Thermal Convection in a 3D Rotating Density-Stratified Giant Planet without a Core

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SUMMARY

Three-dimensional numerical simulations of thermal convection in a fully-convective, non-magnetic, rotating, density-stratified, spherical fluid body are presented. Discontinuous axially-aligned vortices spiral prograde momentum away from the axis of rotation as a result of vorticity generated by fluid flowing through the density-stratification. The convergence of this nonlinear Reynolds stress maintains a banded pattern of differential rotation with a strong prograde jet at the equator, without the classical vortex stretching of convective columns. In the parameter regime of this simulation we see a flow structure that preferentially transports heat to high latitudes resulting in a surface temperature variation inverse to that which would be provided by incoming solar radiation for Jupiter.

**Key words:** Heat flow; Heat generation and transport; Planetary interiors.

1 INTRODUCTION

Both Jupiter and Saturn emit more heat than they receive from the sun. The cooling of the planet and its slow gravitational contraction, and possibly helium settling, provide this excess luminosity (Hubbard 1968; Guillot *et al.* 2004). While the solar absorption varies approximately as the solar
angle, Jupiter radiates thermal energy to space fairly uniformly with latitude (Pirraglia 1984). This suggests either poleward heat transport within the atmosphere by meridional circulation or an increase with latitude of the convective heat flux up from the deep interior. Saturn on the other hand has a large-scale thermal asymmetry in its upper troposphere with the northern hemisphere somewhat colder than the southern hemisphere (Hanel et al. 1981; Hanel et al. 1982). In spite of this thermal asymmetry, the zonal wind system appears to be highly symmetric with respect to the equator (Smith et al. 1982), suggesting a latitudinal variation in the internal flux instead of an atmospheric meridional circulation. Indeed constant density simulations in rotating spherical shells show fluid motions in the deep convection zones of the giant planets can redistribute the internal heat flux in a pattern similar to those seen on Jupiter (Aurnou et al. 2008).

This and other previous three-dimensional (3D) simulations of Jupiter’s internal dynamics include large cores; that is, they simulate the dynamics only within a relatively thin outer convective shell. The lower boundary at the base of the modeled convection zone is typically prescribed to be impermeable and having a uniform temperature, entropy or radial heat flux. This set of imposed boundary conditions may not be a good representation of the actual conditions at the chosen depth within Jupiter’s deep convection zone. The size of Jupiter’s solid or non-convecting core is greatly debated with estimates ranging from that of a small core or no core (Guillot et al. 1994) to that of a massive core, 14 to 18 Earth masses (Militzer et al. 2008). Similarly a large range of core sizes is predicted for the extrasolar giant planets, certainly some of which have no core during at least part of their lifetimes. Although the most vigorous convection is likely restricted to the outer 20% of a giant planet’s radius due to the increase in density and electrical conductivity with depth, uniform lower boundary conditions imposed at shallow depths may produce unrealistic constraints on the simulated internal dynamics.

To address this problem we model a fully-convective rotating, density-stratified planet to allow the flow and thermodynamic variables within the deep interior to find their own configuration. We use the anelastic approximation to allow a realistic depth-dependence of the background density. Most previous simulations have employed the much simpler approximation of a constant background density (the Boussinesq approximation). This approximation generates and maintains a
zonal flow via the global mechanism of convective columns spanning the convection zone in the geostrophic approximation (Busse 1994). The spherical surfaces of the planets cause these Busse columns to spiral outwards in the prograde direction, which creates a differential rotation with higher angular velocity on the outer edge of the columns (Busse 2002). Vorticity generated by pairs of Busse columns can maintain a zonal wind pattern similar to those seen in the surface clouds on Jupiter and Saturn, however it is not clear that columns spanning the convection zone can be maintained in a highly turbulent planet such as Jupiter (Glatzmaier et al. 2008).

While some variations on this geostrophic approximation include a density profile within the convective columns (Ingersoll & Pollard 1982), this mechanism neglects an important source of vorticity as it does not account for compressional torque, the expansion and contraction of fluid as it flows through the significant density stratification in a giant planet (Evonuk & Glatzmaier 2007; Glatzmaier et al. 2008). The profiles of density and velocity perpendicular to the rotation axis determine where angular momentum flux converges to maintain prograde zonal flow and where it diverges to maintain retrograde flow. A radial density profile decreasing more rapidly towards the surface, like those of Jupiter and Saturn, can maintain prograde zonal wind near the surface in the equatorial region as observed on these planets.

In this paper we focus on the fluid and heat flows in a three-dimensional density-stratified planet without a solid core. We will compare the relative importance of vortex stretching and compressional torque in the simulation.

