A 2D study of the effects of the size of a solid core on the equatorial flow in giant planets

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Abstract

Zonal winds simulated in two-dimensional computer models of turbulent convection in the equatorial plane of giant planets have greater surface amplitudes for cases with smaller solid cores, and therefore larger buoyancy driving, all other properties being equal. This differential rotation in radius is maintained by the convergence of angular momentum flux, which occurs because of the convective flow that develops due to the effects of planetary rotation and density stratification. The superposition of the convective flow and the stronger zonal flow produces wave-like, instead of cellular convection.

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1. Introduction

Predicting and understanding the interior dynamics of giant planets are best accomplished by studying computer simulations of convection in rotating fluid spheres. We would like to understand how the various profiles of surface differential rotation, i.e., zonal winds, are maintained on our Solar System giant planets and, in particular, how the presence and size of a solid core may affect these profiles.

Measurements of the gravitational fields of Jupiter and Saturn have been used to construct static models which constrain Jupiter’s dense core to less than 14 \(M_{\oplus}\) and Saturn’s core to less than 22 \(M_{\oplus}\) (Guillot, 1999a). However, models without an inner core are also possible for both planets (Guillot, 1999a). Jupiter’s core may have experienced significant core erosion since its formation (Guillot et al., 1994) resulting in either a very small solid core or the complete absence of a solid core. Further, more than a hundred extrasolar giant planets have now been detected with varying masses and orbital parameters (Fischer et al., 2002). Some of these planets may have experienced high rates of core erosion via penetration of downward convection plumes (Guillot et al., 2002) and strong tidal interactions with their host stars, and may no longer possess a solid core.

Several computer models have been used to investigate the origin of the observed zonal winds on our giant planets. Some ignore the dynamics of the deep interior and assume the zonal winds are maintained only by surface meteorology (e.g., Williams, 1979, 1985, 2003; Cho and Polvani, 1996; Allison, 2000). Undoubtedly many of the observed small-scale weather systems on Jupiter and Saturn are shallow atmospheric phenomena; but the large-scale differential rotation profiles likely extend into the deep interior and are maintained by the interaction of planetary rotation and thermal convection well below the surface (Ingersoll and Pollard, 1982). Certainly, the increase in zonal wind speed with depth measured by the Galileo probe supports this theory (Atkinson et al., 1998). In addition, unless a latitudinally dependent thermocline is prescribed to drive thermal winds that match the observed latitudinally banded profile, shallow surface models produce eastward jets in the equatorial region, opposite those observed on Jupiter and Saturn.

Surface zonal wind profiles with eastward jets at the equator and some alternating-directed jets at higher latitudes have been simulated with three-dimensional (3D) computer simulations
of rapidly rotating thermal convection in deep spherical fluid shells (e.g., Christensen, 2001, 2002; Aurnou and Olson, 2001; Aurnou and Heimpel, 2004). These results are encouraging. However, although they account for deep global convection, they have been produced using the Boussinesq approximation, which assumes density is constant, except for small perturbations due to temperature. This assumption is commonly made when modeling, for example, the Earth’s liquid core for which the bottom density is only 20% greater than that at the top. However, density varies by orders of magnitude through giant planets, including our ice planets, and its density scale heights are smallest near the surface (Guillot, 1999a, 1999b). Neglecting planets, including our ice planets, and its density scale heights

However, density varies by orders of magnitude through giant planets, including our ice planets, and its density scale heights are smallest near the surface (Guillot, 1999a, 1999b). Neglecting the significant expansion of rising fluid and the contraction of sinking fluid in these planets results in extremely different convective flow patterns (Evonuk and Glatzmaier, 2004; Glatzmaier, 2005). In addition, because of limitations in computer resources, these giant planet simulations employ unrealistically large thermal and viscous diffusion. This results in large-scale laminar convection, nothing like the broad spectrum of turbulence that surely exists in these fluid planets. The differential rotation produced in these laminar simulations is maintained by large columnar rolls, parallel to the axis of rotation (Proudman, 1916; Busse, 1983), that are confined by impermeable spherical boundaries in the northern and southern hemispheres. Evidence of this classic convective structure has not been observed on the surfaces of giant planets and very likely does not exist in their interiors (Glatzmaier et al., 2005).

