A dynamo model interpretation of geomagnetic field structures

Ulrich Christensen and Peter Olson
Department of Earth and Planetary Sciences, Johns Hopkins University, Baltimore

Gary A. Glatzmaier
Los Alamos National Laboratory

Abstract. We present numerical dynamo models which qualitatively reproduce some structural characteristics of the present-day geomagnetic field. Time-dependent convection in an electrically conducting, rotating spherical shell at Ekman numbers $1-3 \times 10^{-4}$ generates dipole-dominated magnetic fields, which exhibit distinct bundles of strong radial flux at high latitudes, low flux over the poles, and paired spots of reversed flux near the equator, suggestive of the geomagnetic field pattern at the core-mantle boundary. In our model these structures result from high magnetic Reynolds number, helical convection in columns parallel to the spin axis and rising plumes near the poles.

Introduction

Recent three-dimensional numerical dynamo models, which solve the magnetohydrodynamic equations for convection in a rotating sphere, have shown that different styles of convection can produce nearly identical axial dipole magnetic fields [Glatzmaier and Roberts, 1995a,b; Kuang and Bloxham, 1997; Kageyama and Sato, 1997]. Therefore, the relationship between field morphology and the pattern of convection in the Earth’s core is to be found in the deviations from the axial dipole.

Downward continuation of the surface field observed during the last 300 years [Bloxham et al., 1989] reveals that at the core-mantle boundary (CMB) the magnetic flux contributing to the dipole moment is concentrated in distinct patches at about 60° latitude, two each in the northern and southern hemisphere at the same longitudes. The field is low or even reversed above the poles. The equatorial region in the Atlantic hemisphere exhibits pairs of spots, arrayed roughly North-South, with inverted polarity (i.e., with positive radial field $B_r$ in the northern spot and negative $B_r$ in the southern spot). Bloxham [1986] suggested that these spots are caused by expulsion of toroidal field from the core in regions of strong radial flow. Gubbins and Bloxham [1987] related the high-latitude flux patches to the upper and lower ends of convection columns. The results of our numerical dynamo calculations basically support these interpretations.

Numerical model

Following Kageyama and Sato [1997] we calculate simple, fluid-dynamically consistent dynamo models by studying 3D time-dependent thermal convection in a rotating, electrically conducting spherical shell with inner and outer radii $r_i = 0.351$ and $r_o = 1$. Dimensionless parameters are the Ekman number $E = \nu/(\Omega D^2)$, the Prandtl number $Pm = \nu/\eta$ (fixed at 1), the modified Rayleigh number $Ra = g\Delta T D/\nu\eta$ and the magnetic Prandtl number $Pm = \nu/\eta$ (fixed at 2). $D$ is the shell thickness, $\nu$ is kinematic viscosity, $\kappa$ is thermal diffusivity, $\eta$ is magnetic diffusivity, $g$ is gravity at $r = r_o$, and $\Delta T$ is the imposed temperature difference between the inner and outer boundaries. The boundaries are rigid, electrically insulating, and co-rotate.

The basic equations and the numerical method are described in Olson and Glatzmaier [1995]. The technique differs from that of Glatzmaier and Roberts [1995a,b] by an explicit treatment of the Coriolis force, which precludes simulations at much lower $E$, but, because of smaller computational requirements, allows to run these calculations on a fast PC. We also retain the full inertia term in the momentum equation. We use 33 radial grid points, which are distributed non-equidistantly to resolve the Ekman boundary layers, and spherical harmonics up to degree and order $\ell_{max} = 42$ for $E = 3 \times 10^{-4}$ and $\ell_{max} = 53$ at $E = 10^{-4}$. In the latter case we assume twofold symmetry of the solution in longitude ($m_s = 2$), but we solve for the full sphere ($m_s = 1$) in case of the higher Ekman number. The qualitative behavior and the level of magnetic energy, averaged over one dipole decay time $t_d = r_o^2/(\pi^2 \eta)$, were essentially unchanged when we increased $\ell_{max}$ from 42 to 53 in the case with $E = 3 \times 10^{-4}$.

