On the Character and Consequences of Large Impacts in the Late Stage of Terrestrial Planet Formation

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Received February 26, 1999; revised July 6, 1999

We perform three-dimensional N-body integrations of the final stages of terrestrial planet formation. We report the results of 10 simulations beginning with 22–50 initial planetary embryos spanning the range 0.5–1.5 AU, each with an initial mass of 0.04–0.13 $M_\oplus$. Collisions are treated as inelastic mergers. We follow the evolution of each system for $2 \times 10^8$ years at which time a few terrestrial type planets remain. On average, our simulations produced two planets larger than $0.5 M_\oplus$ in the terrestrial region (1 simulation with one $m > 0.5 M_\oplus$ planet, 8 simulations with two $m > 0.5 M_\oplus$ planets, and 1 simulation with three $m > 0.5 M_\oplus$ planets). These Earth-like planets have eccentricities and orbital spacing considerably larger than the terrestrial planets of comparable mass (e.g., Earth and Venus). We also examine the angular momentum contributions of each collision to the final spin angular momentum of a planet, with an emphasis on the type of impact which is believed to have triggered the formation of the Earth’s Moon. There was an average of two impacts per simulation that contributed more angular momentum to a planet than is currently present in the Earth/Moon system. We determine the spin angular momentum states of the growing planets by summing the contributions from each collisional encounter. Our results show that the spin angular momentum states of the final planets are generally the result of contributions made by the last few large impacts. Our results suggest that the current angular momentum of the Earth/Moon system may be the result of more than one large impact rather than a single impact. Further, upon suffering their first collision, the planetary embryos in our simulations are spinning rapidly throughout the final accretion of the planets, suggesting the proto-Earth may have been rotating rapidly prior to the Moon-forming impact event.

Key Words: planetary formation, origin-solar system, impact processes, Moon, rotational dynamics.

1. INTRODUCTION

The standard model for formation of the terrestrial planets describes the growth of planets via the collisional accumulation of smaller rocky planetesimals in the protoplanetary disk. In the 30 years following the work of Safronov (1969), analytic work and numerical simulations have revealed that the process of terrestrial planet formation can be roughly divided into three stages: the accumulation of gas and dust in the protoplanetary disk into kilometer-sized planetesimals, the growth of lunar to Mars-sized planetary embryos, and the final accumulation of the planets via the collision and merger of the embryos until only the final planets remain. Giant impacts between large planet-sized embryos are a natural occurrence in the late stage of the standard model (e.g., Wetherill 1985). The size, number, and type of giant impacts which a planet experiences can have a profound effect on the final characteristics of a planet. Indeed, a giant impact of this type is generally credited with the origin of the Earth/Moon system (Hartmann and Davis 1975, Cameron and Ward 1976).

The earliest stage of terrestrial planet formation involves the growth of kilometer-sized planetesimals from the dust in the protoplanetary disk via gravitational instability (Goldreich and Ward 1973, Ward 1999) and/or collisional sticking (Weidenschilling and Cuzzi 1993). The next stage is characterized by the accumulation of the planetesimal disk into a few tens of lunar to Mars-sized planetary embryos. Analytic modeling of this stage of planet formation was first performed in Safronov (1969). In this and subsequent works (e.g., Greenberg et al. 1978; Stewart and Wetherill 1988; Spaute et al. 1991; Wetherill and Stewart 1989, 1993), the evolution of a thin annulus in the planetesimal swarm has been modeled using methods of gas dynamics, or a
“particle in a box” approach (hereafter PIAB). In this treatment, the particles are assigned a “random” velocity based on the eccentricity and inclination of their orbits \( (v_{\text{ran}} \propto \sqrt{e^2 + i^2}) \) and a collisional cross section \( \sigma_{\text{coll}} \) based on their physical radius \( (R) \) and an enhancement due to gravitational focusing, where

\[
\sigma_{\text{coll}} = \pi R^2 \left(1 + \left(\frac{v_{\text{esc}}}{v_{\text{ran}}}ight)^2\right) \tag{1}
\]

and \( v_{\text{esc}} \) is the escape velocity from its surface. The velocity distribution of the planetesimals evolves due to gravitational interactions and collisions between the particles and is therefore coupled to the mass distribution. As in gas dynamics, the system is driven toward an equipartition of random energy among the planetesimals. This process, also referred to as “dynamical friction,” causes the most massive planetesimals to have the smallest eccentricities and inclinations and reduces the relative velocities between the largest bodies (Stewart and Wetherill 1988). When encounter velocities with the embryos are low and gravitational focusing significant, the largest body then grows more rapidly than any other body (i.e., \( \frac{dM}{dt} \propto M^{1/3} \)) and separates (or “runs away”) from the rest of the mass distribution. One of the underlying assumptions of the PIAB method is that the planetesimal swarm is uniform (in distributions of mass, number, orbital elements, etc.) throughout the annulus of the protoplanetary disk being modeled. Thus, PIAB methods for modeling the growth of planetary embryos start to break down as runaway growth begins and a few large growing embryos render the assumption of uniformity increasingly less valid. However, recent \( N \)-body simulations, which are not subject to this assumption, have also demonstrated that runaway growth of planetary embryos occurs, at least until neighboring embryos become large enough to mutually perturb one another (Kokubo and Ida 1996, 1998).

It has often been assumed that planetary embryos sweep up all of the material in a local region via runaway growth. Whether this actually occurs is determined by the relative velocities between the embryos and the planetesimal swarm. In general, processes that decrease encounter velocities (e.g., gas drag or dynamical friction) tend to enhance the collision cross section of planetesimals. If encounter velocities between planetesimals are reduced to a level that gravitational focusing is significant, embryos will rapidly accrete material via runaway growth. Likewise, processes that increase encounter velocities between embryos will tend to slow or inhibit runaway growth by decreasing collision cross sections. \( N \)-body simulations of the post-runaway phase have demonstrated that mutual interactions between neighboring embryos tend to increase embryo eccentricities and encounter velocities, slowing the runaway growth of planetary embryos (Kokubo and Ida 1998). During this post-runaway (or “oligarchic”) phase growth planetary embryos slowly accrete neighboring planetesimals (i.e., \( \frac{dM}{dt} \propto M^{-1/3} \)) while maintaining a spacing of about 10 Hill radii \( (r_H = a(2m_{\oplus}/3M_\odot)^{1/3}) \), where \( a \) is the embryo’s semi-major axis, \( m_{\oplus} \) is the mass of the embryo and \( M_\odot \) is the mass of the Sun.

To date, the largest scale simulation of the runaway and post-runaway growth of planetary embryos in the terrestrial region has been performed by Weidenschilling et al. (1997). They utilized a hybrid multizone simulation developed in Spaute et al. (1991) to model the orbital and collisional evolution of a planetesimal swarm. Their model treats the smaller planetesimals as a part of a continuum size distribution. Bodies larger than a minimum cutoff are treated as discrete bodies with individual orbits which are tracked probabilistically. The large scale of the protoplanetary disk was modeled by dividing the continuum part of the mass distribution into radial zones, each about 0.01 AU in width. By removing runaway planetesimals from the continuum and handling them individually, their simulation helps to overcome the limiting assumption of uniformity inherent to PIAB methods, and allows for improved modeling of planetesimal growth beyond the onset of runaway growth.

Weidenschilling et al. (1997) used this hybrid multizone model to simulate the growth of planetary embryos in the terrestrial region, beginning with a swarm of kilometer-sized planetesimals distributed between 0.5 and 1.5 AU, with a surface density of 8.4 g cm\(^{-2}\) at 1 AU that declined as \( r^{-3/2} \). Collisions were treated as inelastic mergers. In their simulation \( \sim10^{20} \) g runaway planetary embryos formed after a few times \( 10^4 \) orbital periods. Shortly after the embryos emerged their rate of growth decreased, and their subsequent post-runaway growth occurred over a much longer period of time. After \( 10^8 \) years, nearly 90% of the initial mass was contained in the 22 largest planetary embryos. These embryos had a median mass of 0.10 \( M_\oplus \), or about the mass of Mars. The masses of the final embryos were relatively independent of heliocentric distance and did not increase with orbital radius as predicted by earlier analytic estimates (e.g., Lissauer and Stewart 1993). The mean eccentricities and inclinations of the embryos were low, \( (e) \sim 0.01 \) and \( (i) \sim 0.05^\circ \). The results of Weidenschilling et al. suggest that after about a million years most, but not all, of the mass of the planetesimal swarm is contained in a few tens of planetary embryos on nearly circular orbits.