As 3D anelastic and Boussinesq simulations are currently unable to afford the resolution needed to simulate strongly turbulent convection, here we choose to use high resolution, non-magnetic, 2D numerical simulations to investigate the effect of a solid core on thermal convection in the equatorial plane of a rapidly rotating, density-stratified giant planet.

The 2D constraint removes meridional circulation and the variation of the dynamics with latitude. The non-magnetic assumption removes Lorentz forces, which likely play a significant role in the deep metallic hydrogen interiors of Jupiter and Saturn. Therefore, the solutions presented here are meant to study the sensitivity of the amplitude of the zonal flow to the size of the core instead of providing realistic structures of turbulent convection and zonal flow in the equatorial planes of Jupiter and Saturn.

We examine three cases, two with solid cores, 35 and 10% the total radius, and one without a solid core. We find the opposite result obtained by Aurnou and Olson (2001) who used a 3D laminar incompressible (Boussinesq) model; our 2D turbulent compressible (anelastic) simulations suggest that surface zonal wind amplitudes are greater the smaller the solid core (Fig. 1). Before discussing the flow patterns in greater detail, we will describe our numerical model.

2. Numerical model

We numerically simulate 2D thermal convection in a fluid with an axially symmetric, background thermodynamic state that varies with radius. We use the finite volume method on a uniform Cartesian grid (e.g., Schmalzl and Hansen, 2000; Talon et al., 2003). Here we provide a brief introduction to the equations, the finite volume method, and our specific model parameters.

2.1. The equations

To capture the effects of the large density stratification without having to resolve sound waves, we employ the anelastic approximation of the equations of motion (e.g., Gilman and Glatzmaier, 1981). To afford the required spatial resolution to simulate turbulent convection we solve the equations in two dimensions, radius and longitude. The governing equations for the velocity measured by the Galileo probe at depth (Atkinson et al., 1998).

\[
\frac{\partial (\bar{\rho} v)}{\partial t} = -\nabla \cdot \left[ \bar{\rho} v_i v_j + P \delta_{ij} - 2\bar{\rho} \left( e_{ij} - \frac{1}{3} \nabla \cdot \vec{v} \delta_{ij} \right) \right] - \bar{\rho} \bar{\gamma} \cdot \vec{r} \times \Omega - \rho \Omega \times (\Omega \times \vec{r}),
\]

\[
\frac{\partial (\bar{\rho} S)}{\partial t} = -\nabla \cdot \left[ \bar{\rho} (S + \bar{\rho}) v + \kappa_t \bar{\rho} \nabla S - \frac{C_p \kappa_t \bar{\rho}}{T} \nabla T \right] + \frac{\bar{\rho} \, d\bar{T}}{T} \frac{dT}{dr} \left[ \kappa_t \frac{dS}{dr} + \frac{C_p \kappa_t}{T} \frac{dT}{dr} \right] + \bar{\rho} Q_s,
\]

\[
\nabla \cdot (\bar{\rho} \vec{v}) = 0.
\]

Here, \( \bar{\rho} \) is the background state density as a function of radius, \( \vec{v} \) is the velocity vector, \( P \) is the pressure perturbation,
\( v \) is the turbulent viscous diffusivity, \( g \) is the background state gravity as a function of radius, \( \Omega \) is the rotation rate oriented perpendicular to the plane of the simulation, \( r = r \hat{r} \) is the radial position vector, \( S \) is the specific entropy perturbation, \( C_p \) is the specific heat capacity at constant pressure, \( \kappa \) is the turbulent thermal diffusivity, \( \rho \) is the density at the center of the disk, \( \rho_t \) is the density at the outer boundary of the disk, and \( r_t = 6.87 \times 10^9 \) cm for a generic giant planet. We set the gradient of the density to be zero at \( r_t \), and the value \( \rho_t \) is calculated by specifying the number of density scale heights, \( N_\rho = 4 \), where
\[
\rho_t = \rho_0 e^{-2N_\rho} = 7.65 \times 10^{-2} \text{ g cm}^{-3}.
\]
Without a density stratification, as in Boussinesq simulations, Coriolis forces in this 2D geometry would always be exactly canceled by part of the pressure gradient so rotation would have no effect on the flow. However, in our anelastic simulations Coriolis forces play a significant role in the structure and evolution of the flow due to the expansion of rising fluid and contraction of sinking fluid.

