NUMERICAL SIMULATIONS OF STELLAR CONVECTIVE DYNAMOS.
II. FIELD PROPAGATION IN THE CONVECTION ZONE

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ABSTRACT

We present numerical simulations of nonlinear, three-dimensional, time-dependent, giant-cell stellar convection and magnetic field generation. The velocity, magnetic field, and thermodynamic variables satisfy the anelastic magnetohydrodynamic equations for a stratified, rotating, spherical shell of ionized gas. The interaction of rotation and convection produces a nonlinear transport of angular momentum that maintains a differential rotation in radius and latitude. At the surface, our simulated angular velocity peaks in the equatorial region in agreement with Doppler measurements of the solar surface rotation rate; below the surface, it decreases with depth in agreement with what is inferred from the rotational frequency splitting of solar oscillations. The interaction of rotation and convection also maintains left-handed helical fluid motions in the northern hemisphere and right-handed motions in the southern hemisphere. Magnetic fields are generated by the shearing and twisting effects of the differential rotation and helical motions and are destroyed by eddy diffusion. They in turn feedback onto the velocity and thermodynamic fields via the Lorentz force and Joule heating. Although we have not continued the computation long enough to simulate a complete magnetic cycle, our solutions demonstrate how the induced magnetic fields propagate away from the equator in the opposite direction inferred from the solar butterfly diagram. We suggest that, instead of operating in the turbulent convective region, the solar dynamo may be operating at the base of the convection zone where our simulated helicity has the opposite sign and a smaller amplitude.

Subject headings: convection — hydromagnetics — stars: interiors — stars: magnetic fields — Sun: interior

I. INTRODUCTION

We would like to simulate the Sun’s magnetic cycle with our dynamo model in order to better understand the physics behind this long-observed phenomenon. The solar butterfly diagram is a record of where, in solar latitude, sunspots have been observed as a function of time. The record clearly depicts the 11 yr period in the total sunspot area and the equatorward drift of the active solar latitude during each 11 yr. The Sun’s 22 yr magnetic cycle (Hale 1924) was discovered by observing the polarity configuration of sunspot groups. Sunspots presumably result from the emergence of buoyant flux loops that have broken away from a main toroidal magnetic field below the surface. Two oppositely directed toroidal magnetic fields, one in each hemisphere (Babcock 1961), are assumed to propagate from midlatitude to the equator in 11 yr, followed by a similar scenario during the next 11 yr with the opposite polarity (Yoshimura 1976b).

Kinematic dynamo models (Soward and Roberts 1977; Moffatt 1978; Parker 1979; Stix 1981; Krause and Rädler 1981) obtain an axisymmetric magnetic field from a linear magnetic induction equation that is tuned by independently parameterizing the effects of differential rotation and helicity. With this freedom a highly parameterized kinematic model can generate a dynamo wave that propagates toward the equator with about the right period. Yoshimura (1983) has taken kinematic dynamo theory a step further by solving a three-dimensional, linear, magnetic induction equation. However, the velocity field is prescribed instead of being obtained from the hydrodynamic equations.

Dynamic dynamo models simultaneously and self-consistently solve the magnetohydrodynamic equations for the velocity, thermodynamic, and magnetic fields with full nonlinear feedback. The first dynamic dynamo simulations (Gilman and Miller 1981; Gilman 1983) were for a Boussinesq fluid which has no basic density stratification. Although this model successfully simulates self-excited dynamo action and magnetic cycles, the magnetic fields propagate away from the equator with a period an order of magnitude shorter than the observed 22 yr solar period.

We examine in this paper the properties of a dynamic dynamo for an anelastic gas with a significant density stratification. The model, numerical method, and solutions are described in Paper I (Glatzmaier 1984) and are briefly reviewed in § II. In § III we discuss the maintenance of the differential rotation and helicity and, in § IV, the maintenance of the magnetic fields.

II. THE MODEL AND SOLUTIONS

We model a spherical shell of ionized gas constrained by the solar gravity, luminosity, composition, and average rotation rate. For this study, we have set the top and bottom boundaries at 93% and 46% of the solar radius, respectively. There are seven pressure scale heights across this shell with the outer two-thirds (in radius) superadiabatic, i.e., convectively unstable, and the inner one-third subadiabatic. The magnetohydrodynamic equations are solved within the anelastic approximation which filters out the short time scale, small-amplitude acoustic waves while retaining the effects of a large density stratification.

