Symmetry and stability of the geomagnetic field

R. S. Coe¹ and G. A. Glatzmaier¹

Received 22 August 2006; revised 5 October 2006; accepted 9 October 2006; published 8 November 2006.

[1] Computer simulations of the geodynamo reveal a strong inverse correlation between stability and equatorial symmetry of the simulated field. This result is consistent with the paleomagnetically inferred degree of symmetry of Earth’s magnetic field during the past 150 Ma, which reversed polarity more frequently when it was more symmetrical. A geodynamo simulation with solid inner core much smaller than it is today produced an exceptionally antisymmetric field, raising the possibility that reversals may have been much less common in the distant geologic past than more recently. Paleomagnetic evidence suggests that this might well be true. Citation: Coe, R. S., and G. A. Glatzmaier (2006), Symmetry and stability of the geomagnetic field, Geophys. Res. Lett., 33, L21311, doi:10.1029/2006GL027903.

1. Introduction

[2] The average reversal rate has varied enormously over the long term, from around 5 reversals per Myr during the past 10–20 Myr to as low as 0.05/Myr during the interval from about 125 to 84 Ma that includes the Cretaceous Normal Superchron (CNS) [Ogg et al., 2004]. What could be the reason for such great differences in long-term geomagnetic field stability? Scientists have long suggested that reversals would occur less frequently during periods of greater dipolarity, that is, when the axial dipole field is larger on average relative to the non-axial-dipole (NAD) field (the rest of the field) [Cox, 1969]. This condition would be signaled in the rock record by smaller paleomagnetic secular variation, defined as angular dispersion with time of paleomagnetic directions about the AD field direction or, equivalently, by dispersion of virtual geomagnetic poles (VGP) about the geographic pole. Indeed, such appears to be the case: estimates of angular standard deviation of VGP are significantly smaller around CNS time than before and after when the reversal rate was much higher [Irving and Pulliaah, 1976; McFadden et al., 1991]. This may be due mainly to a stronger AD component during the CNS rather than weaker NAD components. Indeed, recent paleointensity studies in the 0–160 Ma time interval suggest that the average field may have been systematically stronger when the reversal rate was lower [Tarduno et al., 2002; Tauxe and Cottrell, 2005].

2. Symmetry and 0–150 Ma Reversal Rate

[3] McFadden et al. [1991] demonstrated a more subtle but very strong relation between reversal frequency and the latitudinal dependence of VGP dispersion, \( S(\lambda) \), a relation that Merrill and McFadden [1988] had predicted three years earlier. They showed that the simple relation \( S = (a^2 + \langle b \lambda^2 \rangle)^{1/2} \), where \( a \) and \( b \) are empirical constants, provides a good fit to their hemispherically averaged VGP dispersion of 0–5 Ma lava flows collected between latitudes \( \lambda = 0 \) to \( \pm 70^\circ \) [McFadden et al., 1988]. Fitting this relation to the VGP dispersion from lava flows older than 5 Ma, they found a correlation between reversal rate and the ratio \( b/a \). The maximum in \( b/a \) occurs during a window that includes the CNS when the reversal rate was lowest, and the minimum occurs around 15 Ma when the reversal rate was highest (see Figure 1). A more recent analysis of VGP dispersion specifically during the CNS supports their original finding [Tarduno et al., 2002].

[4] The motivation for this prediction came from properties of spherical harmonic terms in the expansion of the geomagnetic potential, \( V \), where the geomagnetic field is given by \( B = -\nabla V \). Above Earth’s core,

\[
V = R_0 \sum_{n=1}^{\infty} \sum_{m=-n}^{n} \left( \frac{R_k}{r} \right)^{n+1} \left[ g_m^n(t) \cos(\phi \theta) + h_m^n(t) \sin(\phi \theta) \right] \frac{P_m^o(\theta)},
\]

