Class 17 Notes: Core Collapse Supernovae

Our topic today is the post-main sequence evolution of massive stars, culminating in their deaths via supernova explosion. Supernovae of this type are called core collapse supernovae, to distinguish them from supernovae that occur due to accretion onto a white dwarf that pushes it above the Chandresekhar limit.

Before beginning, I will warn you that we are now entering into an area of active research where there are still very significant uncertainties. What I will tell you is the best of our understanding today, but significant parts of it may well turn out to be wrong. I will try to highlight the areas where what I say is least certain, and I will point out a couple of places where statements asserted quite confidently in your textbook have turned out to be incorrect.

I. Post-main sequence evolution

A. Mass Loss

One important effect that distinguishes the evolution of massive stars from that of lower mass stars is the importance of mass loss, both on the main sequence and thereafter. Low mass stars do not experience significant mass loss before the AGB phase, but massive stars, as we have already seen, can lose mass while still on the main sequence, and can lose even more after they leave it.

Like other aspects of stellar mass loss, the exact mechanisms are not understood. Very massive stars, those above 85 $M_\odot$ or so, lose mass in a rapid and unstable manner. We have already encountered one star like this: η Carinae. This is an example of a type of star called a luminous blue variable, or LBV.

[Slide 1 – η Car]

As these processes reduce the star's mass, its atmosphere becomes less and less dominated by hydrogen, eventually reaching $X \approx 0.1$ or even less. We see these stars are somewhat lower mass (but still very massive) stars whose atmospheres are dominated by helium rather than hydrogen. These are called Wolf-Rayet stars, and they are effectively the bare cores of massive stars. Stars from $10 - 85$ $M_\odot$ skip the LBV phase and go directly to the Wolf-Rayet phase.

Stars become WR's while they are still on the main sequence, i.e. burning hydrogen in their centers. Stars in this case are called WN stars, because they are Wolf-Rayet stars that show large amounts of nitrogen on their surfaces. The nitrogen is the product of CNO cycle burning, which produces an equilibrium level of nitrogen above the amount that the star began its life with.

WR stars continue to lose mass rapidly, often producing spectacular nebulae that look like planetary nebulae. They shine for the same reason: the expelled gas is exposed to the high energy radiation of the star, and it floresces in response.

[Slides 2, 3 – WR nebula NGC 2359, WR 124]

Mass loss continues after the star exhausts H and begins burning He – at this point the surface composition changes and we begin to see signs of 3α burning. These are WC stars. The continuing mass loss removes the enhanced nitrogen from the CNO cycle, and convection brings to the surface the result of 3α burning, which is mostly carbon. Very rarely, we see WR stars where the carbon is being blown off, and the surface is dominated by oxygen.

The mass loss can be quite dramatic – 100 $M_\odot$ stars are thought to get down to nearly 30 $M_\odot$ by the time they evolve off the main sequence.

B. Movement on the HR diagram

While these stars show dramatic mass loss, their luminosities do not evolve all that much as they age. That is for the reason we mentioned last time in the context of low mass stars' luminosity evolution: the role of radiation pressure. The luminosity varies as $L \propto \mu^4\beta^4$, and β is in turn given by the Eddington quartic:

$$0.003\left(\frac{M}{M_\odot}\right)^2 \mu^4\beta^4 = 1 - \beta = \frac{L}{L_{Edd}}$$

For very massive stars, the first term is dominant, so $\mu\beta$ is roughly constant, and L is too. This is simply a reflection of the fact that very massive stars are largely supported by radiation pressure. As a result, their luminosity is equal to the Eddington luminosity, which depends only on total mass, not on composition.

[Slide 4 – Meynet & Maeder tracks]

This non-evolution of the luminosity continues to apply even after these stars leave the main sequence. As the stars develop inert ash cores and burning shells like lower mass stars, they cannot increase in luminosity, but they can increase in radius and go to lower effective temperature. The net effect is that they move along nearly horizontal tracks on the HR diagram. The slide shows the latest Geneva models.

As you can see, the luminosities increase less and less for stars of higher and higher masses, and instead they evolve at constant luminosity. Thus massive stars never have a red giant phase, since that would require an increase in luminosity.