The final stage of planetary formation begins as the embryos sweep up the remainder of the swarm of smaller planetesimals and perturb each other into crossing orbits where they suffer a series of large impacts. The accretion of the final planets is complete after a few times \( 10^8 \) years (e.g., Wetherill 1992). Three-dimensional modeling of this stage has been pioneered by Wetherill (1985, 1992, 1994, 1996) using a Monte Carlo method (Opik-Arnold scheme) to perform hundreds of simulations. In this scheme, the orbital elements of each embryo are used to determine the probabilities of encounters with other bodies. When an encounter is predicted, a random number is used to determine whether a collision has occurred or to determine the perturbations to the embryo’s orbital elements resulting from the close encounter. Because the individual trajectories of each body are not followed directly, this method is fast enough to track the evolution of several hundred bodies in three dimensions over timescales of \( 10^8 \) years. However, the Monte Carlo
method does not explicitly track encounter dynamics, account for phase-dependent resonances between the embryos (mean-motion or secular), or correctly follow correlated encounters between bodies. Until recently, direct \(N\)-body simulations of the late stage, which while not subject to these limitations of the Monte Carlo method, have been limited to two-dimensional studies which artificially reduce the accretion timescale (Lecar and Aarseth 1986, Beaugé and Aarseth 1990, Alexander and Agnor 1998).

Chambers and Wetherill (1998) (hereafter CW98) performed three-dimensional \(N\)-body orbital integrations of the late stage of planet formation. Their simulations began with a system of 24–56 initial embryos in the terrestrial region (0.5–1.8 AU) with masses that increase with orbital radius and range from about 0.02\(M_{\oplus}\) near 0.5 AU to 0.10\(M_{\oplus}\) near 1.8 AU. In addition, they performed simulations with an initial swarm of embryos that extended into the asteroid belt. In these runs the initial masses were as large as 0.35\(M_{\oplus}\) at 4 AU. The initial separation of the embryos was 7–10 mutual Hill radii; collisions were treated as inelastic mergers. In two-thirds of their simulations, Jupiter and Saturn were added with their present masses and orbital elements. Their results demonstrate that a system of well-separated embryos on initially circular orbits quickly becomes dynamically excited, achieving crossing orbits after a few times \(10^5\) years. They also found secular perturbations between embryos and with the giant planets and not close encounters between embryos to be the primary dynamical mechanism responsible for the evolution of the eccentricity and inclinations of the embryos. Their simulations typically yielded 2–3 final terrestrial planets with eccentricities and inclinations that were substantially larger than those of Earth and Venus. This particular result indicates that the origin of the relatively low eccentricity orbits of Earth and Venus is not yet understood in the context of the standard model and is especially significant, given the potential dependence of climate and habitability on orbital eccentricity. This issue is discussed further below.

The late stage of planetary formation is of particular importance because it is during this period that many characteristics (e.g., mass, spacing, orbital elements, spin angular momentum, and presence of impact-generated satellites) of the final planets are determined. For example, growing planetary embryos acquire their spin angular momenta as they accrete mass from planetesimals and other embryos. The relative motions of the bodies at impact determine the contributions of each accreted body to the spin angular momentum of the final planet. As an embryo grows, both the average impact velocity and average moment arm of colliding bodies increase, making the rotation state of a planet determined primarily during its final accretion (Dones and Tremaine 1993a).

Large collisions during the late stage have been used to explain several features of the Solar System including the melting of the Earth’s core and removal of its primordial atmosphere (Wetherill 1985), the high density and iron content of Mercury (Benz et al. 1988), the high obliquity of Uranus, the formation of the Pluto/Charon binary (e.g., Stern et al. 1997 and references therein) and the formation of the Earth/Moon system (Hartmann and Davis 1975, Cameron and Ward 1976). Simulations of single-impact events have been performed using smoothed-particle hydrodynamic methods (hereafter SPH) to assess the feasibility of giant impacts to explain some of the above claims and to characterize the type of impacts which are most likely responsible (e.g., Benz et al. 1986, 1987, 1989; Cameron and Benz 1991; Slattery et al. 1992; Cameron 1997; Cameron and Canup 1998). Most notably, works modeling both the lunar-forming impact event (e.g., Cameron 1997, Cameron and Canup 1998) and the subsequent accretion of the Moon from the impact-ejected debris (Canup and Esposito 1996, Ida et al. 1997, Kokubo et al. 1998) have yet to identify a single impact which can account for the masses of the Earth and Moon and the current angular momentum of the Earth/Moon system.

In this paper we present a suite of \(N\)-body orbital integrations of the late stages of terrestrial planet formation. The calculations we present here are broadly similar to those of CW98 and Wetherill (1985, 1992, 1994, 1996) in initial conditions and assumptions. However in our analysis we have examined the dynamical characteristics of the individual impact events (e.g., impactor velocity, encounter angular momentum) and their implication for the final rotation states of the terrestrial planets, and in this respect our study is novel. In particular our simulations were performed to address the following questions within the context of the standard model of planet formation: What are the typical masses and angular momenta that characterize the giant impacts which occur during the late stages of planet formation? How likely is it for a planet to experience multiple giant impacts? What net contribution do giant impacts make to a planet’s final spin angular momentum state? How likely is the impact or series of impacts, necessary for the formation of the Earth/Moon system?

The method and initial conditions employed in our simulations are outlined in Section 2. Our main results are presented in Section 3 and discussed in Section 4. We examine the assumptions and approximations made in this study in Section 5 and our conclusions are summarized in Section 6.

2. Method and approximations

2.1. The integrator

Tracking the evolution of gravitating bodies for more than \(10^8\) orbital periods has recently become feasible due to both the improved performance of computer workstations and the development of symplectic integration algorithms for the gravitational \(N\)-body problem. Wisdom and Holman (1991) first developed Mixed Variable Symplectic (MVS) methods for use in Solar System dynamics to study the long-term orbital stability of the planets. The main virtue of MVS methods is that they conserve the energy of the system to a very high degree, so long as there
are no close encounters between massive bodies. In addition, because MVS methods use a global timestep, all action–reaction pairs of forces cancel to within machine precision. The $N$-body integrations of terrestrial planet formation presented in CW98 used the MVS algorithm when bodies were well separated and a variable timestep Burlisch-Stoer integrator to follow the system when two of the bodies were experiencing a close encounter. Switching between the integration methods resulted in a secular growth in the energy error, typically one part in $10^2$–$10^3$ over $10^8$ years (CW98).

The newly developed Symplectic Massive Body Algorithm (SyMBA) (Duncan et al. 1998) is based on MVS methods but can also handle close encounters in a completely symplectic manner. Because of this feature, integrations performed using SyMBA do not suffer any secular increase in the energy error due to switching integration methods. Also, like MVS methods, action–reaction pairs of forces cancel to within machine precision. Thus, the angular momentum of the system is well conserved. Further, during a collision or close encounter the angular momentum of the encounter will be well conserved, allowing it to be effectively resolved. Despite these advantages, the SyMBA integrator is not without its limitations. The main drawback of SyMBA is that it does not handle close encounters with the Sun very well. The global timestep of the simulation must be less than about a 20th of the orbital period at the periapse distance of an orbit for the periapse passage to be sufficiently resolved. Furthermore, the use of an adaptive timestep to accurately follow passages at small perihelion distances would destroy the symplectic nature of the algorithm (Gladman et al. 1991).

We use the SyMBA integrator to follow the dynamical evolution of a few tens of planetary embryos during the final accretion of the terrestrial planets. By beginning with initial conditions motivated by the results of recent simulations of the middle stage of planet formation (Weidenschilling et al. 1997), we have attempted to self-consistently model the growth and collisional evolution of the planet-sized impactors credited with, for example, the origin of the Earth/Moon system. Our orbital integrations explicitly track the trajectories and interactions of all bodies at all times, and thus, when a collision occurs the dynamical properties of the impact, such as impactor mass, velocity, and orientation, are easily determined. Because the SyMBA algorithm recursively subdivides the effective timestep during an encounter, collisions are detected when bodies overlap by no more than a few percent of their radii. The main advantage of direct integration over statistical methods is that no assumptions or parameterizations about how bodies gravitationally interact are needed. The obvious drawback of such an approach is that it is relatively computationally expensive, limiting the number of bodies and the dynamical timescales that can be explored.

The fact that for large numbers of bodies the computational cost of our calculation scales as $N^2$ (where $N$ is the number of bodies followed) places a practical constraint on the number of bodies which can be used. For this reason, we did not include any residual population of smaller planetesimals left over from the runaway and post-runaway stages of embryo growth in our simulations. Also, collisional fragmentation of embryos was not included, since any process which would tend to increase the number of bodies in a simulation is not computationally feasible. This is not to say that there is no population of small bodies or that a significant amount collisional fragmentation does not occur during the late stage of planet formation, and these issues are discussed later in Section 5. In these simulations, when two embryos collided they were merged inelastically. The radius of the new body was computed by adding the volumes of the colliding bodies and assuming uniform density and spherical shape for the new combined body.

