Local-angle domain illumination for full-wave propagators
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Summary
We propose an efficient split-step local plane wave decomposition method to obtain the local-angle domain (LAD) illumination in the frequency domain for full-wave propagators. For 2D model, firstly we decompose the full waves along vertical direction using 1D decomposition to separate the downgoing and upgoing waves; then apply 1D decomposition along horizontal direction to the separated downgoing or upgoing waves to get corresponding LAD illuminations. The proposed method can obtain the illumination for turning waves, refracted and reflected waves. It can be extended to 3D case with 1D decomposition along vertical direction and 2D decomposition along horizontal coordinates. We exemplify the method with directional illumination (DI) and acquisition dip response (ADR) analysis in 2D SEG/EAGE salt model. We compare the results with those from more expensive 2D local slant stack (LSS) decomposition method. The results from the proposed method are almost the same as those by the 2D decomposition method. Further we investigate the influence of multiples on the illumination strength.

Introduction
Illumination analysis can be used for survey design (e.g., Li and Dong, 2006). An uneven illumination also produces a distorted image. Illumination analysis is a powerful tool to study the influences of acquisition geometry and overburden structures on the image (e.g., Jin and Walraven, 2003; Xie et al., 2006). More accurate amplitude variation with angle (AVA) could be obtained if the image is corrected with angle dependent correction factor (e.g., Wu et al., 2004; Cao and Wu, 2005; Cao and Wu, 2008a).

Traditionally, illumination analyses have used the ray-based method (e.g., Schneider and Winbow, 1999; Bear et al., 2000; Muerdter et al., 2001; Muerdter and Ratcliff, 2001a, b). The ray-based method is convenient and can provide both intensity and directional information carried in the wavefield. However, the high-frequency asymptotic approximation and the caustics inherent in ray theory may severely limit its accuracy in complex regions (e.g., Hoffmann, 2001). Furthermore, the ray method seemingly can provide angle-dependent illumination at any point; however this over-precise illumination map does not reflect the real behavior of the wavefield since it violates the Heisenberg uncertainty principle: The position and direction of a wavefield cannot be specified accurately at the same time (Wu and Chen, 2006). To obtain reliable and frequency-dependent illumination, we need a wave theory based method.

One-way wave equation based propagators are widely used in imaging and illumination analysis. Although they neglect multiples, they properly handle multi-forward-scattering phenomena. They can extrapolate the wavefield very fast and with accurate phase within certain angle range. This makes them suitable for seismic migration. However, the inability to provide localized-angle information prevented them from being utilized for illumination calculation. Recently developed techniques, such as the LSS (e.g., Xie and Wu, 2002) and the beamlet decomposition (e.g., Wu and Chen, 2002), can decompose the wavefield into local plane waves, which are simultaneously localized in space and direction. These methods are independent on the extrapolators. The LAD illuminations have been obtained by LSS (Xie et al., 2003, 2004, 2006), Gabor-Daubechies frame (GDF) beamlet decomposition (Wu and Chen, 2002, 2006; Cao and Wu, 2008a), and local exponential frame (LEF) beamlet decomposition (Mao and Wu, 2007; Cao and Wu, 2008a).

However, the amplitude of the conventional one-way wave propagator is not accurate (e.g., Zhang et al., 2003, 2005; Wu and Cao, 2005; Cao and Wu, 2005, 2008b). Even with some corrections (e.g., Zhang, 1993; Zhang et al., 2003, 2005; Kiyashchenko et al., 2005), the one-way propagator still cannot give accurate wavefield amplitude in complex models with sharp contrast. The numerical implementations based on the one-way wave equation with z-axis as the preferred propagation direction always have inherent limitation in wide-angle accuracy. Therefore, the illumination analysis with one-way propagators may cause errors in the acquisition survey design and true-reflection imaging correction in the complex model.