C. Internal structure: the onion model

The internal structure of a massive star near the end of its lifetime comes to resemble an onion. In the center is an ash core, with the type of ash depending on the star's evolutionary state. At first it is helium, then carbon, etc., until at last the core is composed of iron. The temperature is high enough that the core is never degenerate until the last stages in the star's life, when it consists of iron.

Above the core is a burning shell where the next lowest $\mathcal{Z}$ element in the burning chain burns. Thus above an iron core is a silicon burning layer. As one moves farther outward in the star, one encounters the next burning shell where a lower $\mathcal{Z}$ element burns, and so forth until one reaches the hydrogen burning layer and the hydrogen envelope, if any is left, above it.

[Slides 5, 6 – schematic representation and numerical computation from Heger et al. of onion structure]

The onion structure grows until the star develops an iron core. Since iron is at the peak of the binding energy curve, it cannot be further burned. A star with an iron core and an onion structure around it is known as a supernova progenitor.

II. Supernovae

A. Evolution of the Core

Now consider what happens in the core of a supernova progenitor. The iron core is much like the helium core that we discussed in the context of lower mass stars: it has no nuclear reactions, so it becomes isothermal. If it gets to be more than roughly 10% of the stellar mass, it will exceed the Schönberg-Chandrasekhar limit and begin contracting dynamically. If it becomes degenerate, degeneracy pressure can slow the collapse, but if the core exceeds the Chandrasekhar mass of 1.4 $M_\odot$, electron degeneracy pressure cannot hold it up and then the core must contract.

Contraction creates two instabilities. First, at the high pressures found in the core, heavy nuclei can undergo reactions of the form

$$I(\mathcal{A},\mathcal{Z}) + e^- \;\rightarrow\; J(\mathcal{A},\mathcal{Z}-1) + \nu_e,$$

i.e. nucleus I captures a free electron, which converts one of its protons into a neutron. We'll discuss why these reactions happen in a few moments. Reactions of this sort create an instability because removing electrons reduces the number of electrons, and thus the degeneracy pressure. The loss of pressure accelerates collapse, raising the pressure again and driving the reaction to happen even faster.

Second, since the gas is degenerate, its pressure is unrelated to its temperature. As it collapses, its temperature rises, but this does not halt the collapse because it doesn't raise the pressure. Once the temperature exceeds about $6-7\times 10^9$ K, photons are able to start photodisintegrating iron via the reaction

$$^{56}\mathrm{Fe} + 100\ \mathrm{MeV} \;\rightarrow\; 13\,{}^{4}\mathrm{He} + 4n.$$

As the equation indicates, the reaction is highly endothermic, absorbing about 100 MeV from the radiation field, or about 2 MeV per nucleon, each time it happens. In effect, all the energy that was released by burning from He to Fe is now given back. The loss of thermal energy also accelerates collapse, which leads the core to contract more, which accelerates the reaction, etc.

This is an ionization-like process, which serves to keep $\gamma_a < 4/3$, in the unstable regime where collapse cannot be halted. Once the collapse proceeds far enough, an even more endothermic reaction can take place when photons begin to disintegrate helium nuclei:

$$^4\mathrm{He} + 27\ \mathrm{MeV} \ \rightarrow\ 2p + 2n.$$

This reaction absorbs 27 MeV each time it happens, or $6 - 7$ MeV per nucleon.

The disintegration of He creates a population of free neutrons and protons. Normally free neutrons spontaneously decay into a proton plus an electron plus a neutrino:

$$n \ \rightarrow\ p + e^- + \overline{\nu}_e.$$

The free neutron lifetime is 614 seconds, and the reaction is exothermic (as it must be, since it is spontaneous). The energy released can be determined just by the difference in mass between a proton, $m_p = 1.67262 \times 10^{-24}$ g, and a neutron, $m_n = 1.67493 \times 10^{-24}$ g:

$$\Delta E = (m_n - m_p)c^2 = 1.3\ \mathrm{MeV}.$$

However, conditions in the core are very different from those in free space. The electrons are highly relativistically degenerate. Consider what this means energetically. Back at the beginning of the class, we showed that, for a population of degenerate electrons, they occupy all quantum states up to a maximum momentum