In the simulations presented here global timesteps of 2.7 and 5.5 days were used. In each simulation, none of the embryos’ periheleion distances became small enough to affect the energy conservation of the simulations. Figure 1 shows the relative energy error between collisions as a function of time from one of our simulations with the larger global timestep. In all of our simulations the energy error is generally no larger than one part in $10^6$.

2.2. Initial Conditions

We have performed a total of 10 simulations of the late stage of terrestrial planet formation. These simulations follow the orbital evolution of a few tens of planetary embryos through close encounters and collisions for $2 \times 10^9$ years until only a few planets remain. The initial conditions of the simulations were chosen to be as consistent as possible with the end results of middle stage planet formation simulations. Our simulations used two different initial size distributions of embryos. The spatial distribution and orbital elements of the embryos were roughly the same in both cases; a general description of our initial conditions is summarized in Table I.
TABLE I
Initial Conditions

<table>
<thead>
<tr>
<th>Condition</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Semi-major axes</td>
<td>0.5–1.5 AU</td>
</tr>
<tr>
<td>Surface density of planetesimal disk</td>
<td>$\sim 8.0 \text{ g cm}^{-2}$ at 1 AU</td>
</tr>
<tr>
<td>Embryo eccentricities</td>
<td>$\sim 0.01$</td>
</tr>
<tr>
<td>Embryo inclinations</td>
<td>$\sim 0.05^\circ$</td>
</tr>
<tr>
<td>Bulk density of solid material</td>
<td>$5.4 \text{ g cm}^{-3}$ (simulations 1–8, 10)</td>
</tr>
<tr>
<td></td>
<td>$3.0 \text{ g cm}^{-3}$ (simulation 9)</td>
</tr>
</tbody>
</table>

Simulations 1–8: Embryos produced by Weidenschilling et al. (1997). As mentioned earlier, the work of Weidenschilling et al. (1997) represents the largest scale study of the formation of planetary embryos performed to date. We have completed eight integrations beginning with the 22 most massive embryos produced by the full terrestrial zone simulation of Weidenschilling et al. (1997). The embryo masses and orbital elements were taken directly from the end of the multizone simulation. The median mass of the 22 largest embryos was $\sim 0.10 M_\oplus$. The total mass of the 22 embryos was $\sim 1.77 M_\oplus$ and represents 88% of the initial system mass used in the multizone calculation. The initial embryos were typically separated by about 10 Hill radii. Because the multizone simulation followed the orbits of the embryos in a statistical manner, its output contains no information about the phases of each embryo’s orbit. Thus, we chose the initial phases at random and in each of the simulations the phases were computed with a different random number seed. These simulations, when combined with the multizone simulation of embryo growth, represent the first simulations of planet formation to self-consistently follow the growth of planets from kilometer-sized planetesimals under the simplifying assumption of inelastic mergers.

Simulations 9 and 10: Smaller initial embryos. One of the goals of our study has been to characterize the effect of a series of giant impacts on the final planets. The planetary embryos produced by Weidenschilling et al. (1997) are already as large as the impactors that may have been responsible for the formation of the Earth/Moon system. So, in these two simulations, we considered a larger number of smaller initial embryos to give us slightly better resolution of impactors and to investigate the contribution of smaller impactors to the final characteristics of the planets. These simulations began with 50 embryos, each with a mass of $0.04 M_\oplus$. They were initially distributed such that the surface density varied as $r^{-3/2}$ with a value of $8.0 \text{ g cm}^{-2}$ at 1 AU. Their initial spacing in semi-major axes of the initial embryos was about 5 Hill radii. Again, the initial phases were chosen at random.

3. RESULTS

The work of CW98 discusses the dynamical evolution of a system of embryos in the terrestrial region at some length. We have performed a detailed comparison of our results with theirs and have found general agreement with their conclusions (e.g., in the rate of increase of $(e)$ and $(i)$, in the planet growth rates, and in the final configurations of the planets, etc.) for simulations with similar initial conditions. A discussion of the dynamical evolution of our simulations and the final planets produced by them follows.

Figures 2 and 3 show the evolution of the system of embryos for two simulations. With no background population present to damp their motions, mutual gravitational interactions between the embryos cause the initially small mean eccentricity of the system to quickly increase (see Fig. 4). After a few times $10^5$ years, eccentricities are large enough so that orbits cross and the embryos begin to experience close encounters and collisions. In all cases the mean eccentricities of our simulations grow to values much higher than our initial eccentricities in less than $10^6$ years. In this respect the dynamical evolution of our simulations is not particularly sensitive to the initial eccentricities of the embryos, and is very similar to the evolution of CW98’s simulations that began with embryos in circular orbits. Accretion proceeds until only a few planets remain on well-separated orbits. On average, our simulations yield two terrestrial planets
more massive than 0.5M_⊕ (one simulation with one m ≥ 0.5M_⊕ planet, eight simulations with two m ≥ 0.5M_⊕ planets, and one simulation with three m ≥ 0.5M_⊕ planets) and an occasional smaller planet between 0.5 and 1.5 AU (see the last frames of Figs. 2 and 3). The most massive planets tend to form in the inner half of our simulated region (0.5–1.0 AU) where the surface density and the collision frequency are the largest. In the outer region, the embryos require a longer time to collide and the final planets tend to be smaller in this region. At the end of our simulations, there are often a few residual initial bodies that have been scattered outward to 1.5–3.0 AU in addition to the planets in the terrestrial region. These bodies are sometimes on crossing orbits, indicating that the accretion process in this region may not be complete by the end of our simulations.

The process by which the final planets acquire their mass in simulations that begin with 22 initial embryos (simulations 1–8) is somewhat different than the simulations that began with 50 embryos (simulations 9 and 10). With initial spacings of about 10 Hill radii, the embryos of simulations 1–8 are fairly well separated and can accommodate some increase in their eccentricities before beginning to suffer close encounters. As a few of the initial ~0.10M_⊕ embryos begin to collide and merge in the inner part of the disk, they accrete most and sometimes all of their mass by colliding with initial embryos that have not yet suffered a collision. Usually only 3–4 of the initial bodies ever grow larger than 0.20M_⊕ in these simulations. The final planets in simulations 1–8 grow simply by accreting the initial bodies and are occasionally hit by another growing planet which has accreted a few of the initial embryos. Because the initial bodies of simulations 9 and 10 are spaced by only ~5 Hill radii, their encounter and collision timescales are much shorter than those of the initial bodies of the first eight simulations. Further, when the number of bodies in simulations 9 and 10 has been reduced to 22 (the number of initial bodies in simulations 1–8) there is a range of embryo masses wider than the nearly unimodal initial distribution of simulations 1–8. At this time, the embryos in simulations 9 and 10 typically have eccentricities significantly larger than the initial eccentricities of simulations 1–8. After about 10–40 million years in simulations 9 and 10, there are 4–5 embryos more massive than 0.20M_⊕ with the single most massive body typically located in the inner region of the disk. Unlike simulations 1–8 where the large embryos at this point tend to be fairly isolated, the largest embryos of simulations 9 and 10 are still in close proximity to one another (see Figs. 2 and 3). It is therefore not surprising that the final planets of these simulations suffer more collisions with larger impactors. It should be noted that processes which will tend to decrease the stability time of the initial embryos (e.g., perturbations from Jupiter and Saturn) could promote the growth of larger impactors and fewer but larger final planets in our simulations, and may blur the distinction between the way that planets acquire their final mass in simulations 1–8 and that in simulations 9 and 10.

Figure 5 shows the distribution of final planets produced in all ten of the simulations. This plot illustrates the tendency to form at least one planet with mass approaching 1M_⊕ near 1AU in the terrestrial region. This seems to be a general result of studies of the late stage (Wetherill 1985, 1992, 1994, 1996, and CW98). Figure 6 shows the mass and time-averaged eccentricity of the final planets. The eccentricities of planets with mass approaching 1M_⊕ is typically 0.05–0.15, a value several times larger than

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**FIG. 3.** The evolution of the mass distribution from simulation 9 with 50 initial bodies.

**FIG. 4.** The mass weighted mean eccentricity of the initial protoplanets of simulation 1 is shown as a function of time.
that of the Earth or Venus.\footnote{It should be noted that the orbital elements of the terrestrial planets oscillate due to perturbations between the planets, and the eccentricities of Earth and Venus are currently near their minimum values. Integrating the present Solar System forward reveals that the eccentricities of Earth and Venus can reach values as large as 0.06 primarily due to perturbations from the gas giant planets, and the time-averaged eccentricities of Earth and Venus are roughly 0.03 and 0.04 respectively. If the terrestrial planets, with their present-day orbital elements are integrated and perturbations from the giant planets are excluded, the eccentricities of Earth and Venus seldom venture above 0.02 and each has a time-averaged eccentricity of about 0.014, which is comparable to their present values.} We will discuss this result in detail below.

