in both numerator and denominator of all terms in equation (9). The units of \( K \) are \( ^\circ \text{C}^{-1} \), and the value of \( K_m \) or \( K_0 \) was varied over a range sufficient to produce unusually low to unusually high values of TRM in a 0.5 Oe (0.5 \( \times 10^{-4} \) T) field. Note that the highest blocking temperature is less than the Curie temperature, a physical and mathematical necessity. \( X \) is dimensionless and is assumed negligible compared to TRM susceptibility in the low Curie point case, not a bad assumption because \( Q \approx 20 \) for median valley basalts. For the high Curie point case \( X \) is approximated by two straight lines, a reasonable approximation (see Nagata 1961, p. 143) that simplifies the calculations considerably by allowing direct integration of (9b), thereby reducing the numerical integration from a double to a single integral.

Numerical calculations were performed for a range of demagnetizing factors, intrinsic TRM susceptibilities and \( Q \) ratios sufficient to include essentially all igneous rocks of interest. The results are very conveniently summarized with reference to the limiting case of constant \( J_0 \) and constant \( X \) discussed in detail above (Figs 1−5). The low Curie point case yields TRM directions that could be obtained very simply from equation (15) by replacing the intrinsic susceptibility \( k \) with an equivalent \( k \) only 0.85 times as large. What is surprising and gratifying is the accuracy with which the simple formula with the equivalent \( k \) reproduces the directions obtained by numerical integration using the real \( k \). For the entire range of parameters tested (0 < \( k < 0.17 \), 0 < \( N_3/N_1 < \infty \), 0° < \( \theta < 90° \)) the agreement is within 0.1°.

Likewise, the high Curie point case with \( Q = \infty \) yields TRM directions the same as if an equivalent \( k \) only 0.68 times as large as the real \( k \) were used in (15). When the induced susceptibility is not negligible, an equivalent \( Q \) that is 0.95 times the real \( Q \) allows simple estimation of TRM directions with (15) that lie within 0.3° of the numerically calculated ones for \( Q \) ranging all the way down to unity. The fact that the equivalent \( Q \) is so close to the actual \( Q \) is a reflection of the fact that \( X \) first increases with heating above room temperature before falling nearly to zero at the Curie temperature.

A third case intermediate between the two extreme cases was also studied. This one had a Curie temperature of 240°C and a blocking temperature function with a subsidiary peak near room temperature in addition to the main peak 30°C below the Curie temperature. For this case the numerical calculations are fitted by using (15) with an equivalent \( k \) of 0.79 times the real \( k \).
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Thus, in summary, the effect on TRM direction of the demagnetizing field arising from shape anisotropy can be estimated easily with equation (15) for a wide variety of basalts by using an equivalent $k$ ranging from 0.68 to 0.85 times the real $k$. If the magnetic properties necessary to determine the equivalent $k$ are not known, an estimate of 0.8 times the real $k$ would most likely serve as a reasonably good approximation. If $Q$ is small enough that the effect of weak-field susceptibility is not negligible, an equivalent $Q$ the same as the real $Q$ is also probably a fairly decent approximation.

Discussion

**General**

Shape anisotropy could only affect the NRM directions of igneous rocks with sufficiently high TRM susceptibility, $k$. Plutonic rocks characteristically have NRMs in the $10^{-4}$ emu cm$^{-3}$ ($10^{-1}$ Am$^{-1}$) range, corresponding to an average value of $k$ around 0.001. For such rocks the maximum possible effect is negligible.

Subaerial basalts usually have NRMs in the $10^{-3}$ emu cm$^{-3}$ (1 Am$^{-1}$) range, although there is a great deal of variation (Prévot & Grommé 1975). An average value of 0.01 is a reasonable estimate. Using an equivalent average $k$ of 0.008 in (15), we find that the largest average deviations of NRM direction from the geomagnetic field direction would be 1.4°. Such a small deviation is insignificant for most palaeomagnetic studies. Averaged over a great many lava flows, however, the effect might be detectable because the deviations would be towards consistently shallower inclinations. It is interesting that the hemispherical asymmetry in the palaeomagnetic inclination anomaly reported by Merrill & McElhinny (1977, p. 315 and table 3) is consistent in sense and order of magnitude with the shallowing of inclination in subaerial lava flows predicted in this paper. It is possible that this asymmetry arises from shape anisotropy in lava flows and inclination error (King & Rees 1966) in sediments rather than from an asymmetry in the geomagnetic field.

