Study of the influence of propagator amplitude correction on image amplitude using beamlet propagator with local WKBJ approximation

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Summary

Localized WKBJ correction based on Local Cosine Basis (LCB) beamlet propagator is applied in post- and pre-stack migration to study its influence on image amplitude. 2D SEG/EAGE salt model is used for demonstration. The results indicate that localized WKBJ correction have some improvement on the image amplitude though not very dramatic for 2D SEG/EAGE salt data with relatively small-offset acquisition. The image after acquisition aperture correction is greatly improved in the entire model. The acquisition aperture correction has larger effect on the image amplitude for migration with limited acquisition aperture in general heterogeneous media.

Introduction

Traditional one-way wave equation based migration can provide a reflector map consistent with the real subsurface structure, but provides unreliable reflection (or scattering) strength (or image amplitude) of the reflectors. True-reflection imaging tries to give not only correct location but also correct image amplitude of the reflectors. This bridges the traditional imaging methods and the direct inversion of medium parameters. The factors influencing the image amplitude include propagator errors (e.g., focusing and defocusing by heterogeneity, geometrical spreading, path absorption and path scattering loss, numerical dispersion and numerical anisotropy), acquisition aperture effect, and imaging condition, etc..

Among these factors, the image amplitude errors caused by one-way wave propagators have been extensively studied recently (e.g. Zhang, et al., 2003, 2005; Zhang, et al., 2004; Cao & Wu, 2005). The original one-way wave equations cannot provide accurate amplitude even at the level of leading order asymptotic WKBJ or ray-theoretical amplitudes (Zhang, et al., 2003). WKBJ amplitude is then introduced into the original one-way wave propagators (e.g. Zhang, et al., 2003, 2005; Wu & Cao, 2005; Cao & Wu, 2005; Luo et al., 2005). Traditionally WKBJ solution is derived by asymptotic approximation in smoothly varying \( c(z) \) media, where \( c(z) \) is the wave speed at depth \( z \) (e.g. Morse and Feshbach, 1953; Aki and Richards, 1980; Clayton and Stolt, 1981; Stolt and Benson, 1986). It has also been obtained by introducing an extra amplitude term based on the transport equation of high-frequency asymptotics to the traditional one-way wave equations (Zhang, 1993; Zhang, et al., 2003). WKBJ solution is also derived from the conservation of energy flux in smooth \( c(z) \) media and generalized to general heterogeneous media in local angle domain by introducing the concept of “transparent boundary condition” and “transparent propagator” (Wu & Cao, 2005; Cao & Wu, 2005). In general heterogeneous media, there is no global wavenumber, so beamlet method, which has the localized wavenumber and location information, could be used to implement the localized WKBJ correction. Although the transparent boundary condition does not reflect the physical reality, it may be useful and preferred for some inversion or true-reflection imaging procedure because we can conserve all the energy collected by the receiver array to the maximum degree. The concept of transparent propagator is similar to the flux-normalized propagator for general heterogeneous media introduced by Wapenaar and Grimbergen (1996). This transparent boundary condition could be the best strategy for imaging and inversion since we do not want to have further loss of energy for the already weak signals during the imaging process (Wu et al., 2004).

With the “true-amplitude” one-way wave equations, better image amplitude is obtained in common-shot migration (Zhang, et al., 2003, 2005; Cao & Wu, 2005) and common-angle gathers (Zhang, et al., 2004). Most true-amplitude propagators are formulated and implemented in space domain. The localized WKBJ correction in general heterogeneous media proposed by Wu & Cao (2005) is in local angle (wavenumber) domain. Numerical results demonstrated that WKBJ correction can improve the image amplitude in smooth \( c(z) \) media though not as much as acquisition aperture correction does (Cao and Wu, 2005). The calculated impulse responses in general heterogeneous media demonstrate the effect of localized WKBJ correction on wavefield amplitude (Luo et al., 2005). In this paper, we apply the localized WKBJ correction based on LCB beamlet propagator in post- and pre-stack migration to study its influence on image amplitude.

We begin with a brief review of the theory of localized WKBJ correction based on beamlet propagator and the theory of acquisition aperture correction in local angle domain. Then we apply this beamlet propagator with localized WKBJ correction to post- and pre-stack migration
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for 2D SEG/EAGE salt model to study its influence on image amplitude. And we also compare the influences of localized WKBJ correction and acquisition aperture correction on image amplitude.