In all of the simulations we tracked the spin angular momentum of the embryos that resulted from all of their collisions. We assumed that each embryo had no spin angular momentum at the start of a simulation. When a collision occurred, the spin angular momentum of the merged body was determined by adding the spin angular momenta of the two colliding bodies to the orbital angular momentum of the two bodies about their center of mass. The rotation periods of the growing planets were computed by assuming that the bodies were spheres of uniform density.

Table II lists the mass, semi-major axis, time-averaged eccentricity, time-averaged inclination, obliquity, and rotation period of all of the bodies remaining at the end of the simulations. The rotation periods of these “final planets” are typically close to, and occasionally less than, the rotational stability limit.\footnote{Here we define the stability limit as the point when the rotation period equals the orbital period at the surface of the body. For the densities used here, ρ = 3.54 g cm$^{-3}$, $T_{\text{stable}} = 1.9$, 1.4 h respectively.} This clearly unphysical result is a consequence of the simplifying assumptions made here, in particular the assumption of complete merger during all collisions. This issue is discussed in more detail in Section 5.1.

The obliquities of the final planets are quite large. This result is a general feature of planets formed from the accretion of a few embryos with a significant fraction of the final planet mass. During the late stage, the scale height of the protoplanetary disk is generally greater than the Hill radii of the growing planets. As a result, embryos can enter their mutual Hill sphere with any orientation. When all impact orientations are equally likely, the probability that the angular momentum of a collision is oriented with a particular polar angle in the range $\varepsilon \to \varepsilon + d\varepsilon$ from an arbitrary $z$ axis is

$$p(\varepsilon) \, d\varepsilon = \frac{1}{2} \sin \varepsilon \, d\varepsilon. \quad (2)$$

The obliquity distribution of planets that form from large impacts with random orientation will also have this distribution, and indeed our final planetary obliquities are well described by this basic function (see Fig. 7).
TABLE II
Final Planets

<table>
<thead>
<tr>
<th>Simulation</th>
<th>Mass ((m/M_{\oplus}))</th>
<th>(a) (AU)</th>
<th>((e))</th>
<th>((i)) (°)</th>
<th>Obliquity (°)</th>
<th>(T_{\text{spin}}) (h)</th>
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<tr>
<td>1</td>
<td>0.92</td>
<td>0.62</td>
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<td>0.59</td>
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<td>0.24</td>
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<td>0.02</td>
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<td>2</td>
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Note. The “—” symbol is used for the obliquity of bodies that never experienced a collision and had no spin angular momentum.

FIG. 8.  The impactor mass, impact velocity, and angular momentum of each collisional encounter are shown as a function of the time at which the impact occurred. All of the collisions which occurred in our 10 simulations are shown. Collisional encounters with angular momentum greater than or equal to the Earth/Moon system are classified as “potential moon–forming impacts.”

3.1. Giant Impacts

Figure 8 shows the impactor mass, the ratio of the impact velocity to the two-body escape velocity, and the angular momentum of the two colliding bodies about their center of mass as a function of time for each collision in all of the simulations presented (the “impactor” is always the less massive of the two colliding bodies). Figure 9 gives a breakdown of the impact velocity and collision angular momentum as a function of the mass of the impactor.

As a simulation proceeds, embryos collide and merge and a spectrum of embryo masses and therefore impactor masses develops (see Fig. 8a). Scattering events and distant interactions between bodies of differing mass tend to affect the orbit of the smaller body more strongly than the larger body. This causes the smaller bodies in the distribution to have relatively larger eccentricities than the more massive bodies. This dynamical characteristic is easily seen in Figs. 2 and 3 between \(1 \times 10^6\) and \(40 \times 10^6\) years. Early in a simulation when the mass distribution is effectively unimodal, encounter velocities are low and collisions typically occur near the two-body escape velocity. As the eccentricities and inclinations of the smaller embryos increase, so do their encounter and impact velocities. Figure 8b shows that after a few million years, at which time the embryo
masses span about a factor of five, some impacts occur at several times the escape velocity. By comparing Fig. 8b and Fig. 9a, the impactors which arrive at these high velocities are easily identified as the smallest bodies in the mass distribution at late times.

Of particular interest are the types of impacts which could have resulted in the formation of the Moon and the Earth/Moon system’s relatively large angular momentum. Recent works studying the accretion of the Moon from an impact-generated disk show that a significant fraction of the disk material is scattered back onto the Earth via interactions with the growing Moon and indicate that the accumulation of the Moon is only about 50% efficient. If true, this requires that the proto-lunar disk contain at least two lunar masses of material. The manner in which two colliding bodies process the energy and angular momentum of an impact is dependent on both the dynamical properties of the collisional encounter and the physical properties of the bodies (e.g., size, material, sound speed, equation of state, degree of differentiation). Given the complexity of impact processes, it is perhaps not surprising that no simple scaling relation between the dynamical and physical properties of a collision and the mass of a circumplanetary disk produced by it has been identified. SPH studies of the Moon-forming impact event (Cameron and Canup 1998) indicate that sufficiently massive disks are produced in collisions that involve a total mass of about $0.65 M_\oplus$ and the angular momentum of the Earth/Moon system ($1 L_{\oplus} M$) or a total mass of $1 M_\oplus$ and twice the angular momentum of the Earth/Moon system. It should be noted that due to the complexity of the phenomena and the computational requirements of SPH (and other methods), studies of the Moon-forming impact to date have only examined a relatively small number of different types of collisions. Also, the effect that the assumptions and approximations made in these studies (e.g., equation of state, number of particles) have on the outcome of these simulations is only beginning to be carefully examined. As further refinements are made to SPH models, and additional types of collisions are explored, our understanding of the type of circumplanetary disk produced by an impact may change.

In an attempt to roughly classify those collisions which may result in the formation of a moon similar to the Earth’s, we have described collisions where the angular momentum of the encounter equals or exceeds the angular momentum of the Earth/Moon system as “potential moon-forming impacts.” This classification reflects our rudimentary knowledge of the impact dynamics required to form a satellite and is different than the “giant impacts” of Wetherill (1985, 1992), which were classified by the impactor mass. In the 10 simulations presented here, there were a total of 20 potential moon-forming impacts (see Figs. 8c and 9b). In addition, 25% of the final planets with masses more than $0.50 M_\oplus$ experienced more than one potential moon-forming impact. After $10^7$ years, impactors and targets have grown large enough that potential moon-forming impacts begin to occur. These high angular momentum collisions continue to occur until about $10^8$ years when the final planets are well separated and accretion is nearly complete (see Fig. 8c). It should be noted that if the initial distribution of embryos had been extended into the present-day asteroid belt, the accretion process may have lasted longer as embryos could continue to be scattered into the terrestrial region after $10^8$ years. The calculations of Wetherill (1992) and CW98 show that in this situation, the period during which the largest impact events occur spans up to $3 \times 10^8$ years. In simulations 1–8 the number of moon-forming collisions which occurred in each simulation ranged from 0 to 3 with an average of about 1.6 per simulation. In simulations 9 and 10, which began with smaller embryos and slightly more total mass, there were a total of 4 and 3 moon-forming collisions respectively.

As seen in Fig. 9a, the impact velocity of the potential moon-forming collisions tends to be slightly above the two-body escape velocity ($v_{\text{esc}}$). With the exception of one ($0.04 M_\oplus$) impactor that hit with a velocity of $3.44 v_{\text{esc}}$, the moon-forming impacts in our simulations occurred with velocities in the range $1.00–1.63 v_{\text{esc}}$ with an average of $1.20 v_{\text{esc}}$.

### 3.2. Following the Growth of a Planet

Here we examine the accumulation of mass and angular momentum of our individual final planets. Our treatment of collisions as inelastic mergers requires collisions to conserve both mass and angular momentum. In our simulations each initial embryo began with no spin angular momentum. When two bodies collided, the spin angular momentum of the spherical merged body was determined by adding the spin angular momenta of each of the colliding bodies to that of the collisional encounter about their center of mass. In doing so, we are able to track the changes in the spin angular momentum of a growing planet; however we are not allowing this angular momentum to evolve or precess in any way between collisions. Figure 10 shows the evolution of the mass, magnitude of the spin angular momentum in units of the angular momentum of the Earth/Moon system, obliquity, and rotation period of a typical Earth-like
FIG. 10. The mass, magnitude of the spin angular momentum in units of the Earth/Moon system, obliquity, and rotation period of a typical planet (the 0.73 \( M_\oplus \) planet from simulation 3) are plotted as a function of time. The dotted line of the rotation period vs time graph indicates the rotational stability limit. The small oscillations in the obliquity are due to the motion of the body’s orbit; the spin axis of the body remains fixed in the inertial frame between collisions. Planet \((m \geq 0.50M_\oplus)\) produced by our simulations. This figure illustrates several general characteristics of late stage planetary growth in our simulation. Here each collision results in a net increase in the magnitude of the spin angular momentum of the growing planet. More than half of our final planets with \(m > 0.5M_\oplus\) acquired their spin angular momentum without experiencing a significant decrease in its magnitude. Also, the collision and merger of the growing planet with other bodies reorients the direction of the spin angular momentum vector or obliquity. The obliquity of the growing planets are generally large. After suffering their first collision, the embryos are almost always rotating rapidly, making the rotation period of the planets short and occasionally less than the rotational stability limit. This outcome is consistent with the analytic estimates of Dones and Tremaine (1993a) for planets that experience large impacts.