By assuming a hydrostatic and adiabatic background state, the density profile is used to solve for the gravity and temperature profiles in the 2D disk for a perfect gas:
\[
\tilde{g}(r) = \frac{4\pi G}{r} \int_0^r \tilde{\rho}(r) r \, dr,
\]
\[
\tilde{T}(r) = T_0 \left( \frac{\tilde{\rho}(r)}{\rho_0} \right)^{\gamma_G},
\]
where \( G \) is the gravitational constant and \( T_0 = 1.66 \times 10^4 \) K is the temperature at the center of the disk. We model the equatorial plane of the planet with the axis of rotation directed out of the plane of the simulation and chose the rotation rate of Jupiter, \( \Omega = 1.77 \times 10^{-4} \) s\(^{-1}\).

Our disk sits inside a square, Cartesian-gridded box. The outer boundary of the disk is made essentially impermeable and free-slip by making the region outside the disk, but inside the box, strongly subadiabatic (i.e., stable against thermal convection) and setting the viscosity in this region to zero. In addition, the upper 10% of the disk (in radius) is made subadiabatic. Consequently, upwelling convective plumes penetrate only very slightly into the subadiabatic outer layer. This is actually a more realistic representation of the top of the convective region in a giant planet than the traditional way of imposing a strictly impermeable outer boundary. Rogers and Glatzmaier (2005a, 2005b) have shown that simulations of convection between impermeable boundaries result in significantly different fluid flow structure than simulations with stable, subadiabatic layers adjacent to the boundaries.

The inner boundary for the two cases with solid cores is impermeable and non-slip. The first case has a large core, 35% of the total radius. The second case has a small core, 10% of the total radius, a distinct possibility for Jupiter. The third case has no core, representing a jovian planet that has completely eroded away the core with which it formed.

For all cases, an axially symmetric heating term, \( Q_s \), is applied to the central part of the disk out to 35% of the disk radius. \( Q_s \) has a maximum of \( 10^{-1} \) erg g\(^{-1}\) m\(^{-1}\) K\(^{-1}\) s\(^{-1}\) at the disk center and smoothly drops to zero at 35% of the disk radius; its radial gradient vanishes at both the center and 35% radius. We chose to heat the central region of the disk to mimic the heat release due to gravitational contraction in a giant planet, which increases with depth. Unlike the traditional method of setting a
thermal boundary condition at the core boundary, the heat equation (Eq. (2)) is solved throughout the disk, including the solid core if it exists. The turbulent diffusivity, \( \kappa_t \), is set to zero in the solid core; and the radiative diffusivity, \( \kappa_r \), is zero outside the core. The distributed heating is identical in all cases; so the average total heat flow, convective plus diffusive, at a given radius is the same. However, in the regions where there is a solid core, advection is not permitted, leaving diffusion as the only mechanism to transport the heat outward. For all cases, the Prandtl number in the convection zone is \( \text{Pr} = \nu/\kappa_t = 0.1 \). We assume the viscous and thermal diffusivities are constants, as is the specific heat capacity. The outer boundary of the disk is held at zero entropy perturbation.

Table 1 shows the values of the Ekman number, the Rayleigh number, the Reynolds number and the convective Rossby number for the three cases where \( g_{\text{mid}} \) is the gravity at mid-depth through the convection zone, \( \Delta S \) is the change in entropy from the bottom of the convection zone to the top, and \( D \) is the depth of the convection zone. This convective Rossby number is a measure of convective driving relative to Coriolis effects. The final entry to Table 1 shows the product of the convective Rossby number squared with the negative two-thirds root of the Ekman number. Christensen (2002) postulates that when this quantity is greater than 200 simulations are in the asymptotic regime with regard to decreasing the viscous and thermal diffusivities. All three of our simulations are within this regime.

3. Results

Fig. 1 shows the zonal wind velocities as a function of depth for the three simulations with a horizontal line demarcating the interface between the stable outer radiative zone of the planet and the convective interior. We see that increasing the size of the solid core leads to smaller surface wind velocities. This result is opposite of that obtained by Aurnou and Olson (2001) who compared cores 75 and 35% the total radius in a 3D Boussinesq model and found greater zonal flow velocities in their case with the larger core. One reason, besides density stratification, that our results may disagree, is that we chose to use the same heat source in our cases whereas Aurnou and Olson (2001) chose to use the same Rayleigh numbers in their cases. The different magnitudes of the surface zonal winds in Fig. 1 exist because of different flow patterns at depth. Decreasing the size of the core in our simulations has the effect of increasing the Rayleigh number as the convective zone increases (Table 1). In smaller convective zones viscosity does more work because of increased curvature of the flow and buoyancy does less work because of the reduced radial extent. Increasing the Rayleigh number in 3D rotating convection with a constant background density also leads to an increase in the observed zonal flow velocities (Aubert et al., 2001, 2003; Christensen, 2002).