The full-wave propagator e.g., finite-difference (FD) and finite-element method, can simulate the accurate wave behavior in complex media. It provides more accurate and complete information in illumination analysis for survey design and true-reflection imaging correction. Considering the expensive computation for time-domain full-wave modeling, huge storage requirement for output seismograms needed in time-domain illumination calculation, and the fact that the illumination is frequency-dependent, we propose to analyze the illumination in the frequency domain. We can use frequency domain full-wave forward modeling methods (e.g., Lysmer and Drake, 1972; Marfurt, 1984; Marfurt and Shin, 1989; Jo et al., 1996; Shin and Sohn, 1998; Stekl and Pratt, 1998; Min et al., 2000) to calculate the wavefield for interested frequencies. Using 1D decomposition technique LSS, Luo et al. (2004) compared the single frequency illumination using one-way wave propagator and full-wave propagator (where single frequency full waves are extracted...
Full wave illumination

from time-domain FD modeling). However, artificial interference patterns exist in the illumination results from the full-wave propagator. The reason is LSS was applied only along the horizontal coordinate. The downgoing and upgoing waves are mixed together, making the amplitude of the illumination incorrect and causing the interference patterns. Xie and Yang (2008) and Yang et al. (2008) proposed an illumination method which uses the full-wave FD method as the propagator and uses a time domain local slowness analysis method to determine the angle information and calculate the illumination. It is particularly useful to provide illumination analysis for reverse time migration.

Here we propose an efficient split-step method using 1D/2D decomposition to obtain the LAD illumination in the frequency domain for the 2D/3D full-wave propagator. We show illumination results in 2D SEG/EAGE salt model (Aminzadeh et al., 1994; Aminzadeh et al., 1995) with the proposed method and compare them with the results from more expensive 2D LSS method. We also discuss the influence of multiples on the illumination.

Split-step decomposition for full waves

The local plane wave decomposition techniques, such as LSS method and beamlet decomposition method, were applied along the horizontal coordinate(s) (e.g., Xie and Wu, 2002; Wu and Chen, 2002; Xie et al., 2003). In this case, the local plane wave for a given local horizontal wavenumber includes not only the wave with positive vertical wavenumber (propagating downward) but also corresponding negative vertical wavenumber (propagating upward). In one-way propagators, waves only propagate along one primary direction; therefore decomposition techniques applied only along horizontal coordinate is appropriate. However, for the full wave propagators, they include both the down- and up-going waves. The decomposition applied only along horizontal coordinate will mix the down- and up-going waves, resulting in incorrect illumination amplitude and artificial interference patterns in the illumination map for full wave propagators (see, e.g., Figure 1; in all figures, red and blue mean strong and weak illumination respectively).

For 2D model, one direct way to obtain the LAD illumination for full waves is using 2D LSS or beamlet decomposition; however, 2D decomposition costs much more computation than 1D decomposition. Here we propose a split-step decomposition method to obtain the LAD illumination for the full-wave propagator: firstly, we decompose the full-wave along vertical direction using 1D technique to separate the downgoing and upgoing waves; then apply 1D decomposition along the horizontal direction to the separated downgoing or upgoing waves to obtain corresponding LAD illuminations. This method can be extended to 3D case with 1D decomposition along vertical direction and 2D decomposition along horizontal coordinates.

For the first step in the proposed method, we need to separate the waves with positive and negative vertical wavenumbers. With beamlet decomposition, it is very efficient. Both GDF and LEF beamlets bear uniquely defined directional localization; so they can be used for this step. However, GDF beamlet decomposition can provide more accurate directional wavefield than LEF beamlet decomposition (Cao and Wu, 2008a). Therefore, we use GDF beamlet decomposition in this step. For the second step, we can use either the LSS method or more efficient method with GDF beamlet decomposition (Cao and Wu, 2008a).

Local-angle domain illumination results

We exemplify the method with illumination, including DI and ADR, for the full-wave propagator in the 2D SEG/EAGE salt model. In the following, we will only consider the dominant frequency (15 Hz) for both DI and ADR calculations. Figure 2 shows the DI results for incident angles 0º, +40º and -40º from the proposed method. The results are similar to those obtained from the much more expensive 2D LSS method (Figure 3). DI for incident angles from -60º to +60º at different location along depth level z=3.2675 km shows that these LAD wavefield obtained from the split-step decomposition method are indistinguishable from those by the 2D LSS method (Figure 4). For ADR, the results for dip angles 0º, +30º and -30º from the proposed method (Figure 5) are also similar to those obtained by the 2D LSS method (Figure 6). ADR for dip angles from -60º to +60º at different location along depth level z=3.2675 km also show that the result from the split-step method is almost the same as that from the 2D LSS method (Figure 7).
Full wave illumination

Figure 2: DI for full waves from the proposed method for different incident angles: (a) 0°; (b) +40°; (c) -40°.

Figure 3: DI for full waves from 2D LSS for different incident angles: (a) 0°; (b) +40°; (c) -40°.