Not surprisingly, the contributions to the spin angular momentum of a planet from many large impacts are not always additive. An example of cancellation between spin angular momentum contributions is shown in Fig. 11. During the growth of this planet, the largest impact occurred at \(t = 13 \times 10^6\) years. The protoplanet suffered two subsequent large collisions, the last of which resulted in a net decrease of 0.85\(L_\oplus - M\) in the magnitude of the spin angular momentum. This cancellation of the angular momentum contribution by subsequent impacts was not uncommon in our simulations. Of the 20 final planets with \(m > 0.50M_\oplus\), 40% experienced collisions which decreased the magnitude of the spin angular momentum of the planet by 0.10\(L_\oplus - M\) or more. Furthermore 10% of these planets experienced two collisions of this type and 10% experienced three collisions of this type. Figure 12 illustrates an extreme example of this cancellation. Here the contributions to the spin angular momentum from multiple impacts almost completely cancel, leaving the final planet with a spin angular momentum of only 0.05\(L_\oplus - M\). The obliquity of this planet is strongly retrograde, with a value of 153\(^\circ\), and is reminiscent of the current rotation state of Venus.

Simulations 9 and 10 used a larger number of less massive \((0.04M_\oplus)\) initial bodies. In Fig. 13 the mass and the magnitude of the spin angular momentum of the 0.80\(M_\oplus\) planet from simulation 9 are shown as a function of time. This planet experiences two collisions with an initial embryo of the simulation at about \(32 \times 10^6\) and \(50 \times 10^6\) years which both reduce the planet’s spin angular momentum. The second of these collisions occurred with an impact velocity 1.4 times that of the two-body escape velocity and had a magnitude of 0.55\(L_\oplus - M\). Figures 8c and 9b demonstrate that embryos less massive than \(0.10M_\oplus\) can strike a growing planet with high velocity and make significant contributions to the spin angular momentum of the growing planets.

FIG. 11. The planet mass and spin angular momentum as a function of time for a planet that experienced a net reduction in its spin angular momentum due to cancellation between contributions from multiple impacts are shown. The results shown here are for the 0.85 \(M_\oplus\) planet of simulation 1.
LARGE IMPACTS IN TERRESTRIAL PLANET FORMATION

3.3. Latest and Largest Impactors

Most often the final spin angular momentum state of a planet is determined by the combined effect of the last few collisions and not simply from the single largest impact. In Table III the time of impact, the impactor mass, and collision angular momentum for the largest \( m_1 \), second largest \( m_2 \), and last impactors \( m_{\text{last}} \) to hit final planets with masses greater than \( 0.50 M_\oplus \) are listed (our 10 simulations produced a total of 20 planets with \( m \geq 0.50 M_\oplus \)). Histograms of the mass and angular momentum contributions of the largest impactors to strike these planets are shown in Fig. 14. The largest impactors require time to accumulate and therefore strike the growing planet toward the end of the formation process when the moment arm of the collision will tend to be larger. In general the largest impactors make the greatest contributions to a planet’s spin angular momentum due to both their mass and increased moment arms. The largest and second largest impactors contribute an average of 30 and 19% of the final planet mass respectively. The average contribution to a planet’s spin angular momentum made by the largest impactors was 1.44 \( L_\oplus \), with values fairly evenly spread between 0 and 3.0 \( L_\oplus \). The second largest impactor contributed an average of 0.67 \( L_\oplus \), with a few values approaching 2.0 \( L_\oplus \). For 9 of the 20 planets, the last impactor was the largest. For an equal number of final planets, the last impactor was less massive than both the largest and second largest impactors. In these cases, the last impact contributed an average of only 8.3% of the final planet mass, but contributed an average angular momentum of 0.76 \( L_\oplus \) with values ranging from 0.10 to 1.83 \( L_\oplus \).

For 7 of the 20 final planets more massive than \( 0.5 M_\oplus \), the first potential moon-forming impact was not the last collision they experienced. Five of these planets had at least one more collision with an angular momentum greater than the Earth/Moon system. Further, these planets accreted an average of 31% (range 7–62%) of their final planet mass after experiencing their first potential moon-forming impact. While our statistics are clearly limited by the computational demands of our simulations, our results suggest that the mass and angular momentum of the final planets may be the result of more than one large impact and that moon-forming collisions may occur before planetary formation is completed. We will discuss both of these conclusions in more detail below.
4. ANALYSIS

4.1. Implications for the Origin of the Earth’s Moon

The Giant Impact hypothesis proposes that the Moon accu-
mulated from a circumterrestrial disk of debris that was ejected
into orbit by a collision between the nearly formed Earth and
a Mars-sized impactor. The late stage Monte Carlo calcula-
tions of Wetherill (1992) indicated that impacts by bodies with
mass $m > 0.1M_\oplus$ should occur between $10^7$ and a few times $10^8$ years
from the start of the late stage. The timing of the potential moon-
forming impacts in our simulations are within and toward the
low end of this range, primarily because of the limited radial
extent of our initial conditions. If planetary embryos formed out
into the asteroid belt, the accumulation of the planets could last
longer due to the lower collision frequency in this region and
scattering of embryos by the terrestrial region from the aster-
oid belt. Giant impacts would therefore continue to occur later
in the simulations. The general timing of our largest impacts are
consistent with recent estimates for the timing of the formation
and differentiation of the Moon ($t \sim 50 \times 10^8$ years, e.g., Lee
et al. 1997).

Clearly, two fundamental constraints on models of the for-
mation of the Earth/Moon system are the masses of the bod-
ies and current angular momentum of the system. SPH simu-
lations of the giant impact have typically considered collisions
with impactor-to-target mass ratios of $3:7$ and $2:8$, with no
initial spin angular momentum of the impactor or target, and
with $v_{\text{imp}} = v_{\text{esc}}$ (Cameron 1997, Cameron and Canup 1998).
These studies have investigated a variety of impact angular mo-
menta by changing the impact parameter ($0.5–3.5L_{\oplus-M}$) and
system masses ($0.5–1.0 M_\oplus$) and have demonstrated that glanc-
ing impacts between the two colliding bodies are most efficient
at ejecting a circumterrestrial disk of material into orbit about
the Earth. Recent $N$-body simulations of the accumulation of
the Moon from this disk indicate that a disk mass of at least
two lunar masses is required to form the Moon (e.g., Ida, et al.
1997). The only impacts that SPH simulations have shown to be
able of producing a circumterrestrial disk massive enough
to form the Moon are those with a total mass of $1M_\oplus$, the Earth/Moon system must somehow rid itself of
this excess angular momentum after the Moon-forming event.

As solar tides remove only a small fraction of the system’s an-
gular momentum, this scenario would seem to require that the
Earth experienced another impact to reset the system’s angular
momentum. In the second case, the Earth must acquire another
$\sim 0.35M_\oplus$ of material after the Moon has formed. This scenario
requires that either the system experienced later large impacts, or
that there was a significant amount of material present in a swarm
of smaller planetesimals at the very end of the late stage of planet
formation from which the Earth acquired its additional mass.
The results of our simulations show that multiple impacts may have contributed to the final dynamical state of the Earth/Moon system. First, the angular momenta and masses of the final planets of our simulations are typically not the result of a single collision, and this suggests that impact scenarios with total masses and angular momenta different from that of the Earth/Moon system should be considered. Second, the Moon-forming impact may have occurred at a velocity greater than the escape velocity. Our simulations indicate that the largest impacts occur between 1.00 and 1.63 times the escape velocity. Third, in our simulations planets occasionally experienced glancing collisions at several times the escape velocity with smaller (~0.05M⊕) impactors that were capable of delivering up to 1.1L⊕−M. At present it isn’t clear whether this type of collision would be efficient at producing a massive disk. If this type of collision occurred after the Moon had already formed, it could conceivably reset the spin angular momentum of the Earth. Also, the planetary embryos in our simulations are spinning rapidly throughout the final accumulation stage. In particular, the final planets of our simulations that experienced potential moon-forming impacts had an average angular momentum of ~0.91L⊕−M and were rotating with an average rotation period of 2.4 h prior to these collisions. This result suggests that investigations of moon-forming collisions in which the impactor and/or the target are spinning should be undertaken.