We expand the velocity, magnetic field, and thermodynamic variables in spherical harmonics, \( Y_m^l(\theta, \phi) \), and Chebyshev polynomials, \( \mathcal{T}_n(r) \), with \( 0 \leq |m| \leq l \leq 31 \) and \( 0 \leq n \leq 32 \). Since we have increased the depth of our shell over that used for Paper I, we have increased the number of Chebyshev poly-
nominals (and radial mesh points) from 17 to 33. However, since the full spherical shell solutions have manifested very strong symmetry with respect to the equator (due to the effect of the Coriolis force), we have imposed this symmetry on the solutions presented in this paper in order to conserve computer time. That is, the latitudinal component of velocity and the radial and longitudinal components of the magnetic field vanish in the equatorial plane. A second-order semillicitic time integration scheme is employed; and at each step nonlinear terms are computed in physical space while spatial derivatives are computed analytically in spectral space. The details of our model, the equations, and numerical method are described in Paper I.

Whereas a large spectrum of global fields are explicitly resolved, the nonlinear effects of the smallest, unresolved scales are parameterized via viscous, thermal, and magnetic eddy diffusion. In this study we employ subgrid-scale eddy diffusivities (Smagorinsky 1963) which are proportional to the local resolved velocity deformation. Physically, a greater large-scale velocity deformation should generate larger turbulent velocities which should be more effective in diffusing momentum, heat, and magnetic fields. Our subgrid-scale eddy diffusivity is defined as

\[ \nu = (c\Delta)^2 (2e_{ij} e_{ij})^{1/2}, \]

where \( c \) is the subgrid-scale eddy coefficient, \( \Delta \) is a representative grid interval, and \( e_{ij} \) is the local rate of strain tensor. We either set \( c = 0.2 \) (Deardorff 1971), or we prescribe the value of the viscous diffusivity at the top boundary and, using the above expression, determine the value of \( c \) (as a function of time) which is usually very close to 0.2. Since our implicit treatment of the diffusion terms (Paper I) requires the diffusivities to be functions of radius only, we use the horizontally averaged part of the above expression. If one wanted a subgrid-scale diffusivity that depended on radius, latitude, and longitude, the horizontally averaged part could be treated implicitly while treating the relative variations in latitude and longitude explicitly. We normally average our radially dependent diffusivity over \( \sim 100 \) time steps and then fit it to a smoother polynomial in \( r \) (to avoid numerical instabilities) before updating the three eddy diffusivities. Typically, it decreases monotonically by a factor of \( \sim 50 \) from the top boundary to the bottom due to the effects of the density stratification and the entropy gradient. The relative amplitudes of the viscous \( \nu \), thermal \( \kappa \), and magnetic \( \tilde{\eta} \) diffusivities are established by prescribing the ratios \( \nu/\kappa \) and \( \tilde{\eta}/\kappa \). Typically, at the center of the shell, \( \nu, \kappa, \) and \( \tilde{\eta} \) are \( 1 \times 10^{12}, 5 \times 10^{12}, \) and \( 1 \times 10^{12} \) cm s\(^{-1}\), respectively. Finally, we have found that increasing the amplitude of the diffusivities as a function of the spherical harmonic degree \( l \), i.e., enhancing diffusion for the smaller (resolved) scales, was required at times to prevent the buildup of energy near the cutoff of the spectrum. Although this was done to compensate for the truncation of our spherical harmonic expansions, physically it seems reasonable that eddy diffusion should be more effective when the unresolved eddies and the resolved motions have approximately the same length scales.