Figure 4: DI for incident angles from -60° to +60° at different location along depth level z=3.2675 km: (a) from the proposed method; (b) from 2D LSS.

Figure 5: ADR for full waves from the proposed method for different dip angles: (a) 0°; (b) +30°; (c) -30°.

Figure 6: ADR for full waves from 2D LSS for different dip angles: (a) 0°; (b) +30°; (c) -30°.

Figure 7: ADR for dip angles from -60° to +60° at different location along depth level z=3.2675 km: (a) from the proposed method; (b) from 2D LSS.

Discussion

Above proposed split-step decomposition method can separate the down- and up-going waves. The downgoing waves include not only the primary incident waves but also multiples; and the primary incident waves include the first arrival and multi-arrivals. Figure 8 shows the DI maps for the most energetic waves. Comparison with the DIs for the full downgoing waves (Figure 2) shows that the other arrivals (multi-arrivals and multiples) provide extra illumination to the subsurface. The migration methods based on one-way propagators utilize not only the first arrival but also the multi-arrivals. Therefore the multi-arrivals should be included in the illumination analysis for survey design and true-reflection imaging correction. However, multiples especially the internal multiples should be eliminated since most of the migration methods based on the one-way propagator only utilize primary incident waves. It is
impossible to separate multiples from the other arrivals in the frequency domain. In time domain, we can separate the first arrival or the most-energetic arrival from other arrivals by time windowing. However, we cannot separate multiples from multi-arrivals by time windowing.

In general media, it is hard to obtain the pure multiple data or remove them from the full data. The one-way and one-return boundary element method in the frequency domain (He and Wu, 2007) can calculate the primary transmitted waves and multiples for layered model or inclusion model. Here we exemplify the influence of internal multiples on the illumination strength with a simplified SEG/EAGE salt model. A homogeneous salt body with the same shape as that in the original 2D SEG/EAGE salt model is embedded in a homogeneous background medium. The velocity of the background and the salt are 7800 ft/s and 14700 ft/s respectively. This is a scalar wave model. For acoustic wave model, the salt internal multiples will be weaker since the density of the salt is usually lower than that of the background sediment. DI maps in subsalt region for incident angles -10º and -60º for the primary incident waves and salt internal multiples (Figure 9) show that the maximum amplitude of the primaries is more than six times stronger than that of the multiples for these two angles. Therefore the contribution of the multiples to the illumination strength could be considered as a secondary effect here. However, we can also notice the difference in the spatial distribution of the illumination between the results for multiples and those for primaries. For example, for -60º incident angle, the primaries only strongly illuminate the left part of the subsalt; however, the multiples illuminate the whole subsalt more evenly and provide extra illumination to the shadow in the illumination by primaries (right part in subsalt).

Conclusions

We propose an efficient split-step local plane wave decomposition method to obtain the LAD illumination in the frequency domain for full-wave propagators. It can be used for accurate survey design and true-reflection imaging correction. It can obtain the illumination not only for the downgoing waves but also for the upgoing waves including turning waves, refracted and reflected waves. DI and ADR results from the proposed method in the 2D SEG/EAGE salt model are almost the same as those by the more expensive 2D decomposition method. In a simplified SEG/EAGE salt model, the illumination strength from salt internal multiples could be considered as a secondary effect compared with that from primary incident waves; however, the multiples can illuminate the whole subsalt more evenly and provide extra illumination to the shadow in the illumination by primaries.

The proposed method has the following features: 1. It is in the frequency domain, so it can provide frequency-dependent illumination and the frequency-domain full wave forward modeling is usually efficient and storage-saving to provide the wavefield for given frequencies compared with the time-domain method; 2. It is efficient compared with the full 2D/3D decomposition since we only need three 1D decompositions for 2D model and two 2D plus one 1D decompositions for 3D model; 3. It is independent of the propagator used for full-wave modeling. We conclude that the proposed split-step local plane wave decomposition method for full-wave propagators provides a flexible, efficient and accurate tool for the LAD wave-theory based illumination analysis in complex 2D and 3D models.

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Figure 8: DI for the most energetic waves from the proposed method for different incident angles: (a) 0º; (b) +40º; (c) -40º.

Figure 9: DI for the transmitted waves and salt internal multiples in the simplified SEG/EAGE salt model: (a) transmitted waves for -10º; (b) multiples for -10º; (c) transmitted waves for -60º; (d) multiples for -60º.
EDITED REFERENCES
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