The differential rotation seen in Fig. 1 represents the main part of the flow, as can be seen in the snapshots of the velocity fields in Fig. 2. This differential rotation is large relative to convection because of the vorticity generated by flow through the density stratification and the resulting convergence of angular momentum (Glatzmaier et al., 2005).

Fig. 2. Representative snapshots of the velocity and entropy perturbation fields for the three cases, no core, solid core 10% the total radius, and solid core 35% the total radius with the axis of rotation out of the page. Arrows in the cases with no core and smaller core are scaled the same while the arrows in the case with the larger core are magnified relatively 1.5 times. In the entropy perturbation fields lighter reds indicate hotter material and darker reds indicate colder material. The cases with no core and smaller core are scaled the same while, in the case with the larger core, the color scale is stretched three times relative to that of the other two cases. These snapshots are viewed in the rotating frame from the northern hemisphere, so the planetary rotation is counter-clockwise.
Table 1
Dimensionless parameters

<table>
<thead>
<tr>
<th>Number</th>
<th>No core</th>
<th>10% core</th>
<th>35% core</th>
</tr>
</thead>
<tbody>
<tr>
<td>Ekman Ek = (\nu/(2\Omega D^2))</td>
<td>3.7 \times 10^{-8}</td>
<td>4.7 \times 10^{-8}</td>
<td>9.9 \times 10^{-8}</td>
</tr>
<tr>
<td>Rayleigh Ra = (g_{mid}\Delta SD^3/(C_p\nu\kappa t))</td>
<td>1.0 \times 10^{13}</td>
<td>7.0 \times 10^{12}</td>
<td>5.6 \times 10^{11}</td>
</tr>
<tr>
<td>Reynolds Re = (\nu_{free}/D)</td>
<td>6.3 \times 10^4</td>
<td>6.1 \times 10^4</td>
<td>1.9 \times 10^4</td>
</tr>
<tr>
<td>Convective Rossby Ro_c = (\sqrt{\text{Ek}/\sqrt{Pr}})</td>
<td>0.12</td>
<td>0.12</td>
<td>0.074</td>
</tr>
<tr>
<td>Ro_c^2 Ek^{-2/3} (&gt;200)</td>
<td>1300</td>
<td>1200</td>
<td>260</td>
</tr>
</tbody>
</table>

Fig. 3. The density profile and the inverse density scale height, \(h_\rho\), as a function of radius in the disk. The density is plotted as a solid line and \(h_\rho\) as a dashed line. Note the peak negative value of \(h_\rho\) occurs near the outer boundary.

The density stratification as a function of radius and the inverse density scale height \(h_\rho\), where \(h_\rho = (1/\bar{\rho}) d\bar{\rho}/dr\), are plotted in Fig. 3. The curl of the Coriolis force, which contributes to the time rate of change of vorticity, is \(\nabla \times F_c = 2\Omega h_\rho v_r\), where \(v_r\) is the radial velocity. For this density profile, the maximum negative \(h_\rho\) occurs near the outer rim of the disk. Therefore the dominate process is sinking fluid from the top surface generating positive vorticity as it contracts, causing westward momentum to migrate down and converge in the lower part of the convection zone while eastward momentum migrates upward and converges in the upper part (Glatzmaier et al., 2005). The result is the observed prograde motion (eastward relative to the reference frame) near the outer rim of the disk. Since the zonal flow is much stronger, the convection takes the form of waves instead of cells. These oscillations persist and provide the radial heat transport.

Superimposed on the axisymmetric zonal flow is a small oscillation of the fluid in the radial direction (Fig. 2). The frequency (velocity divided by wavelength) of these oscillations is small compared to the rotation frequency. For the case with the largest core, the frequency of oscillation in the prograde flow is about 0.4% of \(\Omega\) with a wavelength of about \(10^9\) cm. The other two cases have frequency ranges from 2 to 3% of \(\Omega\), with wavelengths in the prograde regions of about \(2 \times 10^9\) cm. These radial flow oscillations are the superposition of convection and zonal flow. Since the zonal flow is much stronger, the convection takes the form of waves instead of cells. These oscillations persist and provide the radial heat transport.