$$p_0 = \left(\frac{3h^3 n}{8\pi}\right)^{1/3},$$

where n is the number density of electrons. If a new electron were to be created by the decay of a neutron, it would have to go into an unoccupied quantum state, and the first available state has a momentum a just above p_0. The corresponding energy is

$$E_0 = pc = \left(\frac{3h^3 n}{8\pi}\right)^{1/3} c$$

in the limit where the electrons are highly relativistic. If we compare this to the energy ΔE that is released by neutron decay, we find that E_0 becomes equal to ΔE when the number density of electrons becomes

$$n = \frac{8\pi}{3}\left(\frac{\Delta E}{ch}\right)^3 = 9.6 \times 10^{30}\ \mathrm{cm}^{-3}$$

If we have one electron per two nucleons (i.e., $1/\mu_e = 1/2$), the average for elements heavier than hydrogen, the corresponding mass density is

$$\rho = n m_{\mathrm{H}} \mu_e = 3.2 \times 10^7\ \mathrm{g\ cm}^{-3}.$$

Once the density exceeds this value, it is no longer energetically possible for free neutrons to undergo spontaneous decay. Instead, the opposite is true, and the reverse reaction

$$p + e^- \rightarrow n + \nu_e$$

begins to occur spontaneously. Each such reaction requires 1.3 MeV of energy, and even further reduces the degeneracy pressure of the electrons. Electron capture by heavy elements earlier on in the collapse occurs for very similar reasons.

The collapse is only halted once another source of pressure becomes available: at sufficiently high density, the neutrons become degenerate. The structure of this degenerate neutron matter is not well understood, and is a subject of active research, but the bottom line of what we understand seems to be that the collapse is halted once the density reaches around 10^{15} g cm^{-3}. The radius of the core at this point is about 40 km, although as the neutron star cools off it eventually shrinks to about 10 km. The density of 10^{15} g cm^{-3} in the core is roughly the density of an atomic nucleus, so the core at this point is a giant atomic nucleus, several km is diameter, with the mass of the Sun.

B. Explosion Mechanism and Energy Budget

All of these processes occur in the core on dynamical timescales. The initial iron core is of order $5,000 - 10,000$ km in radius, and the mass is of order a Chandrasekhar mass, about 1.5 $M_\odot$, so the dynamical time is

$$t_{\rm dyn} \sim \frac{1}{\sqrt{G\rho}} \sim 1 \text{ second.}$$

Thus the core collapses on a timescale that is tiny compared to the dynamical time of the star as a whole – the outer envelope of the star just sits there while the core collapses.

The collapse of the iron core causes the material above it to begin falling, and the exact sequence of events thereafter is somewhat unclear. Your book gives the impression that this is a solved problem, but your book is wrong on this point. Exactly how supernovae work is far from clear. Nonetheless, we can give a rough outline and make some general statements.

First of all, we can figure out the energy budget. Ultimately what drives everything is the release of gravitational potential energy by the collapse of the iron core. It is this sudden energy release that explodes the star. The core has an initial mass of $M_c \approx 1.5\ M_\odot$, and an initial radius $R_c \approx 10^4$ km. The final neutron core has a comparable mass and a radius of $R_{nc} \approx 20$ km. Thus the amount of energy released is

$$\Delta E_{\rm grav} \approx -GM_c^2\left(\frac{1}{R_c} - \frac{1}{R_{nc}}\right) \approx \frac{GM_c^2}{R_{nc}} \approx 3 \times 10^{53} \text{ erg.}$$

Of this, the amount that is used to convert the protons and electrons to neutrons is a small fraction. Each conversion (including the photodisintegration) ultimately uses up about 7 MeV, so the total nuclear energy absorption is

$$\Delta E_{\mathrm{nuc}} = 7 \text{ MeV} \frac{M_c}{m_{\mathrm{H}}} \approx 2 \times 10^{52} \text{ erg} \approx \frac{\Delta E_{\mathrm{grav}}}{15}.$$

Thus only $\sim$ 10% percent of the energy is used up in converting protons to neutrons. The rest is available to power an explosion.

Similarly, some of the energy is required to eject the stellar envelope. The binding energy of the envelope to the core is roughly

$$\Delta E_{\mathrm{bind}} = \frac{G M_c (M - M_c)}{R_c} \approx 5 \times 10^{51} \text{ erg} \approx \frac{\Delta E_{\mathrm{grav}}}{60}.$$

Thus only a few percent of the available energy is required to unbind the envelope.