2 METHODS

Our giant planet study consists of 3D, time-dependent, nonlinear simulations of thermal convection in a rotating sphere of fluid with central gravity. To simulate a fully convective planet we use the finite volume method on a 3D Cartesian grid similar to the two-dimensional model described in Evonuk & Glatzmaier (2006). The benefit of this method over spectral schemes is that the origin is no longer a special location; therefore flow moves freely through the center of the simulation. Benchmarking of the code lemning3D is discussed in Appendix A.

The basic fluid equations solved in lemning3D are the anelastic momentum equation (1), the
heat equation (2) and mass conservation (3):
\[
\frac{\partial}{\partial t}\left(\bar{\rho}\mathbf{u}\right) = -\nabla \cdot \left[\bar{\rho}\mathbf{u}\mathbf{u} + P\delta_{ij} - 2\bar{\rho}\nu\left(e_{ij} - \frac{1}{3}\frac{\partial u_k}{\partial x_k}\delta_{ij}\right)\right] - \bar{\rho}\bar{g}^\mathbf{r} + 2\bar{\rho}\mathbf{u} \times \Omega - \bar{\rho}\Omega \times (\Omega \times \mathbf{r}),
\]
(1)
\[
\frac{\partial}{\partial t}(\bar{\rho}S) = -\nabla \cdot [\bar{\rho}\mathbf{u} - \kappa\bar{\rho}\nabla S] + \frac{\bar{\rho}}{T} \frac{dT}{dr} \left[\kappa \frac{\partial S}{\partial r}\right] + \bar{\rho}Q_s,
\]
(2)
\[
\nabla \cdot (\bar{\rho}\mathbf{u}) = 0,
\]
(3)
where \( t \) is time, \( \rho \) is the density, \( \mathbf{u} \) the velocity vector, \( P \) the pressure, \( \delta_{ij} \) is the Kronecker delta, \( \nu \) the turbulent viscous diffusivity, \( g \) the gravity, \( \Omega \) the rotation rate vector, \( r \) the radial position vector, \( S \) the entropy, \( \kappa \) the turbulent thermal diffusivity, \( T \) the temperature, \( Q_s \) the internal heating function, and the rate of strain tensor \( e_{ij} = 0.5[\partial u_i/\partial x_j + \partial u_j/\partial x_i] \). The over-barred quantities are prescribed reference state variables based on an adiabatic hydrostatic one-dimensional (1D) model and are functions only of radius. We set the viscous and thermal diffusivities to constants in these preliminary simulations.

The giant planets in our solar system have internal heat sources, remnant from their formation as well as from their slow gravitational contraction (Hubbard 1968; Guillot et al. 2004). Therefore our simulation includes a heating source, \( Q_s \), which is axially symmetric with a maximum of 20 ergs \( g^{-1} K^{-1} s^{-1} \) at the sphere’s center, dropping smoothly to zero at 10% of the sphere’s radius.

The entropy perturbation at the surface of the simulation is set to zero.

The density perturbation \( \rho \), is obtained from the linear equation of state:
\[
\rho = \left(\frac{\partial \rho}{\partial S}\right)_P S + \left(\frac{\partial \rho}{\partial P}\right)_S P
\]
with
\[
\left(\frac{\partial \rho}{\partial S}\right)_P = \frac{\gamma_G}{\bar{g}} \frac{d\bar{\rho}}{dr}
\]
and
\[
\left(\frac{\partial \rho}{\partial P}\right)_S = -\frac{1}{\bar{\rho}g} \frac{d\bar{\rho}}{dr}
\]
where \( \gamma_G \) is the Grüneisen parameter, which we set to 0.5, an appropriate value for Saturn (Guillot 1999). We ignore the hydrogen phase boundary as its effects on convection and heat flux are poorly understood.
Our giant planet model is a fluid sphere spanning five density scale heights, \( N_\rho = \ln(\rho_b/\rho_t) = 5.0 \), with \( \rho_b \) and \( \rho_t \) being the density at the origin and the surface of the simulation respectively (Fig. 1). The outer boundary is impermeable corresponding to a pressure of 1 kbar \((10^9 \text{dynes/cm}^2)\) and a density of \(2.89 \times 10^{-2} \text{g/cm}^3\), as suggested from the 1D evolutionary models of (Guillot et al. 1994; Guillot et al. 2004). Due to the jagged nature of the spherical boundary on the Cartesian grid, it is not straightforward to set up free-slip boundary conditions in a meaningful way, therefore we use no-slip conditions and focus on the fluid behavior at a reasonable distance from the outer jagged boundary. We set the Prandtl number, \( Pr = \nu/\kappa \), to 0.1, and the rotation period to 10 hours, which is close to those of Jupiter and Saturn. The Ekman number, an estimate of viscous forces to Coriolis forces, \( Ek = \nu/(2\Omega D^2) \), is \(3 \times 10^{-6}\), where \( D \) is the radius of our simulation corresponding to the radius of Jupiter at 1 kbar \((6.867 \times 10^9 \text{cm})\). Our Rayleigh number, \( Ra = (g_{\text{max}} \Delta S D^3)/(C_p \nu \kappa) \), is calculated from the model results where \( C_p \) is the specific heat capacity at constant pressure and the drop in specific entropy, \( \Delta S \), across \( D \) is related to the drop in the superadiabatic temperature, \( \Delta T \), by \( \Delta S/C_p = \alpha \Delta T \), where \( \alpha \) is the thermal expansion coefficient. The value of \( \Delta S \) varies over time and is a function of the prescribed heating term, \( Q_s \), and the resulting convective patterns in the simulation. The Rayleigh number of our model varies between \(10^8\) and \(10^9\), however the local Rayleigh number is likely smaller in some regions due to varying amounts of numerical diffusion. Table 1 lists the dimensionless numbers for this simulation. The peak Rossby number for Jupiter is 0.012, for Saturn is 0.045, and for our simulation is 0.034.