Fresh submarine pillow basalts such as are erupted at the axis of spreading ridges have values of $k$ that typically range from 0.05 to 0.1 (Carmichael 1970; Atwater & Mudie 1973; Sclater & Klitgord 1973; Johnson & Atwater 1977). Most older sea-floor basalts are much less intensely magnetized than those recently erupted at the ridge axes because of alteration (Irving 1970), but what is important is the intensity they had when they acquired their original TRM. Using an equivalent $k$ of 0.04 to 0.08 and assuming $Q = \infty$, maximum deviations of the NRM direction of 7 to 13° would be predicted (equation (15) and Fig. 5) when the field is inclined 47 to 53° to a thin sheet of the lava. Assuming instead that $Q = 18$ (the average found by Johnson & Atwater (1977)), for the median valley of the Mid-Atlantic Ridge at 37°N), the calculated maximum deviations change only slightly to 7.5 to 14°.

Thus it is conceivable that shape anisotropy could cause a significant deviation of the NRM direction of submarine pillow basalts, and further discussion is warranted. The effect depends critically on the shape of the cooling unit, which is not clearcut for submarine basalts. We can imagine two idealized limiting cases: (1) slow eruption of pillows that cool in isolation and (2) rapid eruption of pillows that form a pile which cools essentially like a single lava flow. In case (1), if the pillows are nearly spherical or if they are nearly cylindrical with axes almost at right angles to the geomagnetic field, no average deviation of the TRM direction will result from the demagnetizing field. In case (2), although pillow rinds form by quenching on contact with the seawater, the bulk of the heat is retained while the pile is formed so that deviation of the TRM direction is governed by the final shape of the pile. Non-uniform cooling, especially if caused by circulation of water through fractures and
connected interstices, would cause a complex distribution of TRM directions except for the special case of a pile with planar top and bottom surfaces and isothermal surfaces that are parallel to these throughout cooling below the Curie point.

If something approximately like case (2) actually occurs at the axis of spreading ridges, then we might expect substantial deviations of NRM direction. The lateral extent of flows in the median valley of the Mid-Atlantic Ridge at 37°N may range up to 600 m (MacDonald 1977). The thickness of flows is not well known, but they are thought to be thin (Aumento et al. 1977). Thus dimension ratios of 1:10 or less are probably common, implying considerable magnetic shape anisotropy (Fig. 3) and the possibility of significantly shallow inclinations of NRM (Figs 4 and 5).

SHALLOW INCLINATIONS OF DSDP LEG 37

The most complete palaeomagnetic study of marine basalts composing layer 2 of the oceanic crust that is available for use in publications is DSDP leg 37 (Ryall et al. 1977). Holes at four sites 30 to 170 km west of the axis of the Mid-Atlantic Ridge at a latitude of 37°N were drilled into the basaltic basement. The deepest hole penetrated 550 m into the basalt. Stable NRM inclinations much shallower on the average than that due to an axial dipole field were found.

Harrison & Watkins (1977) have discussed the possible causes for these shallow inclinations in considerable detail. They concluded that only about 5° of the anomalous shallowing can reasonably be attributed to geomagnetic field behaviour, whereas the average inclination of NRM is 29.5° less than the inclination for an axial dipole field. The unexplained 24.5° of inclination anomaly they attribute to post-emplacement rotations such as those suggested by Moore, Fleming & Phillips (1974), although they point out that there are serious difficulties with this interpretation. On the one hand, bathymetric profiles across the ridge axis to the east reveal tilts of only a few degrees (MacDonald & Luyendyk 1977) rather than the 55° rotation about a direction parallel to the ridge axis that would be required to produce the shallow inclination. On the other hand, surficial samples collected in the median valley at the ridge axis have palaeomagnetic inclinations that cluster close to the inclination of an axial dipole field (Prévot et al. 1976; Johnson & Atwater 1977).