The remaining energy is available to give the envelope a large velocity, to produce radiation, and to drive nuclear reactions in the envelope. We don't have a good first-principles theory capable of telling us how this energy is divided up, but we can infer from observations.

The observed speed of the ejecta is around 10,000 km s^{-1}, so the energy required to power this is

$$\Delta E_{\mathrm{kin}} = \frac{1}{2}(M - M_c) v^2 \approx 10^{51} \text{ erg} \approx \frac{\Delta E_{\mathrm{grav}}}{300}.$$

Finally, the observed amount that is released as light is comparable to that released in kinetic energy:

$$\Delta E_{\mathrm{rad}} \approx 10^{51} \text{ erg} \approx \frac{\Delta E_{\mathrm{grav}}}{300}.$$

Both of these constitute only about 1% of the total power.

So where does the rest of the energy go? The answer is that it is radiated away too, but as neutrinos rather than photons. The neutrinos (produced when the protons in the core are converted into neutrons) don't escape immediately, but they do eventually escape, and they carry away the great majority of the energy with them.

Understanding the mechanism by which the energy released in the core is transferred into the envelope of the star is one of the major problems in astrophysics today. We have a general outline of what must happen, but really solving the problem is at the forefront of numerical simulation science.

Here's what we know: as long as the collapsing core has a pressure set by relativistic electrons, its adiabatic index is $\gamma_a = 4/3$. As it approaches nuclear density and more of the electrons and protons convert to neutrons, it initially experiences an attractive nuclear force that pulls it together, and this has the effect of pushing γ_a even lower, toward 1, and accelerating the collapse. Once the densities get even higher, though, the strong nuclear force becomes repulsive, and γ_a increases to a value $\gg 4/3$.

This is sufficient to halt collapse of the core, and from the perspective of the material falling on top of it, it is as if the core suddenly converted from pressureless foam ($\gamma_a < 4/3$) to hard rubber ($\gamma_a > 4/3$). The infall is therefore halted suddenly, and all the kinetic energy of the infalling material is converted to thermal energy. This thermal energy raises the pressure, which then causes the material above the neutron core to re-expand – it "bounces". The bounce launches a shock wave out into the envelope.

The bounce by itself does not appear to be sufficient to explode the star. The shock wave launched by the bounce stalls out before it reaches the stellar surface. However, at the same time all of this is going on, the core is radiating neutrinos like crazy. Every proton that is converted into a neutron leads to emission of a neutrino, and the collapsing star is sufficiently dense that the neutrinos cannot escape. Instead, they deposit their energy inside the star above the core, further heating the material there and raising its pressure.

The neutrinos are thought to somehow re-energize the explosion and allow it to finally break out of the star. However, there are lots of details missing.

C. Nucleosynthesis

The shock propagating outward through the star from the core heats the gas up to $\sim 5 \times 10^9$ K, and this is hot enough to induce nuclear burning in the envelope. This burning changes the chemical composition of the envelope, creating new elements. Much of the material is heated up enough that it burns to the iron peak, converting yet more of the star into iron-like elements.

I say iron-like because the initial product is not in fact iron. The reason is that the most bound element, ^{56}Fe, consists of 26 protons and 28 neutrons, so it has two more neutrons than protons. The fuel, consisting mostly of elements like ^{4}He, ^{12}C, ^{16}O, ^{28}Si, all have equal numbers of protons and neutrons. Thus there are not enough neutrons around to pair up with all the protons to make ^{56}Fe.

Converting protons to neutrons is via β decays is possible, and in fact it is the first step in the *pp*-chain. However, as we learned studying that reaction, β decays are slow, and in the few seconds that it takes for the shock to propagate through the star, there is not enough time for them to occur.

The net result is that the material burns to as close to the iron peak as it can get given the ratio of protons to neutrons available. This turns out to be ^{56}Ni. This is not a stable nucleus, since it is subject to β decay, but the timescale for decay is much longer than the supernova explosion goes on for, so no beta decays occur until long after the nucleosynthetic process is over.