We start our calculation from a conductive state with a weak thermal perturbation. In the first stage of the calculation, we also impose a simple axisymmetric toroidal magnetic field that is eastward in the northern hemisphere and westward in the southern hemisphere through boundary conditions at $r_i$ and $r_o$. A strong poloidal field builds up within one dipole decay time after which we remove the imposed field. Subsequently, we monitor the field for over three dipole decay times (seven decay times in the full sphere model, which is roughly equivalent to 130,000 years). In the cases reported here, the magnetic energy fluctuated within 50% but did not decay, indicating self-sustained dynamo action.
Table 1. Time averaged results

<table>
<thead>
<tr>
<th>( m )</th>
<th>( E )</th>
<th>( Ra / Ra_{\text{crit}} )</th>
<th>( E_{\text{mag}} )</th>
<th>( E_{\text{kin}} )</th>
<th>( Re_{\text{mag}} )</th>
<th>( \Lambda )</th>
<th>( \Lambda_0 )</th>
<th>( N_u )</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>( 3 \times 10^{-4} )</td>
<td>6.0</td>
<td>2710</td>
<td>775</td>
<td>79</td>
<td>3.25</td>
<td>0.303</td>
<td>2.17</td>
</tr>
<tr>
<td>2</td>
<td>( 10^{-4} )</td>
<td>4.8</td>
<td>2560</td>
<td>960</td>
<td>88</td>
<td>1.02</td>
<td>0.041</td>
<td>1.77</td>
</tr>
<tr>
<td>2</td>
<td>( 10^{-4} )</td>
<td>6.0</td>
<td>6600</td>
<td>1240</td>
<td>100</td>
<td>2.64</td>
<td>0.167</td>
<td>2.09</td>
</tr>
</tbody>
</table>

Results

The magnetic energy \( E_{\text{mag}} \) is approximately equipartitioned between poloidal and toroidal modes and exceeds the kinetic energy \( E_{\text{kin}} \) by a factor of 2.5-5 in our models (Tab. 1). The Elsasser number \( \Lambda = B^2 / (\mu \rho \Omega) \), with \( \mu \) the magnetic permeability, measures the importance of Lorentz forces relative to the Coriolis force. It is of order one when calculated with the \( \text{rms} \)-field in the shell. Its value for the mean field at \( r = r_0, \Lambda_0 \approx 0.1 \), is similar to the value estimated from the geomagnetic field at the CMB.

Outside the inner core tangent cylinder, convection is mainly organized in columns elongated along the spin direction \( \hat{z} \) (Plate 1a,c). This is the preferred structure of rotating non-magnetic convection in a spherical shell [Busse, 1970]. Our present and previous modeling [Olsson and Glatzmaier, 1995] suggests that the basically columnar structure persists in the presence of the magnetic field, even when the Elsasser number exceeds one. As noted before [Busse, 1975; Glatzmaier, 1985; Kageyama and Sato, 1997], there is significant flow in the columns parallel to the rotation axis, equatorward in a positive vortex and poleward in a negative vortex. The combination of columnar and secondary flow means that the motion is strongly helical (Plate 2a). Our and other recent dynamo models generate the poloidal field by a 'macroscopic \( \alpha \)-effect', i.e., through helical advection and twisting of toroidal field lines. The \( \alpha \)-effect is strongest where both the helicity and the toroidal field attain significant amplitudes (Plate 2a,b), which coincides with the two centres of the poloidal field lines.

In our three cases, the dipole contributes about 70% to the field energy at \( r = r_0 \), compared to less than 50% in the geomagnetic field at the CMB. The dipole moment fluctuates within 40%. In the full sphere case, where the dipole is not constrained to be axial, its tilt remains below 6°. The non-dipole part of the field in the full-sphere model is closest to the geomagnetic field and is therefore discussed in detail.

Much of the magnetic flux contributing to the dipole is concentrated in distinct patches centered at 60-65° latitude (Plate 1d), as in the case of the Earth. This is close to, but outside, the tangent cylinder, which intersects the CMB at 70°. Typically we find 3-5 patches in each hemisphere, with a patch of opposite polarity at roughly the same longitude in the other hemisphere (Plate 1b). Comparison of Plates 1c and 1d reveals that these flux concentrations are connected with downwellings. The time-averaged correlation coefficient between \( B_r (r_0) \) and the radial velocity \( u_r \) at \( r = 0.962 \), at latitudes larger than 50°, is -0.62. Because the magnetic Reynolds number \( Re_{\text{mag}} = u_r \text{rms} D / \eta \) is of order 100 in our model, the magnetic flux is partially frozen into the fluid, hence field lines are bundled at points of surface convergence.