where \( r, \phi, \theta \) and \( t \) are radius, colatitude, longitude and time, respectively, and \( P_m^o(\theta) \) are the Schmidt quasi-normalized Legendre functions. These terms divide according to symmetry into two non-interacting “dynamo families” for a class of idealized dynamo models [Roberts, 1971; Gubbins and Zhang, 1993]. The terms associated with gauss coefficients \( g_m^n \) and \( h_m^n \) are antisymmetric about the equator when \( (n + m) \) is odd and are symmetric about the equator when \( (n + m) \) is even. At the equator, only the even terms contribute to the VGP dispersion because the contribution by the odd terms must be zero. Thus \( a \) is a measure of the field arising from gauss coefficients with \( (n + m) \) even. Because the VGP dispersion from even harmonics of the modern field averaged around circles of latitude is almost independent of latitude, Merrill and McFaden [1988] argued that \( b \) is a measure of the field arising from gauss coefficients with \( (n + m) \) odd (excluding the axial dipole term \( g_1^0 \), which contributes nothing to VGP dispersion). In support of this assumption, they pointed out that \( S(\lambda) \) of the longitudinally averaged modern field is not significantly different from that derived from the temporally averaged 0–5 Ma field [McFadden et al., 1991]. More recently, Tauxe and Kent [2004] have successfully modeled VGP dispersion with a giant gaussian process. By introducing a parameter that prescribes the ratio of antisymmetric to symmetric harmonics for each degree \( n \), they are also able to represent the variation of dispersion with latitude.

3. Symmetry and Stability of Geodynamo Simulations

[5] Computer simulations of the magnetohydrodynamic processes in Earth’s core that generate the geomagnetic field provide another way to assess which conditions may be
important determinants of field instability and reversal rate. For various imposed boundary and material conditions in the simulations, relatively long-term behavior can be computed in terms of the time series of gauss coefficients that describes the simulated field anywhere outside the core. By examining the simulated field at the core-mantle boundary (CMB) rather than at the surface, we avoid disproportionately weighting the lower degree harmonics, which attenuate more slowly with distance from the center. The spatial energy density (times twice the magnetic permeability) associated with each harmonic,

\[ W_n = \frac{(n+1)}{2} \left( \frac{g_n^m}{\mathcal{R}_n} + \frac{h_n^m}{\mathcal{R}_n} \right)^2 (\mathcal{R}_n \mathcal{R}_m) \]

provides a convenient measure of its contribution to the field [Loves, 1974].

Glatzmaier et al. [1999] reported results of eight simulations that, though differing only in the pattern of radial heat flux imposed at the CMB, exhibited large differences in geometry, strength, and stability of the magnetic field. The field for the various simulations at the CMB up to harmonic \( n = m = 5 \) ranged in axial dipole spatial energy density \( AD \) and degree of axial dipolarity \( AD/NAD \) by a factor of 42 and 24, respectively (Table 1). Note that the \( W \)-values in this paper are always time-averaged, usually over the entire duration of the simulation, and are truncated at degree and order 5 because hyperdiffusivity was employed and spatial resolution limited in order to achieve usefully long simulations [Glatzmaier and Roberts, 1996]. Above degree 7, spatial energy fell off much faster with \( n \) than below. Thus,

\[ W = \sum_{n=1}^{5} \sum_{m=0}^{n} W_n^m \text{ and } AD/NAD = W_1^0 (W - W_1^0). \]

We note, however, that including terms up to \( n = m = 21 \) had negligible effect on the ratios of values used to compare simulations.

Figure 1. \( b/a \) (squares) is an empirical measure of the relative contribution of odd \( (b) \) to even harmonics \( (a) \) of the time-averaged geomagnetic field inferred from paleomagnetic secular variation (i.e., from \( S = \langle a^2 + (b\lambda)^2 \rangle^{1/2} \), see text). The thin wavy curve is smoothed reversal frequency corresponding to the normal and reversed chronos (black and white bars) of the polarity time scale [McFadden et al., 1991]. CNS: Cretaceous Normal Superchron.

Figure 2. Time-averaged spatial energy density at the core-mantle boundary (CMB) associated with each harmonic degree and order up to \( n, m = 5, 5 \) for five simulations in Table 1. These cases have different patterns of imposed CMB heat flux, but all have zonal and equatorial symmetry and the same total heat loss from the core. Note that the most stable case, E, consistently favors harmonic terms with \( n + m \) odd, as does usually the second most stable case, D.