4.2. Comparison with the Terrestrial Planets

Each of our simulations produces about two planets larger than 0.5M⊕ and often an additional smaller planet between 0.5 and 1.5 AU on noncrossing orbits. While the absence of a Mercury-like planet (aMercury = 0.39 AU) is likely due to our choice of initial conditions (i.e., an inner disk edge of 0.5 AU), this general result is grossly similar to the mass and number of the terrestrial planets. However, our planets are different from the terrestrial planets in a number of important respects.

The obliquities and spin periods of the final planets produced by our simulations are quite different from those of the present-day terrestrial planets (see Table II). However, the spin states of the terrestrial planets have undergone significant evolution from their primordial values. Mercury and Venus have rotation periods much longer than those of Earth and Mars. Due to their relatively close proximity to the Sun, the rotation rates and obliquities of the inner planets have undergone significant tidal decay and any memory of their primordial values has been erased (see, e.g., Dones and Tremaine 1993a,b). Earth and Mars have orbits large enough that solar tides are ineffective in altering their spins appreciably. Thus, the magnitude of the spin angular momentum of the Earth/Moon and martian systems can be considered primordial. While the magnitude of the Earth/Moon system’s angular momentum has been roughly constant since its formation, the rotation period of the Earth has increased as tidal interactions have transferred angular momentum from the Earth’s rotation to the Moon’s orbit. Just after the Moon forming impact event nearly all of the system’s angular momentum resided in the Earth’s spin. The rotational period of the Earth in this configuration is about 4.1 h.

The obliquities of the terrestrial planets have also evolved from their primordial values through spin–orbit coupling. Mutual gravitational interactions between the planets cause both their orbits and spin axes to precess. When the spin axis precesses much more rapidly than the orbit, the spin axis tends to precess about the orbit normal. When the orbit precesses much faster than the spin axis, the spin axis tends to precess about the average position of the orbit normal or the invariant plane of the Solar System. When the spin axis precession rate is commensurate with the precession rate of one of the planet’s orbit, the spin axis can fluctuate with large amplitudes. Ward (1973) was the first to point out the importance of these spin–orbit resonances for the obliquity evolution of Mars. Laskar and Robutel (1993) and Touma and Wisdom (1993) have demonstrated that due to overlapping spin–orbit resonances the obliquity evolution of Mars is formally chaotic with possible variations from nearly 0° to ~60°. Furthermore Laskar and Robutel (1993) also show that in the absence of massive natural satellites all of the terrestrial planets could have experienced large-scale obliquity variation in the past. Due to the relatively large mass of the Moon, Earth’s spin axis precesses very rapidly and avoids the orbital precession frequencies of the planets (Ward 1974). It is largely for this reason that Earth’s spin axis currently precesses about the orbit normal with a roughly constant obliquity. Thus, with the exception of the Earth, none of the terrestrial planets retain any memory of their primordial obliquities, and only the Earth/Moon and martian systems have roughly maintained the magnitude of their initial spin angular momenta. It is therefore possible that the primordial obliquities of the terrestrial planets were much different than their current values and may have been comparable to those of the final planets produced by our simulations.

While the Moon currently serves to keep the Earth’s obliquity relatively constant, just after its formation the Moon was a primary perturber of Earth’s spin axis. Touma and Wisdom (1994) show that the recession of the lunar orbit and tidal interactions cause the obliquity of Earth to evolve from an early post-impact value between 8° and 18° to the present 23.5° value (see also Goldreich 1966). When the spin angular momentum of a planet is acquired by large, randomly oriented impacts as we have modeled here, the probability distribution of planetary obliquities is given by Eq. (2). Integrating this relation gives a probability that the Earth’s post-impact obliquity was in the range suggested by Touma and Wisdom (1994) of about 1 in 12, although we note that all such lunar recession calculations assume that the lunar-forming impact was the last impact to significantly affect the Earth’s spin and/or obliquity.

The average rotation period of final planets larger than 0.50M⊕ was about 1.8 h in our simulations, a value just above the stability limit. These periods are also about half of the estimated primordial rotation period of the Earth and less than a
12th that of Mars (4.1 and 24 h respectively). One reason for the rapid rotation produced by our simulations is our assumption of inelastic mergers. High-velocity glancing (high angular momentum) collisions may result in a significant amount of escaping material that would tend to carry away excess angular momentum. In the next section we will discuss the issue of fragmentation in more detail but, suffice to say, that the efficiency at which colliding planetary embryos retain the mass and angular momentum involved is not yet well known and merits further investigation. Also, due to strong cancellation between angular momentum contributions from individual planetesimals, mass accreted from a large number of much smaller bodies will tend to slow the rotation period of the final planets.

The martian rotation period at 24 h is so much longer than the average value of our simulations that its origin must be addressed. It is conceivable that the slow rotation of Mars is the result of the strong cancellation between the angular momentum contributions of multiple impacts as shown in Fig. 13. It should also be noted that Mars is about the same size as the embryos produced by the runaway and post-runaway growth of planetesimals (Weidenschilling et al. 1997) and therefore the size of the initial bodies in our simulations. It has been suggested that Mars could be an unaccreted embryo which survived the late stage of planet formation without colliding with another body (CW98). In this case, Mars acquired its spin angular momentum during the runaway and post-runaway stages of planet formation rather than during the late stage. Recent simulations of the accretion of spin angular momentum from a nonuniform disk of planetesimals (Ohtsuki and Ida 1998) indicate that a growing planet can acquire a significant amount of prograde spin angular momentum by accumulating material from the edges of a gap in the planetesimal disk surrounding the embryo. This mechanism is capable of producing rotation periods even shorter than 24 h and is a viable candidate for explaining the length of the martian day.

In 8 of our 10 simulations, the two largest planets with \( m \geq 0.50M_\oplus \) are adjacent to one another and are separated by an average of 49 mutual Hill radii. This value is a little less than twice the current separation of the Earth and Venus. The large separation of our final planets is almost certainly related to the planets' large eccentricities \((\sim 0.10)\). Early in our simulations the mutual interactions between embryos excites them into higher eccentricity orbits where they collide and merge. These high eccentricities drive the accretion of the planets until their orbits are separated such that they can no longer perturb each other strongly. Thus, with no significant mechanism for damping the final eccentricities of the planets, it should come as no surprise that most of the planets produced by our simulations have large eccentricities. This was also a general result of the simulations performed by CW98. This basic result has two possible implications. First, it could be that the nearly circular orbits of the Earth and Venus are simply rare (Chambers 1998). It has been suggested that the fact that we live on a planet with a low eccentricity may be an observational bias on our part, for if the Earth had a more eccentric orbit it could potentially have larger seasonal variations and the type of life that evolved on such a world could be substantially different than that which inhabits the Earth (Chambers 1998). If systems with a single massive planet with low eccentricity are rare, the requirement of two adjacent, low eccentricity, \( m \geq 0.50M_\oplus \) planets (e.g., Earth and Venus) would seem to make the terrestrial system extremely rare. Another possibility is that we are missing an important piece of physics in our simulations. Recall that we have neglected any background population of smaller planetesimals that may have persisted from earlier stages of accretion and the replenishing of this background via fragmenting collisions in the late stage. The effects of such a background would generally be to decrease embryo and planet eccentricities via dynamical friction and decrease embryo and planet spin rates by contributing mass without significantly adding to the spin angular momentum. Given that the high eccentricities of the final planets produced by our simulations could be the result of one of our simplifying assumptions, it would seem a bit premature to interpret this result as indicative of the uniqueness of the Earth's orbit.

5. DISCUSSION

In the simulations presented here we have made significant simplifying assumptions, including a choice of initial conditions and our treatment of collisions as inelastic mergers without any appreciable fragmentation. In this section we discuss how each of these assumptions may have influenced the results presented.

5.1. Effects of Nonmerging Collisions

To get an idea of the extent to which collisional fragmentation takes place during the late stage, we have made estimates using existing scaling laws for describing collision outcomes. We compare our estimates with the results of recent SPH simulations of the Moon-forming collisional event, estimate the amount of material that would be dispersed in the collisions which occur in our simulations, and discuss the implications of fragmentation in the late stage.