\( B_r \) is comparatively small in the polar regions in the model with \( E = 3 \times 10^{-4} \) (Plate 1c) where the flow includes weak polar upwelling plumes. Considering the longitudinally and time-averaged radial field \( B_r \) as a function of latitude, its polar value is 55% of the maximum (at about 65°). In times of more intense polar upwelling, the ratio drops below 40%. In the two cases with \( E = 10^{-4} \) the regions inside the tangent cylinder do not actively convect. Here the polar field strength is 87% and 77%, respectively, of the maximum. The weak polar minimum in those two cases occurs because the field is generated outside the tangent cylinder. In the model with \( E = 3 \times 10^{-4} \) the polar upwellings advect the field lines away from the pole and the polar minimum is more pronounced.

Another feature of the magnetic field in Plate 1b are flux 'spots', occuring in bipolar pairs aligned north-south across the equator, with a polarity opposite to...
Plate 2. (a) Longitudinally averaged helicity \( H = u \cdot \omega \), where \( \omega \) is vorticity, in the full sphere model. The average helicity, normalized with the product of rms-velocity and vorticity, is \(-0.41\) in the northern hemisphere and \(+0.37\) in the southern hemisphere. (b) Axisymmetric part of the toroidal field (blue: westward, red: eastward), contour interval is 0.18. Field lines show the axisymmetric poloidal field. (c) \( \omega \)-effect, i.e. azimuthal average of \( rB_r \frac{\partial (r^{-1} u_\phi)}{\partial r} + r^{-1} \sin \theta B_\theta \frac{\partial (\sin \theta r^{-1} u_\phi)}{\partial \theta} \)

that of the dipole field. These spots form in regions of strong radial flow near the outer surface (Plate 1a). Similar spots in the geomagnetic field have been interpreted as being due to the expulsion of toroidal field in an upwelling [Bloxham, 1986]. However, simple columnar upwelling should produce east-west aligned bipolar pairs of spots, similar to sun spots. To understand the north-south alignment, we must consider the field structure near the outer surface.

The toroidal field is concentrated in bundles at low latitude near the outer surface, with opposite polarity in the two hemispheres (Plate 2b). In addition, there are reversed bundles inside the tangent cylinder. The stretching of poloidal field lines by differential rotation (\( \omega \)-effect) is a possible source of toroidal field. However, while the calculated \( \omega \)-effect (Plate 2c) could explain, by its sign and distribution, the bundles in the tangent cylinder, it has the wrong sign for the more important field bundles near the equator. A detailed analysis (not presented here) reveals that this component of the field is also created by an \( \alpha \)-effect mechanism, as in the Busse dynamo [Busse, 1975]. At the prograde edges of the azimuthal field bundles in Plate 3 concentrations of radially-inward field are induced by downwelling flow. The reversed flux spots occur where this field leaks across the outer boundary. Complementary flux spots do not occur at the retrograde edge of the azimuthal field bundles because the flow there diverts the bundle toward higher latitude.

At \( E = 10^{-4} \) and \( Ra = 4.8 Ra_{crit} \), \( B \) and \( u \) are perfectly symmetric about the equator. The radial field at the surface still exhibits flux concentrations near the tangent cylinder. However, as in the simulations of Kageyama and Sato [1997] with \( E = 8 \times 10^{-4} \), the strongest concentrations of \( B_r \) occur at low latitude and equatorial reversed spots are not observed. When \( Ra \) is increased to \( 6 Ra_{crit} \), the symmetry is weakly broken and

Plate 3. Colors indicate temperature in the equatorial plane for the full sphere model. Arrows show \( dB_h/dz \), where \( B_h \) is the horizontal part of the field. Because \( B_h \) is nearly antisymmetric about the equators, the arrows may also be taken for the horizontal field slightly north of the equator (south of it, the field direction would be reversed). Zero longitude is at nine o’clock. Strong inward field in the lower left quadrant corresponds to the pair of flux spots seen in the right part of Plate 1b.
the field morphology is intermediate between that of
the other two cases. Usually the maximum of \( B_r \) is
at low latitudes, but occasionally the high-latitude flux
patches dominate and paired spots of inverse flux occur
at the equator. Hence, there seems a trend towards the
more Earth-like field morphology seen at \( E = 3 \times 10^{-4} \)
with increasing Rayleigh number.