Table 1. Characteristics of the Average Field for Each Simulation

<table>
<thead>
<tr>
<th>Case</th>
<th>CMB Heat Flux Pattern</th>
<th>( AD \times 10^8 \text{ nT}^2 )</th>
<th>( AD/NAD^a )</th>
<th>( O/E^b )</th>
</tr>
</thead>
<tbody>
<tr>
<td>A</td>
<td>( P_{1,0} )</td>
<td>23.1</td>
<td>0.25</td>
<td>0.63</td>
</tr>
<tr>
<td>B</td>
<td>( P_{2,0} )</td>
<td>9.6</td>
<td>0.21</td>
<td>0.69</td>
</tr>
<tr>
<td>C</td>
<td>( P_{2,0} )</td>
<td>103.0</td>
<td>0.43</td>
<td>0.81</td>
</tr>
<tr>
<td>D</td>
<td>( P_{2,0} )</td>
<td>281.0</td>
<td>3.49</td>
<td>2.49</td>
</tr>
<tr>
<td>E</td>
<td>( P_{4,0} )</td>
<td>403.0</td>
<td>4.94</td>
<td>19.16</td>
</tr>
<tr>
<td>F</td>
<td>( P_{4,0} )</td>
<td>97.4</td>
<td>2.60</td>
<td>0.60</td>
</tr>
<tr>
<td>G</td>
<td>( P_{4,0} )</td>
<td>133.0</td>
<td>3.60</td>
<td>1.06</td>
</tr>
<tr>
<td>H</td>
<td>( P_{4,0} )</td>
<td>31.5</td>
<td>0.38</td>
<td>0.53</td>
</tr>
<tr>
<td>S</td>
<td>( P_{4,0} ) (small IC)</td>
<td>225.0</td>
<td>3.21</td>
<td>7701</td>
</tr>
</tbody>
</table>

\(^a\)Average axial dipole spatial energy.  
\(^b\)Axial-dipole to non-axial-dipole energy.  
\(^c\)Ratio of odd to even spatial energy (see text).  
\(^d\)Corresponding associated Legendre polynomial.  
\(^e\)Averages over the time before F dynamo crashed.  
\(^f\)Low-order pattern of seismic velocity in deep mantle.  
\(^g\)Small inner core.
with \( O/E = 0.6 \). Thus one might expect \( F \) to exhibit the most unstable field behavior. Indeed, although \( F \) reversed only twice, it oscillated wildly in direction and intensity and crashed the first time it reversed, about halfway through the simulation; its magnetic field energy fell three orders of magnitude at that time and never recovered, continuing down another order of magnitude by the end. Case D, the second most antisymmetric with \( O/E = 2.5 \), reversed once quickly after only 20,000 simulated years and then maintained a highly stable constant polarity for the remaining 80,000. Its CMB heat-flux opposite, case C, underwent four reversals and two excursions in the same length of time. The uniform heat flux simulation, case G, was of intermediate stability, reversing twice. Thus, the \( O/E \) ratio does a good job of predicting relative field stability for the family of simulations with zonal, equatorially symmetric CMB heat flux. Cases A, B and H, characterized by heat flux distributions that lack equatorial symmetry, zonal symmetry or both, are comparable in \( O/E \) ratio and field stability with the less stable cases of the five previously discussed simulations (F and C, Table 1). It is worth noting that the \( O/E \) ratio is a better predictor of instability for case \( F \) than the other commonly assumed stability parameters in Table 1—axial dipole energy \( AD \) and dipolarity \( AD/AD_{NAD} \)—c.f. cases A, B, C and H. The strong correlation between the \( O/E \) ratio and stability of the simulated field is entirely consistent with the paleomagnetic observations of Earth’s magnetic field, discussed above: times of higher reversal rate are times of relatively greater symmetric (even) harmonic content (Figure 1) [McFadden et al., 1991].