While extrapolations based on laboratory experiments have proved very useful in providing a basic understanding of accretion dynamics and the potential role of competing processes (e.g., Greenberg et al. 1978, Wetherill and Stewart 1993), recent work on collisional modeling has focused on extending the studied size and impact range through the use of hydrocode simulations of impacts (e.g., Ryan and Melosh 1998). A key parameter that determines the fate of a collision is the critical specific energy, \( Q^* \), which is defined as the energy per unit mass required to fragment and disperse half of the target mass (note that this is sometimes now referred to as \( Q_{fr}^* \)). Figure 6 of Love and Ahrens (1996) suggests that the fraction of the target dispersed by a collision scales linearly with the specific energy of
the impact \( \left( \frac{m_{\text{esc}}}{m_i} \right) = \frac{Q}{Q^*} \), where

\[
Q = \frac{1}{2} \left( \frac{m_i m_t}{m_i + m_t} \right) v_r^2
\]

is the specific energy \( m_{\text{esc}} \) is the mass dispersed, \( m_i \) is the impactor mass, \( m_t \) the target mass, and \( v_r \) is the impact velocity. Using this linear expression together with the \( Q^* \) scaling relations of Love and Ahrens (1996) and Melosh and Ryan (1997), we have estimated the fraction of material escaping from each of the collisions that occurred in a typical simulation. The fraction of material dispersed as a function of the total mass involved in each collision is plotted in Fig. 15. The fraction of mass dispersed from each collision is in the range of 5–20% and 0.3–3% respectively for the two models. This would seem to indicate that even the largest and lowest velocity collisions in our simulations result in the dispersal of a few percent of the mass of the colliding embryos in the form of small bodies. Whether the rate that material is returned to the planetesimal swarm via collisional fragmentation is sufficient to maintain a substantially massive background is not well understood and should be studied.

As mentioned earlier, occasionally the angular momentum of a collisional encounter is large enough that merging the two colliding bodies results in a combined body that is rotationally unstable. Clearly an inelastic merger in this case is unphysical. To get a more realistic picture of the outcome of such a collision we turn to one of the recent SPH impact simulations of Cameron and Canup (1998). We examine a simulated collision with a 0.70\( M_\oplus \) target and a 0.30\( M_\oplus \) impactor with 3.2 times the angular momentum of the Earth/Moon system. If this collision were treated as an inelastic merger and all of the angular momentum was transferred to a sphere of uniform density with 100% efficiency, its rotational period would be about 1.5 h. In the SPH simulation of this collision roughly 3% of the total mass escapes the system carrying with it about one-quarter of the system angular momentum. An additional 3% of the mass is lifted into orbit above the proto-Earth carrying with it about 15% of the system angular momentum. When the rotation period of the Earth is calculated given the remaining angular momentum and assuming a spherical shape, a stable rotational period of about 2.5 h results. Roughly 40% of the system angular momentum is not accreted by the merged body. The results of this SPH simulation illustrate that even small fractions of escaping mass can remove significant amounts of collisional angular momentum. Also the picture of angular momentum accretion is complicated further by targets and impactors which are initially rotating. Certainly the amount of material and angular momentum escaping a collision will also depend on whether the colliding bodies were rotating prior to the impact. In the absence of a better understanding of how angular momentum is processed by such collisions it would seem reasonable to assume that our neglect of fragmentation results in a systematic overestimation of the angular momentum retained by a merged body by as much as 50% or more for the most glancing and high-velocity collisions. The rotation periods of our Earth-like final planets are roughly half as long as that of the Earth/Moon system just after the Moon-forming event. Correcting for our assumption of pure accretion of angular momentum may well account for this difference.

The qualitative behavior of a planet’s angular momentum shown in Figs. 10–13 (i.e., significant contributions from multiple impacts, cancellations, and large obliquities) is likely most sensitive to the number of large impacts. So long as the dispersal of a few percent of the mass involved in a collision does not reduce the number of large impacts, we expect that the spin angular momentum of a growing planet would behave in a fashion similar to the results presented here even with a better treatment of collision outcomes, although our future work will address this issue quantitatively.

In general, our neglect of any background population of planetesimals which could damp embryo eccentricities through dynamical friction and our use of inelastic mergers, rather than fragmenting collisions which would tend to replenish the planetesimal swarm, lead to an overestimation of the encounter and impact velocities between embryos and to an overestimation of the spin angular momentum retained by two colliding bodies. Monte Carlo simulations of the late stage that included a parameterization of fragmenting collisions (Wetherill 1992) suggest that
the final planet architectures are not significantly influenced by fragmentation. However, the techniques used in that study were not well suited for modeling embryo–swarm interactions such as dynamical friction. Thus it is possible that the dynamical evolution of the forming planets may be qualitatively changed by the presence of a low mass planetesimal swarm.

As a first-order exploration of this scenario we have developed a simple simulation to roughly account for presence of the planetesimal swarm and the effect of dynamical friction acting on a system of planetary embryos. The drag term of Chandrasekhar (see, e.g., Eq. 7–14 in Binney and Tremaine 1987) was adapted to model a system of planetesimals and was added to the acceleration of each embryo. This term is

$$\frac{d\mathbf{v}_e}{dt} = -\frac{4\pi \ln(1 + \Lambda^2) G^2 M_e \sigma}{v_r^2 \epsilon_{\text{rms}}} \left( e^{ef}(X) - \frac{2X}{\sqrt{\pi}} e^{-x^2} \right) \mathbf{v}_e,$$

where $M_e$ is the embryo mass, $v_r$ is the embryo’s velocity relative to the local circular orbit ($v_r = v_e - v_c$), $\sigma$ is the local surface density, $X = v_r/\sqrt{2\epsilon_{\text{rms}}} v_c$, and $\epsilon_{\text{rms}}$ is the root mean square eccentricity of the planetesimal swarm. Here we have assumed that the root mean square inclination of the planetesimals is one-half of the root mean square eccentricity, which is valid in the high-velocity regime of late stage simulations. The parameter $\Lambda$ measures the radial extent of the planetesimal disk that the embryo interacts with and is approximately described by the following relation where $e$ is the embryo eccentricity:

$$\Lambda = (e^2 + \epsilon_{\text{rms}}^2)^{1/2} e^2 \left( \frac{M_\odot}{M_e} \right).$$

In using this drag term, the surface density ($\sigma$) and dispersion velocity ($\epsilon_{\text{rms}}$) of the planetesimal swarm were treated as constants. The $N$-body simulations of Tanaka and Ida (1997) demonstrate that embryos separated by $\sim 10$ Hill radii are efficient at scattering planetesimals into gaps cleared in the disk by the growing embryos and suggest that the surface density of the planetesimal swarm is smoothly varying with heliocentric distance, at least at the beginning of the late stage. Our treatment of $\sigma$ and $\epsilon_{\text{rms}}$ as constants overestimates the dynamical effect of the planetesimal swarm and allows the swarm to act as an infinite reservoir of angular momentum. We tested this method of modeling dynamical friction with a planetesimal swarm by performing a direct comparison with the $N$-body simulations of Ida and Makino (1992) and found excellent agreement with their rates of eccentricity decay for embryos embedded in a swarm of planetesimals.

To examine the dynamical influence of the planetesimal swarm in our late stage simulations, we have performed preliminary $N$-body simulations beginning with the initial planetary embryos of simulation 1 subject to dynamical friction with a planetesimal swarm with surface density $1.0$ g cm$^{-2}$ at 1 AU and rms eccentricities of 0.05 and 0.30 as modeled by Eq. (3). These backgrounds roughly approximate the material remaining in the planetesimal swarm at the end of the Weidenschilling et al. (1997) multizone simulation (recall that 88% of the system mass was contained in the 22 largest embryos). In these preliminary calculations the inclusion of Eq. (3) in the acceleration of the embryos tended to prevent the type of eccentricity growth shown in Fig. 4 and tended to prevent, or at least postpone, the onset of close encounters between embryos. Although this coarse treatment of the planetesimal swarm does not allow for several relevant processes (e.g., embryos accreting and depleting the planetesimal swarm, the increase of the rms eccentricity of the planetesimal swarm via interactions with the embryos, the clearing of gaps in the disk), the results of these calculations support the idea that planetary embryos could potentially remain dynamically coupled to a low mass planetesimal swarm. Perhaps the multizone simulation of Weidenschilling et al. (1997) is not yet complete in the sense that the embryos will continue to deplete the planetesimal swarm until they do dynamically decouple from it signaling the onset of the late stage. However, if collisional fragments can effectively replenish the planetesimal swarm, then planetary embryos may remain coupled to the swarm during the entire final stage of planet formation. This scenario is supported by the two-dimensional simulations of Alexander and Agnor (1998) that included a simple fragmentation model. In their simulations a few percent of the mass involved in each collision was dispersed in the form of collisional fragments. These fragments quickly replenished the planetesimal swarm such that it contained up to 50% of the system mass when the simulations were terminated.

Thus the persistence and dynamical influence of a planetesimal swarm in the late stages of planet formation may be important to the development and final configuration of the planets. Our future work will involve a study of interactions between planetary embryos and low-mass planetesimal swarms in an effort to better understand the dynamical role played by the swarm during the late stage of planet formation.