Our diffusivities, \( \nu \) and \( \kappa \), set to \(5 \times 10^{10} \text{cm}^2/\text{s}\) and \(5 \times 10^{11} \text{cm}^2/\text{s}\) respectively, are a crude representation of the effects of the small-scale turbulence we cannot afford to resolve. Their large values force us to drive convection with a larger than realistic luminosity in order to obtain realistic surface wind speeds. This is a common problem due to limitations in computational resources. The spatial resolution in our simulation is \(400^3\) in Cartesian coordinates. The model was run for more than 17,000 numerical time-steps corresponding to 0.001 viscous diffusion times, approximately 56 convective turnover times, or about 500 10-hour rotation periods.

In this exploratory simulation we neglect magnetic fields and investigate how a rotating, density-stratified, convecting fluid sphere transports heat and angular momentum.
3 RESULTS

An equatorial slice through the simulated planet shows a typical pattern of the axial vorticity, $\omega_z$ (Fig. 2a), where vorticity is the curl of the velocity, $\omega = \nabla \times u$. This snapshot shows positive vorticity material (red) spiraling towards the surface of the model and negative vorticity material (blue) moving towards the center of the simulated planet. The convergence of prograde (positive vorticity) material at the surface in the equatorial plane acts to support a prograde jet in the equatorial region (Fig. 2b). A meridional slice through the simulation shows the extent of these features along the $z$-axis, the axis of rotation. As expected in a rotating system, columnar structures form aligned with the axis of rotation. However they do not tend to span the convective zone uniformly; rather they form disconnected features which move radially with time (Fig. 2c).

The resulting pattern of tangential velocity, while maintaining a prograde equatorial jet and a primarily retrograde central region, is complicated and fluctuates through time, frequently displaying additional prograde jets near the surface and occasionally at depth. The strongest prograde features are seen near the surface in the equatorial region. Smaller prograde features form at higher latitudes and extend into the outer 30% of the planet but do not span the convection zone (Fig. 2d). Near the surface (at 90% of the radius) an alternating pattern in the zonal velocity with latitude can be seen when averaged in time and longitude (black line, Fig. 3). The zero in Fig. 3 is chosen based on the volume-integrated angular momentum, while the zeros in similar plots for Jupiter and Saturn are chosen based on the rotation rates of the deep seated magnetic fields as the interior profiles of angular momentum for these planets are unknown. The zonal flow pattern of the simulation is very time dependent, a representative time shot (red line, Fig. 3) demonstrates that the number and direction of jets can vary greatly at higher latitudes. These changes occur on timescales on the order of 100’s of planetary rotations.

Since the simulation includes a large density variation in the background profile, both classic vortex stretching and compressional torque are viable mechanisms for maintaining and generating this zonal flow structure. Vortex stretching relies on quasi-geostrophic columnar features spanning the convection zone interacting with the convex curvature of the outer boundary to converge angular momentum. While the bulk of the fluid appears quasi-geostrophic we see evidence for
non-geostrophic flow in Fig. 2(d) where fingers of prograde material penetrate the convective region at mid and high latitudes. As seen in individual vortex features in the fluid (Fig. 4), most columnar features are tightly constrained near the equator and at low latitudes, however as these features extend in the z-direction, away from the equator, they expand into less dense regions and are sheared apart. This inhibits the existence of long convective columns and the vortex stretching mechanism for maintaining zonal flow. However, since the compressional torque mechanism does not require long convective columns, it can and does maintain zonal flows.