Localized WKBJ correction based on beamlet propagator

The original WKBJ correction in smoothly varying $c(z)$ media can be written as

$$ P_2 = \frac{\cos \theta_2 \rho_2 c_2}{\cos \theta_1 \rho_1 c_1} = \frac{\rho_2}{\rho_1} \frac{k_1 (c_1)}{k_2 (c_2)}, \quad (1) $$

where $P, \rho, c, \theta, k$ are pressure, density, velocity, propagation angle and global vertical wavenumber, respectively (see Figure 1).

Equation (1) can be generalized to general heterogeneous $c(x, z)$ media based on beamlet propagator in local angle domain (Wu & Cao, 2005; Cao & Wu, 2005; Luo et al., 2005). The total wavefield can be decomposed into beamlets (e.g. Wu et al., 2000)

$$ u(x, z, \omega) = \sum_m \sum_n u(\vec{x}_m, \vec{z}_n, z, \omega) b_{mn}(\vec{x}_m, \vec{z}_n), \quad (2) $$

where $b_{mn}$ is the decomposition basis vector (beamlet), $u(\vec{x}_m, \vec{z}_n)$ is the coefficient of the decomposed beamlet located at space $\vec{x}_m$ and wavenumber $\vec{z}_n$, where $\vec{x}_m = n \Delta x$, $\vec{z}_n = m \Delta z$.

The localized WKBJ correction based on beamlet can be written as

$$ \frac{u(\vec{x}_m, \vec{z}_n, z + \Delta z, \omega)}{u(\vec{x}_m, \vec{z}_n, z, \omega)} = \frac{\rho(\vec{x}_m, \vec{z}_n, z + \Delta z)}{\rho(\vec{x}_m, \vec{z}_n, z)} \frac{k_j(\vec{x}_m, z + \Delta z)}{k_j(\vec{x}_m, z)}, \quad (4) $$

where $k_j(\vec{x}_m, z)$ is the window location $\vec{x}_m$ dependent vertical wavenumber at depth $z$, which satisfies

$$ k_j^2(\vec{x}_m, z) = \omega^2 / \varepsilon^2(\vec{x}_m, z) - \varepsilon^2. \quad (5) $$

Figure 1: Diagram for WKBJ correction.

Acquisition aperture correction in local angle domain

Wu et al. (2004) proposed an amplitude correction method with acquisition aperture correction in local angle domain. The numerical examples showed significant improvement in both the total strength images and the angle-dependent reflection amplitudes, which demonstrated the significance of aperture correction in true-reflection imaging. The imaging condition (for a single frequency) in local angle domain (Wu & Chen, 2002, 2006) can be written as,

$$ L(\vec{x}, \vec{y}, \vec{z}) = 2 \sum_{x_1} G_j(\vec{x}, \vec{y}; x_1) \int_{\Delta x_1} dx_2 \frac{CG_j(\vec{x}, \vec{y}; x_2)}{\partial z} \tilde{u}(x_1, x_2) \quad (6) $$

where $G_j$ is Green’s function used in the imaging process, which could be different from the Green’s function of forward modeling; $\vec{y}$ and $\vec{z}$ are the source and receiving angles, respectively; “*” stands for complex conjugate; $G_j(\vec{x}, \vec{y}; x_1)$ is the incident wavefield in the local angle domain at the imaging point $\vec{x}$; and the integral is a back propagation Rayleigh integral, $\tilde{u}(x_1, x_2)$ is the spatial receiver aperture and $u(x_1, x_2)$ is the recorded scattered waves at receiver $x_2$ from the source at $x_1$ on the surface. The relevant amplitude correction factor matrix $Fa$ for above imaging condition (6) is,

$$ \left| F_a(\vec{x}_1, \vec{y}_1; \vec{x}_2, \vec{y}_2) \right| = 2 \sum_{x_1} \left| G_j(\vec{x}_1, \vec{y}_1; x_1) \right| G_j(\vec{x}_1, \vec{y}_2; x_1) \left| \frac{\int_{\Delta x_1} dx_2 \tilde{u}(x_1, x_2)}{\partial z} \tilde{u}(x_1, x_2) \right|^{1/2}, \quad (7) $$

where $G_j$ is the Green’s function in forward modeling. Since the superior performance of the dip-angle domain correction scheme (Wu & Luo, 2005), we will apply the acquisition aperture correction in dip-angle domain here.