### Discussion

Some of our parameters are quite remote from Earth
values. In particular, our Ekman number is of order
\( 10^{-4} \) compared to \( 10^{-14} \) in the core and the magnetic
Prandtl number is of order one compared to \( O(10^{-5}) \)
for liquid metals. Using \( t_d \) for scaling time implies that
our model magnetic field has 1/100 of the Earth's field
intensity. However, we obtain in our model the correct
balance between the dominant Coriolis and Lorentz for-
tices. The magnetic Reynolds number is slightly low, but
of the right order of magnitude for the core, suggesting
that the length scale of the model magnetic field agrees
with that of the geomagnetic field. The restriction to
moderate parameters enables us to solve the dynamo
equations without further parameterizations. In parti-
cular, we do not need to employ hyperdiffusivities (\( \nu, \kappa \)
and \( \eta \) increase with harmonic degree), as was necessary
in the recent models by Glatzmaier and Roberts [1995b]
and Kuang and Bloxham [1997].

The qualitative correspondence between field struc-
tures seen in our full sphere model and the geomagnetic
field at the CMB induces us to speculate about a similar
origin in terms of the large-scale flow pattern. The flux
patches at high latitudes in the geomagnetic field may
indicate downwelling near the CMB. The secondary flow
along convection columns points away from the outer
boundaries in a positive vortex. Therefore our model
supports the conjecture [Gubbins and Bloxham, 1987]
that the geomagnetic flux concentrations mark the up-
and lower ends of columns with cyclonic sense of
rotation that extend through both hemispheres of the
core.

Low flux at the Earth's poles may indicate polar up-
wellling, which would agree with the seismologically
inferred faster rotation of the inner core relative to the
mantle [Song and Richards, 1996; Su et al., 1996; Crea-
ger, 1997]. This superrotation is thought to reflect a
mean eastward circulation of fluid at the bottom of the
outer core, to which the inner core is magnetically
coupled [Glatzmaier and Roberts, 1996; Aurnou et al.,
1996]. A warm plume at the pole drives, by a thermal
wind effect, an azimuthal circulation that varies from
eastward near the inner core boundary to westward at
the CMB.

The existence of north-south aligned geomagnetic bi-
polar flux spots could hint at toroidal field bundles in
the core similar to those seen in our model. The spots
would indicate the location of radially inward flow. In
the model, these spots are more strongly centered at
the equator than they are in the Earth (where they can
be 40° off), which is possibly due to a higher degree of
equatorial symmetry in our model.

Other dynamo models show field structures similar to
those in Plate 1 (compare figure 2 of Kuang and Blox-
ham [1997]), suggesting that they are robust features.
The ability of recent dynamo models to account for par-
icular features of the geomagnetic field morphology is
surprising, given the large discrepancy between the va-
ues of some model parameters and Earth values. At
worst, this agreement might be fortuitous. At best, it
might indicate that the important ingredients for un-
derstanding the geodynamo, to first order, are only the
convection of electrically conducting fluid under the in-
fluence of rotation and the spherical shell geometry.

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U. Christensen, Institut für Gephysik, Universität
Göttingen, Herzberger Landstrasse 180, 37075 Göttingen,
Germany (e-mail: urc@willi.uni-geophys.gwdg.de)

P. Olson, Department of Earth and Planetary Sciences,
Johns Hopkins University, Baltimore MD 21218) (e-mail:
olson@gibbs. eps.jhu. edu)

G.A. Glatzmaier, Institute of Geophysics and Planetary
Physics, Los Alamos National Laboratory, Los Alamos NM
87545 (e-mail: gag@lanl.gov)

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