Energy and Power

Newton’s 2\textsuperscript{nd} law of motion states that \( \text{Force} = \text{Mass} \times \text{Acceleration}, \)

\[
[F = m \cdot a], \quad [\text{N}] = [\text{kg}] \cdot [\text{m} \cdot \text{s}^{-2}]
\]

The force required to lift a mass (overcome its weight) on Earth is

\[
F = m \cdot g
\]

where \( g \) is the acceleration due to gravity. On Earth’s surface, \( g = 9.8 \text{ m} \cdot \text{s}^{-2}. \)

\textit{Work} equals force times distance (displacement, \( d \)),

\[
[W = F \cdot d], \quad [\text{J}] = [\text{N}] \cdot [\text{m}]
\]

Joule is the S.I. unit of energy describing \textit{mechanical work}, \textit{heat}, and \textit{electricity}. The mechanical definition of a Joule is the amount of \textit{energy} expended, or \textit{work} done, to exert a \textit{force} of 1 Newton 1 meter in the direction of the \textit{force}.

\[1 \text{ J} = 1 \text{ N} \cdot \text{m}\]

\textbf{Ex:} Assuming that \( g = 10 \text{ m} \cdot \text{s}^{-2}, \) how many Joules of energy are required to lift 1 kg of water 1 m straight up?

\textbf{A:} First find the force required to overcome the weight of the water:

\[
F = m \cdot g \Rightarrow F = (1 \text{ kg}) \left(10 \frac{\text{m}}{\text{s}^2}\right) \Rightarrow F = 10 \frac{\text{kg} \cdot \text{m}}{\text{s}^2} \Rightarrow F = 10 \text{N}
\]

Next, calculate the work required to exert the above force over 1 meter of lifting:

\[
W = F \cdot d \Rightarrow W = (10 \text{ N})(1 \text{ m}) \Rightarrow W = 10 \text{ J}
\]

Thus, it requires 10 J of energy to lift 1 kg of water 1 m vertically.

\textbf{Ex.} On Earth, it requires \(~10 \text{ J}\) of energy to lift 1 kg, 1 m vertically. How many Joules of energy are required to lift 3 kg of water 2 m?
A: Let $w$ be the number of Joules required to do the work. We can use the above definition of work:

$$W = m \cdot g \cdot d$$

$$w = (3 \text{ kg}) \cdot \left( \frac{10 \text{ m}}{s^2} \right) \cdot (2 \text{ m})$$

$$w = \left( \frac{60 \text{ kg} \cdot \text{m}^2}{s^2} \right)$$

$$w = 60 \text{ J}$$

OR

$$w = \left( \frac{10 \text{ J}}{1 \text{ kg} \cdot 1 \text{ m}} \right) \cdot (3 \text{ kg}) \cdot (2 \text{ m})$$

$$w = \left( \frac{10 \text{ J}}{1 \text{ kg} \cdot 1 \text{ m}} \right) \cdot (3 \text{ kg}) \cdot (2 \text{ m})$$

$$w = 60 \text{ J}$$

Power $[W]$ is the rate at which work is done, i.e. the amount of work done / energy expended per unit time.

$$Power = \frac{work}{time}, \quad [W] = \left[ \frac{\text{J}}{\text{s}} \right]$$

thus, 1 Watt = 1 Joule per second. Lifting 1 kg per second converts 10 J of energy to 10 W of power.

Ex. On Earth, it requires $\sim 10 \text{ J}$ of energy to lift 1 kg 1 m. How many Watts of power are required to lift a 3 kg weight 2 meters per second?

A: Let $p$ be the power (number of Watts) required. Using the value for work calculated above:

$$p = \frac{work}{time} \rightarrow p = \frac{60 \text{ J}}{1 \text{ sec}} \rightarrow p = 60 \text{ W}$$