In a more quantitative approach we can compare the topographic scaling of the jet features in the simulation to the predicted Rhines scaling for Boussinesq and anelastic fluid spheres. We plot the modified length scale, $L\sqrt{\Omega/D\langle U \rangle}$, where $L$ is the width of the zonal feature and $\langle U \rangle$ is its mean velocity for the simulation in black in Fig. 5. The red line in Fig. 5 shows the modified length scale predicted for a geostrophic Boussinesq simulation, $L\sqrt{\Omega/D\langle U \rangle} = \pi \sqrt{h/(D|dh/ds| \sin^2 \theta_{lat})}$, where $h$ is the height of the fluid column, $|dh/ds|$ is the change in the height of the column as it moves perpendicular to the axis of rotation, and $\theta_{lat}$ is the latitude of the jet (Heimpel et al. 2005). The orange line in Fig. 5 is the modified length scale predicted for a geostrophic anelastic fluid, $L\sqrt{\Omega/D\langle U \rangle} = \pi \sqrt{M/(D|dM/ds| \sin^2 \theta_{lat})}$, where $M$ is the mass per cross sectional area of the fluid column and $|dM/ds|$ is the change in mass per cross sectional area as the column moves with respect to the axis of rotation (Ingersoll & Pollard 1982). The failure of the Rhines scaling, especially at high latitudes indicates that the zonal flow is not driven solely by vortex stretching.

Since both mechanisms are at play in the simulation, it is of interest to compare their relative magnitudes in the equatorial plane of the simulation, that is, the linear terms for vortex stretching $(2\Omega \partial u_z/\partial z)$ and compressional torque $(-2\Omega (\nabla \cdot u) = 2\Omega h_\rho u_r$, where $u_z$ is velocity directed parallel to the axis of rotation, $h_\rho = \bar{\rho}^{-1}d\bar{\rho}/dr$, and $u_r$ is the radial velocity in the equatorial plane). It is clear that in a Boussinesq simulation, where $\nabla \cdot u = 0$, the compressional torque term would vanish. In the anelastic simulation the contribution of the compressional torque will be greater for a larger number of density scale heights within the planet. This term will make its maximum contribution where the density is changing the most, corresponding to peak absolute
values of inverse density scale height, $h_\rho$. In our simulation, as in Jupiter and Saturn, this occurs near the surface (Fig. 1). A comparison of these terms in the equatorial plane yields values of the compressional torque that are 25 times that of vortex stretching near 90% the radius, where the zonal velocity is plotted in Fig. 3. Even at this relatively modest resolution and parameter regime the compressional torque term, usually neglected, is playing a significant role in the fluid dynamics. The classic vortex stretching term is dominant at smaller radii where the density scale height is large.

The axially aligned vorticity features have the additional property of preferentially transporting heat to higher latitudes. Higher surface temperatures in the model near the poles are maintained by convective heat transport to higher latitudes (Fig. 6a). This would translate to higher radiative heat flux on the surface if our model extended out to an optical depth of unity. The time and longitudinal average of the temperature perturbation with latitude (Fig. 6b), shows a qualitative match for the distribution, lower temperatures near the equator and higher temperatures near the poles, needed to balance the incoming solar absorption and produce the relatively uniform heat flux profile of Jupiter (Conrath & Pirraglia 1983). This is not a result unique to anelastic convection as similar trends have been seen for quasi-geostrophic, thin shell, Boussinesq simulations relying solely on classical vortex stretching (Aurnou et al. 2008).

Curiously, in spite of the higher densities in the central regions of the simulation, the fluid radial velocities at depth are not significantly smaller than those near the surface. In this case the scenario of warm upwelling and cool downwelling is not strictly adhered to. Vigorous convection allows the upwelling regions to transport both warm and cool fluid upwards and downwelling regions to move both cold and warm material downwards. This limits the convective heat flux from becoming too large while maintaining average convective velocities at depth comparable to those near the surface.

4 CONCLUSIONS

While it is certain that Jupiter and Saturn are in a more turbulent regime, and likely in a regime where rotation is more dominant, this simulation provides some insight into the behavior of plan-
etary bodies without solid cores. As in the majority of 3D rapidly rotating Bousinesq simulations, we see in our anelastic simulation a zonal flow pattern maintained with a strong prograde jet at the equator. While classical vortex stretching plays a role in generating vorticity in this simulation, the compressional torque term becomes strongly dominant in the regions where the density scale heights are smallest, i.e., near the surface. The compressional torque should play an ever increasing role for models that are more turbulent, approaching the parameter regimes of the giant planets.