With this brief review of the model, we will give a brief summary of the properties of our numerical solutions. Usually \( \sim 70\% \) of the total kinetic energy (relative to the rotating frame) is in the differential rotation, \( \sim 30\% \) in the non-axisymmetric convection, and \( \sim 0.1\% \) in the meridional circulation. As suggested in Glatzmaier and Gilman (1982), we have found that a Prandtl number \( \nu/\kappa \) less than unity maintains a small meridional circulation relative to differential rotation, as observed on the Sun. The magnetic energy tends to be more time dependent than the kinetic energy. Typically, the total magnetic energy is three orders of magnitude smaller than the total kinetic energy, with \( \sim 85\% \) in the axisymmetric toroidal field, \( \sim 10\% \) in the nonaxisymmetric magnetic field, and \( \sim 5\% \) in the axisymmetric poloidal field.

Our maximum convective velocity is usually \( \sim 200 \) m s\(^{-1}\) with a peak in the energy spectrum (for both \( l \) and \( m \)) typically occurring between wave numbers 10 and 15. This simulated convective velocity is more than an order of magnitude greater than the observational limit on giant cell (Doppler) velocities at the solar surface (Snodgrass and Howard 1984). However, our top boundary has been placed several pressure scale heights below the solar surface (at 93% of the solar radius) in order to avoid the high resolution that would be required near the surface where the pressure scale height becomes very small. In addition, if the giant cell velocity pattern on the Sun is time dependent as our simulations suggest, the detection of giant cells on the solar surface may be quite difficult. Note that the peak occurring at \( l = 11 \) in the frequency splitting of solar oscillations (Duvall and Harvey 1984), which is difficult to interpret as a local peak in the rotation rate, could be a signature of solar giant cell convection below the surface (Weiss 1984).

Our simulated meridional circulation usually peaks at \( \sim 5 \) m s\(^{-1}\) at the top boundary near \( \pm 30^\circ \) latitude where it is directed poleward in both hemispheres. This agrees fairly well with Doppler measurements of the meridional velocity on the solar surface (Snodgrass 1984). However, our simulated meridional circulation is usually composed of at least two major cells below the surface in each hemisphere with a downflow at mid-latitude (Fig. 5 of Paper I). As a result, our simulated temperature profile at the top boundary has a maximum in the equatorial region and minima at about \( \pm 50^\circ \) latitude with a typical temperature difference of \( 2 \) K. This agrees fairly well with recent measurements of the latitudinal temperature variation on the solar surface (Kuhn, Libbrecht, and Dicke 1985).

Our simulated magnetic field strength is \( \sim 50 \) G at the top boundary and has a structure, determined by the giant cells (Paper I), that is similar to the large-scale magnetic field structure observed on the solar surface (Bumba 1976; Howard 1977). The resulting Maxwell stress, which inhibits differential rotation, is typically 10 times smaller than the large-scale Reynolds stress which maintains the differential rotation.

### III. THE VELOCITY FIELD

The two major factors in cyclic dynamo models (Parker 1955) are differential rotation which generates toroidal magnetic fields from poloidal magnetic fields and helicity of the convective motions which generates poloidal fields from toroidal fields. Since the phase propagation of the magnetic field depends on the sign and amplitude of the differential rotation and helicity, we will describe how the effects of rotation, spherical geometry, and density stratification maintain these velocity fields in our simulations. Motions perpendicular to the rotation axis are favored, especially in the equatorial region, because the pressure gradient and buoyancy forces can be balanced more easily by the Coriolis forces which are perpendicular to the rotation axis. Consequently, to first order, convection tends to take the form of north-south rolls sandwiched between meridian planes. However, due to the spherical geometry, the cross sections of these rolls are greatest in the equatorial region.

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where, for a given radius, two meridian planes have their greatest separation. In addition, the Coriolis forces on the circulating fluid tend to increase or decrease the cross section of the roll depending on the sense of the circulation. Consequently, latitudinal pressure gradients are set up that cause the circulating fluid to drift away from the equatorial plane (in the direction of converging meridian planes) if the Coriolis forces are trying to decrease the cross section of the roll, and likewise the fluid drifts toward the equatorial plane if it has the opposite sense of circulation (Glatzmaier and Gilman 1981). This is illustrated schematically in Figure 1a.