Similarly, consideration of the \( O/E \) ratio for a simulation with much smaller inner core than today predicts that the reversal rate was much lower in the distant past than more recently. The time-averaged characteristics of a geodynamo simulation conducted with a small inner core that is only one-quarter of today’s radius are given in Table 1 (case S). This simulation exhibits an \( O/E \) ratio of 7700, far higher than any of the others. It had heat flux uniformly distributed over the CMB and set about 40% higher than for the other cases [Roberts and Glatzmaier, 2001], and ran for 60,000 simulated years. During the simulation the field did not reverse or undergo any large excursions. The \( AD/AD_{NAD} \) ratio was also fairly high, but not as high as for cases E, D and G (Table 1). The extremely high \( O/E \) ratio of case S compared to case G, the uniform heat flux case with today’s inner core radius (Table 1), is clearly a result of its much smaller inner core.

The axial dipole energy \( AD \), dipolarity \( AD/AD_{NAD} \), and \( O/E \) ratio all successfully separate the simulations that experienced more reversals and excursions (A, H, B, F, C) from those that experienced fewer (G, D, E, S). Within these two groups the \( O/E \) ratio does a better job of characterizing the relative stabilities of B, F, G, and probably S, but longer simulations would be needed to obtain enough reversals and excursions to demonstrate the differences quantitatively [Glatzmaier et al., 1999]. More simulations are also needed to establish systematically how small departures in CMB heat flux pattern from zonal and equatorial symmetry affect reversal rate and \( O/E \) ratio.

4. Precambrian Reversal Rate

Over the past 150 Myr Earth’s field reversed on average 1.7 times per million years [Ogg et al., 2004]. The question is, did it actually reverse at a much lower rate in the much more distant past? A crude statistical method used to assess long-term variations in relative reversal rate is to compare the percentage of paleomagnetic studies in different time intervals that have mixed polarity [McElhinny, 1971]. Higher percentage suggests higher reversal frequency, and vice versa. Roberts and Piper [1989] applied this method to studies in the interval 0 to 2.7 Ga (their Figure 37). Their analysis suggests that the average reversal frequency was substantially lower before 1 Ga than after. Although particularly vulnerable to undetected overprinting and inhomogeneity of data quality and sampling density, this method has nonetheless enjoyed considerable success when applied to studies of younger rocks. It indicated reversal frequency minima during the late well documented Cretaceous and Permo-Carboniferous superchrons [McElhinny, 1971], and it has indicated a third minimum during Ordovician time [Johnson et al., 1995] that has recently received direct magnetostratigraphic confirmation of superchron status [Pavlov and Gallet, 2005].

The oldest well documented polarity zonation is recorded in a sequence of late Archean basalts (ca. 2775–2715 Ma), maﬁc tuffs, felsic volcanic and clastic sedimentary rocks, the Fortescue Group of Western Australia [Strick et al., 2003]. A single reversed interval is bounded above and below by normal intervals, one of which has a minimum duration of 20 to 30 Myr. This result is very well supported. The remanence is carried by basalt ﬂows and dolerite dikes, rock types that are known to be among the most stable and resistant to overprinting, and the directions after thermal demagnetization cluster in antipodal groups that pass a reversal test [McFadden and McElhinny, 1990]. The stratigraphy is well deﬁned by geologic mapping and geochemical correlation. U-Pb and Pb-Pb ion microprobe determinations on zircon and baddeleyite establish the age and duration and conﬁrm the inferred stratigraphic sequence. Moreover, the paleomagnetic directions deﬁne a simple, progressive apparent polar wander path. These results establish a minimum reversal frequency of 0.03 Myr⁻¹, a very low value, though the true rate could be higher because polarity zones might have been missed due to gaps in deposition and sampling.

The ca. 2465 Ma Matachewan-Hearst dike swarm, the roots of an early Proterozoic ﬂood basalts province in Canada, records dominantly reversed polarity, but about 25% of the dikes are normally magnetized [Halls, 1991]. On the basis of extensive sampling and consistent cross-cutting relationships, Halls [1991] interprets that only two sequential polarity intervals are recorded within an interval of a few million years, the older zone being reversed. The dikes may span a longer interval up to 30 Ma, as shown by U-Pb ages on baddeleyite and zircon [Heaman, 1997]. A low to very low reversal rate is suggested, depending on the duration of dike injection.