Not all the material in the star is burned to the iron peak. As the shock wave propagates through the star it slows down and heats things up less. The net results is that material farther out in the star gets less burned, so the supernova winds up ejecting a large amounts of other elements as well. Calculating the exact yields from first principles is one of the goals of supernova models.

D. Observations

1. Light Curves

When a supernova goes off, what do we observe from the outside? The first thing, which was only seen for the first time a couple of years ago, is a bright ultraviolet flash from the shock breaking out of the stellar surface. We saw this because Alicia Soderberg, a postdoc at Princeton got very lucky. She was using an x-ray telescope to study an older supernova in a galaxy, when she saw another one go off. The telescope was observing the star as it exploded, and it saw a flash of x-rays as the shock wave from the deep interior of the star reached the surface.

[Slide 7 – Soderberg image]

After the initial flash in x-rays, it takes a little while before the optical emission reaches its peak brightness. That is because the expanding material initially has a small area, and most of that emission is at wavelengths shortward of visible. As the material expands and cools, its optical luminosity increases, and reaches its peak a few weeks after the explosion. After that it decays. The decay can initially take one of two forms, called linear or plateau, but after a while they all converge to the same slope of luminosity versus time.

[Slide 8 – light curve image]

This slope can be understood quite simply from nuclear physics. As we mentioned a moment ago, the supernova synthesizes large amounts of ^{56}Ni. This nickel is unstable, and it undergoes the β decay reaction

$$^{56}_{28}\mathrm{Ni} \;\rightarrow\; ^{56}_{27}\mathrm{Co} + e^+ + \nu_e + \gamma$$

with a half-life of 6.1 days. This is short enough that most of the nickel decays during the initial period of brightening or shortly thereafter.

However, the resulting ^{56}Co is also unstable, and it too undergoes a β decay reaction:

$$^{56}_{27}\mathrm{Co} \;\rightarrow\; ^{56}_{26}\mathrm{Fe} + e^+ + \nu_e + \gamma.$$

This reaction has a half-life of 77.7 days, and it turns out to be the dominant source of energy for the supernova in the period from a few tens to a few hundreds of days after peak. The expanding material is cooling off, and this would cause the luminosity to drop, but the radioactive decays provide a energy source that keeps the material hot and emitting.

By computing the rate of energy release as a function of time via the β decay of cobalt-56, we can figure out how the luminosity of the supernova should change as a function of time. Radioactive decays are a statistical process, in which during a given interval of time there is a fixed probability that each atom will decay. This implies that the number of cobalt-56 decays per unit time that occur in a particular supernova remnant must be proportional to the number of cobalt-56 atoms present:

$$\frac{dN}{dt} = -\lambda N.$$

Here N is the number of cobalt-56 atoms present and λ is a constant. The equation simply asserts that the rate of change of the number of cobalt-56 atoms at any given time is proportional to the number of atoms present at that time.

This equation is easy to integrate by separation of variables:

$$\frac{dN}{N} = -\lambda\, dt \qquad \Longrightarrow \qquad N = N_0 e^{-\lambda t},$$

where N_0 is the number of atoms present at time $t = 0$. The quantity λ is known as the decay rate. To see how it is related to the half-life $\tau_{1/2}$, we can just plug in $t = \tau_{1/2}$:

$$\frac{1}{2}N_0 = N_0 e^{-\lambda\tau_{1/2}} \qquad \Longrightarrow \qquad \lambda = \frac{\ln 2}{\tau_{1/2}}.$$

For ^{56}Co, $\lambda = 0.0089$ / day.

While radioactive decay is the dominant energy source, the luminosity is simply proportional to the rate of energy release by radioactive decay, which in turn is proportional to the number of atoms present at any time, i.e. $L \propto N$. This means that the instantaneous luminosity should follow

$$L \propto e^{-\lambda t} \qquad \Longrightarrow \qquad \log L = -(\log e)\lambda t + \text{constant}.$$

Thus for the cobalt-56-powered part of the decay, a plot of $\log L$ versus time should be a straight line with a slope of

$$-(\log e)\lambda = -0.004 \text{ day}^{-1}.$$

An excellent test for this model was provided by supernova 1987A, which went of in 1987 in the Large Magellanic Cloud, a nearby galaxy. The supernova was observed for more than five years after the explosion, and as a result we got a very good measure of how its luminosity dropped. We can see a clear period when the slope follows exactly what we have just calculated. Once enough of the ^{56}Co decayed, other radioactive decays with longer half-lives took over.