Therefore, as a result of rotation and spherical geometry, the fluid in the outer part of the shell having a component of velocity in the direction of rotation relative to the rotating frame, i.e., a positive contribution to angular momentum, also tends to have a component of velocity toward the equator (Fig. 1a), and fluid having a component of velocity in the opposite direction of rotation relative to the rotating frame, i.e., a negative contribution of angular momentum, tends to drift away from the equator. Consequently, there is a net latitudinal flux of angular momentum, \( \langle r \sin(\theta) \rho v_r v_\theta \rangle \), toward the equator in both hemispheres. (The brackets represent averages in longitude, and the variables have their usual meanings.) In addition, because of mass conservation and density stratification, this effect is greatest in the outer part of the shell where the mass density is small and the fluid velocity is large.

There is also an angular momentum flux in radius, \( \langle r \sin(\theta) \rho v_r \rangle \), which is determined by the cross-sectional shape of the north-south rolls. The rolls tend to be tilted in the direction of rotation (Fig. 3 of Paper I); that is, rising fluid rotates faster than sinking fluid. Consequently, there is a net radial angular momentum flux toward the surface.

It is the convergence of this angular momentum flux that maintains the radial and latitudinal variation of the rotation rate, i.e., the differential rotation. Actually, the profile and time dependence of the differential rotation is also determined by the Coriolis forces resulting from the meridional circulation and the viscous and Lorentz forces which try to maintain solid body rotation. However, the dominating factor is the flux of angular momentum which, in our simulations, converges in the outer part of the shell in the equatorial region.

Therefore, due to the effects of rotation, spherical geometry, and density stratification, large-scale convection maintains, via the nonlinear transport of angular momentum, an angular velocity that peaks in the equatorial region and decreases with depth. A typical profile of our simulated differential rotation is illustrated in Figure 2a where positive (negative) angular velocity relative to the rotating frame of reference is plotted with solid (broken) contours. It is similar to profiles obtained from nonlinear Boussinesq simulations (Gilman and Miller 1981) and from linear anelastic simulations (Glatzmaier and Gilman 1982). The latitudinal variation of our simulated rotation rate at the top boundary agrees, to within \( \sim 10\% \), with Doppler measurements of the solar surface differential rotation (Howard et al. 1983). The radial variation of the simulated rotation rate below the surface is in agreement with a recent analysis (Duvall et al. 1984) of the rotational frequency splitting of solar oscillations (Duvall and Harvey 1984) which suggests that angular velocity in the equatorial region decreases by \( \sim 15\% \) from 93\% to 45\% of the solar radius.

The other important ingredient for a cyclic dynamo is kinetic helicity which is the dot product of velocity and vorticity, \( \langle \mathbf{V} \cdot \nabla \times \mathbf{V} \rangle \). A typical profile of our simulated kinetic helicity, averaged in longitude, is illustrated in Figure 2b. As a result of rotation, spherical geometry, and density stratification, kinetic helicity on the average is negative, left-handed, in the northern hemisphere and positive, right-handed, in the southern hemisphere.

The latitudinal and radial components of this dot product make the major contributions to helicity. The latitudinal contribution can easily be understood by observing in Figure 1a that, to first order, the fluid undergoes left-handed (right-

![Diagram](image_url)
handed) helical motion the northern (southern) hemisphere (Glatzmaier and Gilman 1981). That is, in the northern hemisphere fluid has a latitudinal drift in the opposite direction of its latitudinal component of vorticity; whereas, in the southern hemisphere the latitudinal drift and vorticity are in the same direction.

The radial contribution to kinetic helicity is mainly due to the effects of rotation and density stratification. The Coriolis forces resulting from the expansion of rising fluid cause the fluid to twist as it rises producing negative (positive) vorticity in the northern (southern) hemisphere. The opposite type of vorticity is produced when sinking fluid contracts. As a result, the radial contribution to kinetic helicity also tends to be negative in the northern hemisphere and positive in the southern hemisphere.

However, in the inner part of the shell where the density scale height is large, the relative amount of expansion and contraction is small. In addition, it is there that the fluid horizontally converges as it begins its ascent and diverges as it terminates its descent. Coriolis forces resulting from these converging and diverging motions in the inner part of the shell produce a radial contribution to kinetic helicity that is positive (negative) in the northern (southern) hemisphere. However, in this region, as a result of the large mass density and stable entropy gradient, helicity is relatively small, as indicated by the zero contours in the inner part of the shell in Figure 2b.