Numerical simulations like these need higher resolution and lower diffusivities to more closely approach the turbulent parameter regimes of the giant planets. A significant density stratification is an important step towards the realistic modeling of the giant planets.

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REFERENCES


Figure legends

Figure 1: Plot of the background density profile ($\bar{\rho}$, solid line) and the corresponding inverse density scale height ($h_\rho = d \ln \bar{\rho} / dr$, dashed line) as a function of radius in the simulation. Peak negative values of $h_\rho$ occur towards the surface of the simulation. The rising, expanding material generates negative vorticity due to the Coriolis force while the sinking, contracting material generates positive vorticity (grey arrow). The positive vorticity is advected towards both the inner and outer regions in the equatorial plane, however the dominate process occurs at the outer boundary due to the larger number of density scale heights there (larger negative $h_\rho$), resulting in a surface prograde jet at the equator.

Figure 2: Panel a) shows the equatorial slice as viewed from the north of the vorticity parallel to the rotation axis, vorticity in the z-direction, with red indicating positive vorticity (i.e., in the same sense as the planetary rotation vector) and blue indicating negative vorticity. We see positive vorticity spiraling towards the surface to support the prograde equatorial jet. Panel b) shows an equatorial slice of the zonal velocity with yellow indicating prograde motion and blue retrograde. Panel c) shows a meridional slice illustrating the axial orientation of the spiraling vorticity structures with the same coloring as in (a). Panel d) shows a meridional slice of the zonal velocity with the same coloring as in (b).

Figure 3: The time and longitudinally averaged zonal flow pattern as a function of latitude in black. Note that the zero in the velocity plot is chosen based on the volume integrated angular momentum, while the zero in similar plots for Jupiter and Saturn are chosen based on the rotation rates of the surface magnetic fields as the interior profiles of the angular momentum for these planets are unknown. A representative snapshot of the longitudinal average is shown in red.

Figure 4: Columnar features in the fluid, shown via stream tracers in blue. Near the equator the columns form tight featured aligned parallel to the axis of rotation, however further from the equator the columns are sheared apart as they move into the less dense regions of the fluid.

Figure 5: Comparison of the modified length scales of the zonal flow based on the time averaged mean velocities of the simulation jets for Boussinesq Rhines scaling (red line), the modified
anelastic Rhines scaling for columnar convection (orange line), and the actual length scales seen in the simulation (black line). We see a similar trend to the anelastic disconnected vorticity scaling but with length scales an order of magnitude greater.

**Figure 6:** The temperature perturbation. Panel a) is a snapshot of the temperature perturbation showing a hot (red) isothermal contour extending along the axis of rotation while spiraling towards the surface away from that axis. A near surface radial contour, shown in part, shows higher temperature perturbations in red near the poles and lower perturbations in blue near the equator. Panel b) shows the near surface time and longitudinally averaged temperature perturbation of the simulation as a function of latitude.

**Figure A1:** On the left the mean temperature profile with depth for the benchmark simulation $N_z = 50$ and on the right the RMS profiles with depth for the temperature fluctuations ($T_{rms}$), the horizontal velocity variations ($u_{rms}$), and the vertical velocity variations ($w_{rms}$).

**Tables**

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<th>Table 1: Non-Dimensional Numbers</th>
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<td>Peak Rossby Number (Ro)</td>
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<th>Table A1: Benchmark Simulation Parameters</th>
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<td>Taylor Number (Ta)</td>
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<td>Aspect Ratio (A)</td>
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APPENDIX A: BENCHMARKING OF \textit{lemming3D}

The 3D finite volume code, \textit{lemming3D}, was benchmarked against the spectral codes of Julien et al. (1996) and Kunnen et al. (2006). The case chosen was a Boussinesq, periodic, rotating box simulation with no-slip boundary conditions with parameters shown in Table A1. The simulation was heated from below and cooled from above with random initial perturbations on a smoothly varying background temperature profile to initialize convection. Despite the higher numerical diffusion of the finite volume method we find at low and mid resolution (50 and 100 vertical points on a Chebyshev grid) agreement within less than 1\% for the vertical velocity RMS and 9.7\% for the RMS temperature fluctuations at mid-depth (Table A2). Agreement is also seen for the temperature and RMS profiles with depth (Fig. A1) as well as the convective patterns. At this time there are no non-linear density-stratified benchmark cases although plans are being made to conduct these in the future.