The results of this paper suggest that a part of the anomalous shallowness of the leg 37 samples might be attributable to shape anisotropy if it is assumed that most of the lava cooled below the Curie temperature as thin flows rather than as isolated pillows. Precisely the opposite assumption is then required to explain the absence of an inclination anomaly for the samples from the surface of the median valley referred to in the paragraph above. This is not unreasonable, however, for if any pillows are likely to have cooled as isolated bodies they would be those on the surface. The average NRM intensity of these pillows is 0.024 emu cm⁻³ (24 Am⁻¹) and the average Q is about 18 (Johnson & Atwater 1977). Using the present axial dipole field intensity of 0.53 Oe (0.53 × 10⁻⁴ T) and an average demagnetizing factor for the pillows of 4π/3, we obtain from (13) an estimate of the average k for the pillow basalt at 37°N of 0.064. This corresponds to an equivalent TRM susceptibility of 0.05 (0.8 times the actual k), which predicts an inclination anomaly of 9° due to shape anisotropy in thin flows at this latitude (Fig. 5 and equation (15)). This is 37 per cent of the inclination anomaly not explained by geomagnetic field behaviour (Harrison & Watkins 1977).

Higher TRM susceptibility could account for a greater proportion of the inclination anomaly; k = 0.09 could produce half of the anomaly (ΔI = 12.20) and k = 0.20 the entire anomaly. Such high values of k are not consistent with the NRM intensities of the surficial
samples nor with the average intensity derived by inversion of deep-tow magnetometer data from the median valley (MacDonald 1977). A few cases of very high remanent intensities that probably do correspond to such high values of k have been reported (two samples from the Mid-Atlantic Ridge at 45° N, Irving (1970) and Carmichael (1970); Galapagos spreading centre; magnetic survey by Selater & Klitgord (1973) and one pillow fragment studied by Grommé et al., in preparation). These cases are exceptional, however, and cannot serve as a guide for calculating average inclination anomalies due to shape anisotropy for leg 37 unless it should turn out that even the fresh basalt in the median valley has already lost a lot of its original magnetization. This seems unlikely but should be checked if possible by collecting samples of pillows from the median valley that are a few tens of years old or less.

EXPERIMENTAL TEST

Experiments on thin discs of fresh pillow fragments were attempted to test the theoretical effects of shape anisotropy. They were abandoned in the preliminary stages, however, because the samples altered when heated (in a vacuum) repeatedly above the Curie point. The results are few, incomplete and approximate, but nonetheless are worth a brief discussion.

Three thin discs (dimension ratio 1:4) were tested from three different pillows that were studied by Marshall & Cox (1971, 1972) and Grommé et al. (in preparation). These were cooled in a vacuum from the Curie temperature at various orientations in a known field, and the direction of the resulting TRM was measured. In only one of the three discs (93A-4) was the deviation of TRM direction consistent in both sense and magnitude with that predicted by equation (15) for shape anisotropy. In another disc (104-3) the deviations in TRM direction were larger in magnitude than those predicted for shape anisotropy and inconsistent in direction. It seems as if the sample possessed an intrinsic anisotropy of TRM acquisition considerably stronger than the shape anisotropy. The third disc (17D-1-3) displayed an intrinsic anisotropy of about half the magnitude of the previous sample and a shape anisotropy much less than predicted.

A possible cause of both intrinsic anisotropy and reduced shape anisotropy is short-range interactions between magnetic grains. Short-range interactions within clusters of grains have much greater potential effects than the interactions within a rock with uniformly distributed grains (for which the Lorentz field applies). For example, if the grains are clustered in linear trains, those trains of grains more nearly parallel to the field will carry most of the TRM. The reason is that the first grain blocked in a train that is parallel to the field exerts a positive magnetostatic interaction on the other grains that tends to align them in the same direction. The first grain blocked in a train that is at right angles to the field exerts a negative interaction on its neighbours, the net result being a smaller degree of alignment than the previous case such that these trains contribute less net magnetization to the rock. The positive interaction field reinforces the field within the trains parallel to the field and thus diminishes the effect on the TRM direction of the demagnetizing field arising from shape anisotropy. Moreover, if the trains are not distributed isotropically in orientation, the preferred direction of trains will confer an intrinsic anisotropy on the rock.

Simple trains of grains have been observed in pillows (Marshall, private communication), but more complex, skeletal grains or arrangements of small grains are commoner (Ryall et al. 1977, and many others). Some of the skeletal grains, however, namely those that are cruciform or feathery, might be expected to behave similarly to simple trains.

The deviations of TRM direction associated with the shape and intrinsic anisotropy ranges up to 15° in one pillow sample, 7° in another and 3° in the third. These limited results on only three pillows are not sufficient to draw any general conclusions, but they do suggest
that intrinsic anisotropy of the pillows as well as shape anisotropy of the flows could contribute appreciably to the anomalous inclinations observed in marine basalts.

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References


Effect of shape anisotropy on TRM direction