[Slide 9 – light curve of SN1987A]

The effect is even more prominent in type Ia supernovae, which are produced when a white dwarf is pushed over the Chandrasekhar limit. In that case the nuclear reaction burns a much larger fraction of the star to ^{56}Ni, so its decay into cobalt and then iron completely dominates the light curve.

2. Neutrinos

   Supernova 1987A also provided strong evidence for another basic idea in supernova theory: that supernovae involve the neutronization of large amounts of matter, and with it the production of copious neutrino emission. The first detection of supernova 1987A was *not* its light. The shock wave takes some time to propagate through the star after the core collapses. The neutrinos, however, escape promptly, and on February 23, 1987 the Kamiokande II neutrino detector in Japan and the IMB detector in Ohio both measured a burst of neutrinos that arrived more than three hours before the first detection of visible light from the supernova. Burst is perhaps too strong a word, since the total number of neutrinos detected was 20 – neutrinos are hard to measure! Nonetheless, this was vastly above the noise level, and provided the first direct evidence that a supernova explosion involves release of neutrinos.

3. Historical importance

   A brief aside: because of their brightness and the long duration for which they are visible, supernovae played an important part in the early development of astronomy, and in the history of science in general. In November of 1572, a supernova went off that was, at its peak, comparable in brightness to the planet Venus. For about two weeks the supernova was visible even during the day. It remained visible to the naked eye until 1574.

   The 1572 supernova was so bright that no one could have missed it. One of the people to observe it was the Dane Tycho Brahe, who said "On the 11th day of November in the evening after sunset, I was contemplating the stars in a clear sky. I noticed that a new and unusual star, surpassing the other stars in brilliancy, was shining almost directly above my head; and since I had, from boyhood, known all the stars of the heavens perfectly, it was quite evident to me that there had never been any star in that place of the sky, even the smallest, to say nothing of a star so conspicuous and bright as this. I was so astonished of this sight that I was not ashamed to doubt the trustworthyness of my own eyes. But when I observed that others, on having the place pointed out to them, could see that there was really a star there, I had no further doubts. A miracle indeed, one that has never been previously seen before our time, in any age since the beginning of the world."

   [Slide 10 – plate from Tycho's *Stella Nova*]

   Tycho was so impressed by the event that he wrote a book about it and decided to devote his life to astronomy. He went on to make the observations that were the basis of Kepler's Laws. Kepler himself saw another supernova in 1604. The supernovae played a critical role in the history of science because they provided clear falsification of the idea that the stars were eternal and unchanging, which had dominated Western scientific thought since the time of the ancient Greeks. Previous variable events in the sky, such as comets, were taken to be atmospheric phenomena, and there was no easy way to disprove this. With the supernovae, however, they persisted long enough to make parallax observations possible. The failure to detect a parallax for the supernovae provide without a doubt that they were further away than the moon, in the supposedly eternal and unchanging realm outside the terrestrial sphere.

Unfortunately for us, Tycho's supernova was the last one to go off in our galaxy (unless one went off on the far side of the galactic center, where we wouldn't be able to see it due to obscuring dust). A number of astronomers would very much like there to be another one, since astronomical instrumentation has improved a bit since Tycho's day...

III. Supernova Remnants

The material ejected by a supernova into space slams into the interstellar medium, the gas between the stars, at a velocity up from a few to ten percent of the speed of light. When this collision happens, it creates a shock in the interstellar medium that heats interstellar gas to temperatures of millions of K. The shocked bubble filled with hot gas is known as a supernova remnant, and such remnants can be visible for many thousands of years after the supernova itself fades from view.

The association of these structures with supernovae can be demonstrated quite clearly by looking with modern telescopes at the locations of historical supernovae. For example, remnants have been identified for both Tycho's and Kepler's supernovae, and another for the Crab supernova (named after the constellation where it is located). The Crab supernova was recorded in 1054 by Chinese astronomers – no one in Europe at the time was paying attention to the sky, or if they were, they didn't bother to write it down.