The North American Belt-Purcell Supergroup of silt, sandstone and volcanic rocks, spanning ~70 Myr or more from ~1468 to <1401 Ma, passes reversal and fold tests and records only four main polarity intervals [Elston et al., 2002]. Following their interpretation, one reversed interval appears to comprise about 30 Myr. Overlying this reversed interval and the following normal interval, however, are two thin zones—totaling only about one-tenth of the total
stratigraphic thickness—with mixed polarity zonation that may represent 17–19 chron or subchrons total (R. J. Enkin, personal communication, 2006). The transition between very low and high reversal frequency appears to have been abrupt, unlike the well-known polarity history of the past 150 Myr.

[15] Between about 1150 to 1050 Ma the sedimentary Grand Canyon Supergroup and the Keweenawan volcanic rocks of North America record only two reversals [Elston et al., 2002]. On the other hand, a 50 m composite section of micritic limestone and shale of the Siberian Malgin Formation records 16 reversals from ~1100–1050 Ma [Gallet et al., 2000]. These reversals are cleanly recorded, and the magnetization passes several stability tests. The Keweenawan polarity changes are not full 180° reversals, suggesting an overprint that could not be fully removed, but the Grand Canyon section appears continuous from 1100–1050 Ma and its record correlates well with the Keweenawan record. Most likely the records from the two continents are not truly coeval. Thus, just as for the earlier Belt-Purcell record described above, intervals of very low and much higher reversal rate are apparently juxtaposed.

[16] Finally, a large number of paleomagnetic studies have focused on Grenville time, the ~200 Myr immediately following the Keweenawan interval described above. Dunlop and Yu [2004] conclude that the paleomagnetic evidence supports only four main polarity chrons between 1020 and 820 Ma. They point out that because most of the results involve thermoviscous remanence that was acquired during the uplift and slow cooling of the Grenville orogen, shorter chrons could have been averaged out, but they argue that for this to happen there must have been some long periods of single polarity.

5. Conclusions

[17] Fifteen years ago, Roberts and Piper [1989] suggested that the Precambrian may have been a time of low reversal frequency on the basis of their statistical analysis of percentage of studies showing mixed polarity. Since then five detailed records between 1050 and 2775 Ma have become available [Strik et al., 2003; Halls, 1991; Elston et al., 2002; Gallet et al., 2000], which document around 45 reversals in time intervals totaling 225–250 Myr, an average reversal frequency of 0.2 Myr⁻¹. This is of course a minimum frequency, but a great many reversals would have had to be missed to bring it up to the average frequency of 1.7 Myr⁻¹ that characterizes the recent, best documented period from 0 to 150 Ma. A sixth interval with just four main chrons between 820–1020 Ma [Dunlop and Yu, 2004], although based on slowly acquired remanence that would be expected to smooth the polarity record, also adds credence to the impression of a lower Precambrian reversal rate.

[18] Nonetheless, it is not yet possible to conclude firmly from the paleomagnetic record that the reversal frequency of the ancient Earth’s field was low because of the paucity of long, continuous sections of Precambrian rocks suitable for detailed magnetostratigraphy [Gallet et al., 2000]. Our geodynamo simulations, however, provide an entirely independent line of support for this hypothesis. We have shown that the simulations with more stable fields are those with higher O/E ratios, in accord with the known correspondence between high O/E ratio and low reversal frequency for Earth’s field during the last 150 Myr [McFadden et al., 1991]. The simulation with very small inner core, such as Earth is believed to have had one billion or more years ago, has an extraordinarily high O/E ratio, which also suggests a low Precambrian reversal rate. We note that currently available data do not indicate that the average field intensity was unusually high from 1 to 2.5 Ga [Macoun et al., 2003; Smirnov et al., 2003; Dunlop and Yu, 2004], but may indicate smaller contributions of even harmonics to the field around 2.5 Ga than from 0–5 Ma [Smirnov and Tarduno, 2004]. Thus, symmetry of the time-averaged field may prove to be a more robust predictor of reversal frequency than intensity.

[19] Acknowledgments. This work was supported by NSF grants EAR 9903194, 0221941 and 0310316. Thanks to R.J. Enkin and John Tarduno for helpful discussions and reviews.

References


R. S. Coe and G. A. Glatzmaier, Department of Earth and Planetary Sciences and Institute of Geophysics and Planetary Physics, University of California, Santa Cruz, CA 95064, USA. (rcoe@pmc.ucsc.edu)