IV. THE MAGNETIC FIELD

Although the actual simulated magnetic field is non-axisymmetric (Paper I), we will discuss the evolution of only the axisymmetric part of the magnetic field. The axisymmetric part of the longitudinal component of the magnetic field will be referred to here as the toroidal field, and the axisymmetric part of the radial and latitudinal components of the magnetic field will be called the poloidal field. Simulated toroidal and poloidal fields are illustrated in Figure 3 at ~1 yr intervals. The toroidal field in the direction of rotation, i.e., into the paper, is represented by solid contours and that in the opposite direction of rotation, i.e., out of the paper, by broken contours. The poloidal field is represented by lines of force. Note that the actual simulated lines of force are three-dimensional and non-axisymmetric and that those in Figure 3 only correspond to the axisymmetric part of the poloidal magnetic field. Also note that many thousands of time steps were computed before the first plot in Figure 3a in order to develop the structure of the convection starting from random entropy perturbations, which then developed the differential rotation via nonlinear angular momentum transport, which then developed the magnetic field structure from a random seed field.

The poloidal field is generated by helical fluid motion which twists the toroidal field. This is illustrated schematically in Figure 1b. In the northern hemisphere, the left-handed fluid motion in the convective region twists the toroidal magnetic field into right-handed lines of force, and vice versa in the southern hemisphere. Consequently, in the convection zone magnetic helicity, \( B \cdot (V \times B) \), tends to be positive (negative) in the northern (southern) hemisphere. This is depicted, for example, in Figure 3c where the clockwise poloidal fields have been generated by kinetic helicity operating on the two, main, oppositely directed, toroidal fields in the middle of the shell.

The toroidal field, on the other hand, is generated mainly by differential rotation which shears the poloidal field. It is also generated to a smaller extent by nonaxisymmetric convective motion which twists the nonaxisymmetric magnetic field; however, this effect tends to inhibit magnetic field propagation. Consider again the fields in Figure 3c. Since the equatorial region is rotating faster (Fig. 2a), the northward directed poloidal field near the bottom of the shell is being sheared in a way that produces a longitudinal component of the magnetic field in the opposite direction of rotation (broken contours) in the northern hemisphere and in the direction of rotation (solid contours) in the southern hemisphere. This rotational shearing of the poloidal fields is also destroying the larger, oppositely directed toroidal fields in the middle of the shell on their equatorward sides and enhancing them on their poleward sides. As a result, the toroidal field pattern moves outward in radius and poleward in latitude in both hemispheres.

The magnetic field propagation is illustrated by the sequence of the profiles in Figure 3. Two main oppositely directed toroidal fields appear in Figure 3a, one in each hemisphere. Kinetic helicity is twisting these toroidal fields generating the clockwise poloidal field in the middle of the shell and destroying the counterclockwise poloidal field in the outer part of the shell. As time proceeds (Figs. 3a–3c), the shearing effect of the differential rotation generates small toroidal fields of opposite polarity in the inner part of the shell and causes the main toroidal fields in the middle of the shell to propagate away from the equator. The clockwise poloidal fields also propagate away from the equator because they are maintained against diffusion by kinetic helicity operating on the main toroidal fields. The original toroidal fields eventually diffuse away because the poloidal field lines at high latitude (Figs. 3d–3f) are nearly parallel to the contours of angular velocity (Fig. 2a); that is, differential rotation is unable to shear these lines of force. However, differential rotation continues to shear the poloidal field in the inner part of the shell near the equator, so the new toroidal fields continue to grow (Figs. 3d–3f). Since kinetic helicity is small in the inner part of the shell (Fig. 2b), a new counterclockwise poloidal field will not be generated until the new toroidal fields expand into the outer part of the shell.
Less than a half cycle has been simulated here over a time corresponding to a little over 5 yr. We presume that if the computations were continued a complete cycle would be simulated. Of course, one cannot be certain that the structure and evolution of the fields will continue as in Figure 3 until further computations are performed. However, the problem with these anelastic dynamic dynamo simulations, and with the Boussinesq dynamic dynamo simulations (Gilman 1983), is that the axisymmetric part of the magnetic field propagates away from the equator, in the opposite direction inferred from the solar butterfly diagram.