[Slides 11 - 13 – Tycho's SNR, Kepler's SNR, and the Crab SNR]

We can understand the structure of a supernova remnant using a simple mathematical argument made independently by L. I. Sedov in the USSR and G. I. Taylor in the UK. These authors discovered the solution independently because Taylor discovered it while working in secret on the British atomic bomb project, which was later merged with the American one. It turns out that the problems of a supernova exploding in the interstellar medium and a nuclear bomb exploding in the atmosphere are quite similar physically. Sedov published his solution in 1946, just after the end of World War II, while Taylor's work was still secret.

Consider an idealized version of the supernova problem. An explosion occurs at a point, releasing an energy E. The explosion occurs inside a medium of constant density ρ, and we assume that the energy of the explosion is so large that the pressure it exerts is vastly greater than the pressure in the ambient material, so that the ambient gas can be assumed to be pressureless. This is a very good approximation for both supernovae and nuclear bombs. We would like to solve for the position of the shock front r as a function of time t.

The mathematical argument used to solve this relies on nothing more than fancy dimensional analysis. Consider the units of the given quantities. We have the energy E, density ρ, radius r, and time t, which have units as follows:

$$\begin{aligned} [r] &= L \\ [t] &= T \\ [\rho] &= ML^{-3} \\ [E] &= ML^2T^{-2}. \end{aligned}$$

Here L means units of length, T means units of time, and M means units of mass. Thus a density is a mass per unit volume, which is a mass per length cubed. Energy has units of ergs (CGS) or Joules (MKS), which is a mass times an acceleration times a distance, and acceleration is distance per time squared.

We want to have a formula for r in terms t, ρ, and E. It is clear, however, that there is only one way to put together t, ρ, and E such that the final answer has the units of length! The mass must cancel out of the problem, so clearly the solution must involve E/ρ. This has units

$$\left[\frac{E}{\rho}\right] = L^5T^{-2}.$$

We want to obtain something with units of length, so clearly the next step is to cancel out the T^{-2} by multiplying by t^2. This gives

$$\left[\frac{E}{\rho}t^2\right] = L^5.$$

Finally, to get something with units of L and not L^5, we must take the 1/5 power. Thus, the radius of the shock as a function of time must, on dimensional grounds, be given by

$$r = Q\left(\frac{E}{\rho}\right)^{1/5} t^{2/5},$$

where Q is a dimensionless constant. Similarly, the shock velocity as a function of time must follow

$$v = \frac{dr}{dt} = \frac{2}{5}Q\left(\frac{E}{\rho}\right)^{1/5} t^{-3/5}.$$

Actually solving the equations of fluid dynamics shows that

$$Q = \left[\left(\frac{75}{16\pi}\right)\frac{(\gamma_a - 1)(\gamma_a + 1)^2}{3\gamma_a - 1}\right]^{1/5},$$

where γ_a is the adiabatic index of the gas into which the shock propagates.

Taylor used this solution to deduce the energy of the first atomic explosion at Trinity using nothing but photos of the blast wave at different times that had been published in newspapers and magazines. When he published the result in 1950, a number of people were not happy.

For supernovae we generally can't see them expand – the expansion takes too long. However, we can obtain a relationship we can test between the temperature of the shocked material and the radius of the remnant. At the shock the kinetic energy of the expanding gas is converted into heat, so the temperature at the shock, which is a measure of internal energy per unit mass, is simply proportional to the kinetic energy per unit mass, which varies as v^2. Thus we have

$$T_{\text{shock}} \propto v^2 \propto \left(\frac{E}{\rho}\right)^{2/5} t^{-6/5}.$$

Now let us rewrite this in terms of the radius. Solving the first equation for t, we have

$$t \propto \left(\frac{E}{\rho}\right)^{-1/2} r^{5/2},$$

and plugging this into our equation for the shock temperature, we have

$$T_{\text{shock}} \propto \left(\frac{E}{\rho}\right) r^{-3}.$$

Thus the temperature of supernova remnants should decrease as the third power of their size, assuming roughly constant energy and ISM density. Small remnants such as Kepler's, Tycho's, and the Crab are visible in x-rays, but the rapid temperature drop with size ensures that, once they expand significantly, they cool off too much to be visible in x-rays.