Imaging with localized WKBJ correction based on LCB beamlet propagator

In this part, we will apply the LCB beamlet propagator with localized WKBJ correction to post- and pre-stack migration for 2D SEG/EAGE salt model (Figure 2) to study the influence of localized WKBJ correction on image amplitude. We will also compare its influence with that of acquisition aperture correction on image amplitude.

(1) Post-stack migration

Post-stack migration is a wave back-propagation process, so it can directly show the effect of localized WKBJ correction on the amplitude of the propagator. Figure 3 shows the post-stack migration result before and after localized WKBJ correction. The image amplitude is obviously improved in the entire model, especially that of the steep faults in the sediment, the salt boundary and the subsalt structures. WKBJ solution is a transparent propagator (or energy-conservative propagator), which neglects all the scattering (reflection) loss during propagation so we can conserve all the energy collected by the receiver array to the maximum degree. The side-effect of WKBJ correction is that the artifact is also amplified.

Figure 2: 2D SEG/EAGE salt model
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Figure 3: Post-stack image for 2D SEG/EAGE salt model before and after localized WKBJ correction.

(2) Pre-stack migration

Figure 4 shows the pre-stack migration result with convolution imaging condition before and after localized WKBJ correction. Similar to post-stack case, the image amplitude is improved in the entire model and also with stronger artifact. To make the image amplitude closely represent the reflection strength, the image is normalized with the square of incident wave amplitude in local dip angle domain. The normalized images with/without the localized WKBJ correction (Figure 5) give very similar amplitudes. The smaller-offset acquisition for the SEG/EAGE salt model data (maximum offset 14000 feet) may be the reason of the relatively weak effect of WKBJ correction on image amplitude. The small-offset acquisition can only acquire reflected waves with small reflection angles from the dipping reflectors, which makes the paths of incident wave and reflected wave be quite similar. And the lateral velocity variation is not very dramatic either. Therefore the effect of WKBJ correction for the incident and reflected wave is similar, which makes the image amplitude correction less dramatic. However, we notice the image amplitude increase for the subsalt steep faults. This will be seen clearer later in conjunction with the aperture correction.

Finally, localized WKBJ correction and acquisition aperture correction are both applied to pre-stack migration. The result with both corrections (Figure 6b) shows similar image to that with only acquisition aperture correction (Figure 6a). WKBJ correction also amplifies the artifact in subsalt area. Compared with the image before acquisition aperture correction (Figure 4a), the image after acquisition aperture correction (Figure 6a) is greatly improved in the entire model. The image for the steep faults in the sediment is sharper and more continuous. For subsalt structures, the image along the steep sand structures and the baseline are much more uniformly distributed after the correction. And the noises in the subsalt region caused by salt body multiples are also reduced. This shows that the acquisition aperture correction has larger effect on the image amplitude for 2D SEG/EAGE salt model. This agrees with the result in smooth c(z) media (Cao & Wu, 2005). One noticeable improvement in image amplitudes is the two steep subsalt faults directly beneath the salt body. We have improved the phase accuracy of the LCB beamlet propagator, so that the reflected signals from those reflectors can be better focused.

Figure 4: Pre-stack image for 2D SEG/EAGE salt model before and after localized WKBJ correction.

Figure 5: Pre-stack image for 2D SEG/EAGE salt model before and after localized WKBJ correction. The image is normalized with the square of incident wave amplitude in dip angle domain.
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after back-propagating through the irregular salt body. The WKBJ correction helps in restoring the strength of large-angle signals. As a result, the image amplitudes of these steep faults are increased, although the noise background is also raised.

Conclusion

We apply localized WKBJ approximation based on LCB beamlet propagator in post- and pre-stack migration for 2D SEG/EAGE salt model to study its influence on image amplitude. The results indicate that localized WKBJ correction have some though not very dramatic improvement on the image amplitude for 2D SEG/EAGE salt model. The smaller-offset acquisition and not very dramatic lateral velocity variation for the SEG/EAGE salt model data may be the reason of the relatively weak effect of WKBJ correction on image amplitude. The side-effect of WKBJ correction is the artifact is also amplified. With acquisition aperture correction the image is greatly improved in the entire model. The acquisition aperture correction has larger effect on the image amplitude for migration with limited acquisition aperture in general heterogeneous media. The combined amplitude corrections of localized WKBJ and acquisition aperture can give the best result, especially for the weak steep reflectors directly beneath the salt body in the case of SEG/EAGE salt model.

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References

EDITED REFERENCES
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