These oscillations are expressions of the rich fluid flow structure underlying the much stronger zonal winds. Subtracting the axisymmetric zonal flow as a function of radius from the velocity field clearly illustrates the radial transport in the fluid, shown in Fig. 4 for the case with a large core. The net fluid flow is a
These results differ from those of Aurnou and Olson (2001), who the fluid, resulting in higher velocities. As mentioned above, the buoyancy forces have a larger radial distance to accelerate with the large core. In the cases with deeper convective zones times larger in the cases with small and no core than for the case with positive Reynolds stress $v_{\phi} v_{\phi}$ which transports eastward (prograde) motion outwards and westward (retrograde) motion inwards. Arrow sizes are enlarged 1.3 times relative to those in Fig. 2c and are plotted at 1/8 of the total numerical resolution in each direction. Below are two schematic diagrams of the local generation of vorticity, due to expansion of rising material (negative vorticity) and contraction of sinking material (positive vorticity), and the non-linear convergence of angular momentum responsible for maintaining the zonal winds.

The magnitudes of the zonal flow decrease with large core size. Typical values of the Reynolds number (Table 1) are three times larger in the cases with small and no core than for the case with the large core. In the cases with deeper convective zones the buoyancy forces have a larger radial distance to accelerate the fluid, resulting in higher velocities. As mentioned above, these results differ from those of Aurnou and Olson (2001), who find a larger core has higher zonal winds because the convective columns tilt more strongly and are therefore better organized than those with a smaller core. However, in the turbulent interior of Jupiter, we expect the effects of density stratification, not columnar convection, to maintain zonal winds (Glatzmaier et al., 2005).

As we are restricted to surface and near surface observations of the giant planets, it is important to compare the differences in the fluid velocities at the transition between the convective and stable zones among the three cases. The plot of the zonal wind as a function of depth (Fig. 1) shows lower wind speed at this transition for the case with the larger core, about 15% of that seen in the case with no core. The difference between the small core case and the no core case is minimal. Referring back to Table 1, we see that the two cases with the larger convective Rossby numbers (small and no core) have larger zonal wind velocities.

These 2D simulations only model the equatorial plane, and our heating rate is much higher than that of Jupiter (to compensate for our necessarily larger diffusivities). It is fortuitous then that the zonal wind speeds at the disk’s outer boundary in our small core and no core cases (Fig. 1), are similar to the observed surface zonal wind speed at Jupiter’s equator, $0.76 \times 10^4 \text{ cm s}^{-1}$ (García-Melendo and Sánchez-Lavega, 2001). The simulated wind speeds at the top of the convection zone for the cases with small core and no core are slightly larger than the zonal wind speed measured by the Galileo probe 125 km below the cloud tops just slightly north of the equator, $1.70 \times 10^4 \text{ cm s}^{-1}$ (Atkinson et al., 1998). These values are marked as stars in Fig. 1. Although this is an interesting coincidence, these 2D models are not meant to produce a realistic simulation of Jupiter; 3D models are required to capture the important features of an actual giant planet.

4. Conclusions

Two-dimensional simulations of the convection in the equatorial plane of a Jupiter-like planet, illustrate how differential rotation can be maintained by wave-like convection via the strong interaction of rotation and density stratification. The simulated zonal winds at the top of our turbulent convection zone are prograde because fluid sinking from the surface contracts, generates positive vorticity and a convergence of eastward angular momentum near the surface. This mechanism exists even in the highly turbulent flow regimes expected in giant planets, like Jupiter, and does not require the classic convective columns seen in constant density laminar simulations. Planets with smaller cores produce stronger surface zonal winds (all else being equal) because the fluid is accelerated to greater convective velocities in deeper convection zones. It would, however, be difficult to distinguish between a giant planet with no core and one with a 10% core based on the magnitude of the observed surface zonal winds.

However, since we have assumed two-dimensionality and no magnetic fields (in order to simulate strongly turbulent convection), our solutions are not meant to be conclusive predictions of the flow structure in the equatorial plane of giant planets. Three-dimensional magneto-hydrodynamic simulations of turbulent
convection in density-stratified rapidly rotating fluid planets are needed (Glatzmaier et al., 2005).

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