5.2. Effects of Initial Conditions

Our choice of initial conditions has assumed that all of the small planetesimals of the middle stage have been swept up and accreted by the planetary embryos and the amount of material remaining in the swarm of smaller planetesimals is negligible. Even in the pure accretion scenario employed in the Weidenschilling et al. (1997) calculation that we have used as our initial conditions, about 12% of the mass present is contained in small bodies after one million years. At present, it is not clear what timescale is required for the embryos to accrete this material. Fragmentation during the middle stage will likely complicate matters further. Wetherill and Stewart (1993) performed PIAB simulations of the growth planetary embryos including gas drag and a parameterization of collisional fragmentation. Their results showed that the production of collisional fragments in combination with gas drag tended to extend the period of
runaway growth of planetary embryos. It is yet unclear how the competing processes of fragmentation, gas drag, and accretion will fare in the higher velocity post-runaway growth phase of the planetary embryos. Thus, our understanding of the number and size of the embryos present at the start of the late stage is an open issue and may change in the future.

How will changes in the number and size of the initial embryos affect the results presented here? Our simulations with different spacing of initial embryos seem to suggest that when the late stage begins with a few tens of embryos in orbits spaced by 10 Hill radii or more that there will in general be fewer moon-forming collisions than when the late stage begins with a swarm of smaller more closely spaced embryos. It would seem that the number of high angular momentum collisions with massive impactors tends to decrease with the number of initial embryos. If the late stage begins with fewer, but larger embryos, the likelihood that a planet will experience significant cancellations in its spin angular momentum will decrease with the number of impacts it experiences. However in all of our cases the embryos had short rotation periods and large obliquities after suffering their first collision. It appears that as long as the terrestrial planets accreted from two or more embryos each they were most likely rotating rapidly with large obliquities throughout the late stage.

We have also neglected the presence of the gas giant planets in our choice of initial conditions. Due to the requirement that they form before the dispersal of the gas from the protoplanetary nebula by the onset of the solar wind, it is generally thought that Jupiter and Saturn must have formed on a timescale less than about 107 years. Since the formation time of the terrestrial planets is significantly longer than this, it seems likely that Jupiter and Saturn were present and perturbing the inner Solar System during the accumulation of the terrestrial planets. In the absence of any stabilizing effects (e.g., a background swarm of planetesimals or gas in the protoplanetary disk), perturbations from the giant planets would tend to increase embryo eccentricities and inclinations and decrease the stability of a system of embryos in the terrestrial region (Ito and Tanikawa 1999). This decreased stability could lead to the growth of larger impactors in the region 0.5–1.0 AU of the disk when the initial embryos are large and well separated as in simulations 1–8. The increase in embryo eccentricities resulting from perturbations of the giant planets would most likely increase the number of large impactors, impact velocities, and the number of high-angular momentum collisions. Thus, the gas giant planets could cause an increase in the number of potential moon-forming events in the terrestrial region. These increased eccentricities would most likely tend to drive the accretion process farther, resulting in fewer final planets with eccentricities even larger than those reported here (CW98), enhancing the difference between the eccentricities of Earth-like planets produced by late stage simulations and those of the observed planets.

On the other hand, the increase in impact velocities induced by the gas giants would also tend to increase the amount of material dispersed in fragmenting collisions, returning more material to the planetesimal swarm and potentially enhancing the ability of dynamical friction to increase the orbital stability of planetary embryos in the terrestrial region. Thus to adequately model the role of Jupiter and Saturn in the formation of the terrestrial planets and in the occurrence of large impacts, both the destabilizing effects (perturbations) and stabilizing effects (increased fragmentation and a planetesimal swarm) should be quantified and included. Our future work will address these issues.

6. SUMMARY

The results of our $N$-body simulations of the late stage of planetary accretion are:

- The accretion of a few final planets from a system of a few tens of planetary embryos with initial low eccentricity and low inclination orbits is typically complete after $10^8$ years. The most common outcome is a pair of planets with $m \geq 0.5M_{\oplus}$ between 0.5 and 1.0 AU and a less massive planet near 1.5 AU.
- The most massive final planets typically have larger eccentricities and are more widely spaced than Earth and Venus.
- Simulations which began with Mars-sized embryos with an initial spacing of about 10 Hill radii had an average of 1.6 impacts with angular momenta greater than that of the Earth/Moon system. Simulations which began with smaller, more tightly spaced initial embryos had an average of 3.5 collisions of this type. The number of massive impactors and high-angular momentum collisions that occur in the late stage is thus somewhat sensitive to the number, spacing, and masses of the initial embryos.
- The final mass and angular momentum state of an “Earth-like” planet is not necessarily the result of a single impact. Typically the last 2–3 collisions a planet experienced made significant contributions. The average contributions to the mass and system angular momentum from the largest and second largest impactors were $(m_1) = 0.30M_{\text{final}}(0.24M_{\oplus})$, $(L_1) = 1.44L_{\oplus-M}$, and $(m_2) = 0.19M_{\text{final}}(0.12M_{\oplus})$, $(L_2) = 0.67L_{\oplus-M}$. For about half of our final planets, the last impactor to strike the planet was neither the largest nor the second largest. In these cases, the average mass and angular momentum contributions of the last impact were $(m_{\text{last}}) = 0.08M_{\text{final}}$ with $(L_{\text{last}}) = 0.76L_{\oplus-M}$.
- There can be significant cancellation between contributions to the spin angular momentum of a planet made by multiple large impacts during the late stage.
- Growing planets are generally spinning rapidly throughout the late stage. The final planets of our simulations that experienced potential moon-forming impacts had spin angular momentum in the range of $0.2–3.1L_{\oplus-M}$ with an average of $\sim0.9L_{\oplus-M}$ immediately prior to these high-angular momentum collisions.

These results have a number of implication for the origin of the Earth/Moon system, notably:

- The angular momentum of the Earth/Moon system could be the result of more than one large impact.
The largest impact the Earth experienced was not necessarily the last.

The proto-Earth may have been spinning rapidly prior to the Moon-forming collision.

Thus, when modeling the Moon-forming impact, total masses and angular momenta different from the values of the present-day Earth/Moon system should be investigated. In addition, models with a proto-Earth that is rotating prior to the Moon-forming event should be studied.

There are a number of indications that fragmentation and small bodies may play a more significant role in the late stage of planet formation than previously believed. The assumption of inelastic mergers during the late stage leads to a systematic overestimation of the angular momentum acquired by planetary embryos via collisions. Specifically, this leads to bodies which spin so rapidly as to be rotationally unstable. Better models of fragmentation that describe collisions between large planetary embryos are needed. Furthermore, fragmentation during the middle and late stage of planet formation may blur the distinction between the period of embryo growth and the final accumulation of the planets.

Given the current lack of knowledge of the rate and manner in which collisional fragments are produced during the late stage of planet formation, we are open to a wide range of speculation on how the final accretion of the planets might actually occur. If leftover debris and collisional fragments created during the late stage are quickly swept up by the planetary embryos and never represent a significant fraction of the total mass of the system, the late stage would likely proceed in a fashion similar to the simulations presented here. It is also possible that a swarm of smaller planetesimals left over from the middle stage of planet formation could be maintained by fragmentation in the late stage. In this case the swarm could have a significant influence on the evolution of the planets. Dynamical friction could reduce the eccentricities of the embryos and slow their accumulation process. One could also imagine a case where most of the small bodies in the terrestrial region are initially swept up by the embryos. When the embryos begin to collide, the fragments produced replenish the planetesimal swarm and reduce embryo eccentricities until the background is depleted and the embryos again perturb each other into crossing orbits where they collide and repeat the process. It is conceivable that such a complex cyclic series of events could also have been at work during the formation of the planets.

While the simplified models of planet formation employed here and elsewhere represent a useful tool for exploring many of the processes involved in planet formation, the manner in which the terrestrial planets actually formed was certainly far more complex. The development of more sophisticated models which include fragmentation and the dynamical influence of a planetesimal swarm would seem essential to addressing the differences between the results of numerical simulations and the actual terrestrial planets.

ACKNOWLEDGMENTS

We thank Stu Weidenschilling, Don Davis, Francesco Marzari, and Keiji Ohtsuki for sharing the results of their multizone simulation with us. We also acknowledge Al Cameron for making the results of his numerous SPH simulations available to us, Glen Stewart for help with the dynamical friction calculations, and also Steve Alexander, Martin Duncan, Dan Durda, Larry Esposito, Jack Lissauer and Bill Ward for their comments and helpful discussions. R.M.C. was supported by the Origins of Solar Systems Program; H.F.L. was supported by the Planetary Geology and Geophysics Program and the Exobiology Program.

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