The period of the magnetic cycle will be \( \sim 10 \) yr if the fields continue to propagate at their present rate. Although this is about a factor of 2 shorter than the observed 22 yr period, it is longer than the 1.3 yr period of the Boussinesq simulations (Gilman 1983). Our slower propagation rate is probably due to our smaller kinetic helicity which determines the regeneration rate of the poloidal magnetic field. Our helicity is smaller than that simulated in the Boussinesq (constant density) model because conservation of mass and the density stratification cause helicity to peak near the surface and drop off rapidly with depth (Fig. 2b).

V. SUMMARY

We have described how the effects of rotation, spherical geometry, and density stratification organize the simulated large-scale thermal convection in such a way to maintain the differential rotation and kinetic helicity profiles illustrated in Figure 2. We have also explained how differential rotation shears the poloidal magnetic field generating a toroidal mag-
netic field and how helical fluid motions twist the toroidal field generating a poloidal field. Finally, we have shown that, because our simulated angular velocity decreases with depth and our simulated kinetic helicity in the outer part of the shell is negative (positive) in the northern (southern) hemisphere, the axisymmetric part of our simulated magnetic field propagates away from the equator in apparent disagreement with what is inferred from the solar butterfly diagram.

Those who model kinematic dynamos would probably argue that we have not simulated the correct differential rotation profile, for if angular velocity increased with depth, the fields would probably propagate toward the equator. However, as explained above, we do not have the freedom to specify the differential rotation and helicity profiles; we solve the full MHD equations for the velocity, thermodynamic, and magnetic fields simultaneously with full nonlinear feedback (Paper I). We have used many hours of computer time trying to find, by varying the viscous, thermal, and magnetic diffusivities, a
three-dimensional, self-consistent solution that has angular velocity increasing with depth through the convection zone with a surface differential rotation profile that matches that observed on the Sun. We have never found such a solution. The closest we have come was a differential rotation profile similar, in the outer two-thirds of the shell, to that illustrated in Figure 2a but with a slight increase in angular velocity with depth in the inner third of the shell. However, this differential rotation profile eventually evolved into the one in Figure 2a which has very little radial variation of angular velocity in the inner part of the shell. In addition, our simulated differential rotation profile is in agreement with solar Doppler velocity measurements (Howard et al. 1983) and with what is inferred from the frequency splitting of solar oscillations (Duvall and Harvey 1984; Duvall et al. 1984).

Although our simulated toroidal magnetic field propagates away from the equator, the evolution of the radial component of the longitudinally averaged field at the surface is in agreement with solar observations. On the Sun, the radial component of the field propagates toward the equator at low
latitudes and the opposite polarity propagates toward the poles at high latitudes (Yoshimura 1976a, b; Stix 1976a). In addition, when the longitudinal component of the field is in the direction of rotation, the radial component is directed downward; and vice versa (Stix 1976b). Both of these observational constraints are satisfied by our simulations (Fig. 3).

There is a hypothesis that may be worth considering. The toroidal magnetic field may propagate parallel to the axis of rotation, away from the equator, deep below the solar surface and, as suggested in Figure 3, intercept the surface first at midlatitude followed by a phase propagation on the surface toward the equator. This presumes that the toroidal field has to be at a threshold strength at a given depth below the surface before a flux tube can break away and form a sunspot. This hypothesis is only suggested by these simulations and certainly not demonstrated.

On the other hand, if the magnetic field were concentrated in small flux tubes between cells throughout most of the solar convection zone as it is observed to be on the solar surface (Stenflo 1976), our model of large-scale continuous fields would be inappropriate for this region. However, since the velocity and magnetic fields would be decoupled to some extent under these conditions, the solar dynamo may be operating with large-scale fields in the less turbulent transition region at the base of the convection zone (Parker 1975; Durney 1976; Galloway and Weiss 1981). Since our kinetic helicity has the opposite sign in this inner part of the shell (Fig. 2b), magnetic fields should propagate toward the equator in this region. In addition, since the amplitude of helicity is smaller in this region, the period of the magnetic cycle should be longer, i.e., closer to the solar period. We describe our numerical investigation of this hypothesis in Paper III (Glatzmaier 1985).

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