Class 15 Notes: Stars Before the Main Sequence

In the last class we discussed the structure of hydrogen-burning stars, which are the ones that constitute the main sequence. This is the phase in which stars spend the majority of their lives. Starting in this class and for the remainder of the term, we will discuss stellar evolution to either side of the main sequence. Today’s topic is the nature of young stars, those that have not yet reached the main sequence. The next three classes will then discuss the diverse fates of stars after they leave the main sequence.

I. Star formation

A. Molecular Clouds

Any discussion of the early evolution of stars must begin with the question of how stars come to be in the first place. The topic of star formation is a vast and active field of research, and there are still numerous unanswered questions about it. However, we can sketch enough of the rough outline to get a basic picture of what must happen. In some sense, this will give us an idea of the initial conditions for a calculation of stellar evolution.

Stars form out of the interstellar medium, a diffuse gas (mean number density \( n \sim 1 \text{ cm}^{-3} \)) that fills the space between the stars. Most of this gas is atomic or ionized hydrogen with low density, but in certain places it collects into giant clouds. For reasons we will discuss in a few moments, these clouds are the places where stars form.

In these clouds the density is much higher, \( \sim 100 \text{ cm}^{-3} \), and the gas is predominantly in the form of molecular hydrogen (\( \text{H}_2 \)). These clouds are typically \( 10^4 - 10^6 \text{ M}_\odot \) in mass, but they occupy a tiny volume of the galaxy, because they are so much denser than the gas around them.

[Slide 1 – molecular clouds in the galaxy M33]

We detect these clouds mostly by the emission from the CO molecules within them. CO molecules can rotate, and their rotation is quantized. Molecules that are rotating with 1 quantum of angular momentum can spontaneously emit a photon and stop rotating (giving their angular momentum to the photon). These photons have energies of \( 4.8 \times 10^{-4} \text{ eV} \), so the corresponding frequency is

\[
\nu = \frac{E}{h} = 115 \text{ GHz}.
\]

This is in the radio part of the spectrum, and these photons can penetrate the Earth’s atmosphere and be detected by radio telescopes, which is how maps like
the one I just showed can be made. In addition to this molecular line, there are
many more that we can use, involving both different transitions of CO and of
other molecules – thousands have been detected.

These clouds are extremely cold, typically around 10 K, mainly because the CO
molecules are very efficient at radiating away energy. The clouds are also very
dusty, and the dust makes them opaque in the optical. We can see this very
clearly by comparing images of a galaxy in optical and CO emission – the places
where there are clouds show up as dark dust lanes in the optical, because the dust
absorbs all the optical light.

[Slide 2 – CO and optical images of M51]

If we zoom in to look at a single one of these clouds, we see that they are messy,
complicated blobs of gas with complex structures. These complex structures are
caused by the fact that the gas is moving around turbulently at speeds of several
kilometers per second. To make matters even more complicated the clouds are
also magnetized, and the motion of the gas is controlled by a combination of
gravity, gas pressure, and magnetic forces.

[Slides 3 and 4 – the Pipe nebula and the Perseus Cloud]

We know that stars form inside these molecular clouds because we can see them if
we look in the right way. The dusty gas is opaque in the optical, but dust absorbs
infrared light less than optical light. As a result, if we look in infrared we can see
through the dust. This is only possible from space, since the Earth’s atmosphere
is both opaque and blindingly bright in the infrared, but the Spitzer telescope
makes it possible. Infrared images of these dark clouds reveals that they are filled
with young stars.

[Slide 5 – the W5 region in optical and IR]

B. Jeans instability

So why do stars form in these cold, dense clouds, and seemingly only in them?
The basic answer is Jeans instability, a phenomenon first identified by Sir James
Jeans in 1902. The Jeans instability can be analyzed in many ways, but we will
do so with the aid of the virial theorem.

Consider a uniform gas cloud of mass $M$ and radius $R$. The density is $\rho =
3M/(4\pi R^3)$, and the density and temperature are very low, so the gas is non-
degenerate and non-relativistic. Therefore its pressure is given by the ideal gas
laws:

$$P = \frac{R}{\mu} \rho T.$$

A subtle point here is that the value of $\mu$ for an interstellar cloud is different than
it is for a star, because in a star the gas is fully ionized, while in a molecular cloud
it is neutral, and the hydrogen is all in the form of $H_2$. This configuration has
$\mu = 2.33$. 

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If this (spherically-symmetric) cloud is in hydrostatic equilibrium, it must satisfy the virial theorem, so the cloud must have

\[ U_{\text{gas}} = -\frac{1}{2} \Omega. \]

The gravitational potential energy is

\[ \Omega = -\alpha \frac{G M^2}{R}, \]

where \( \alpha \) is our standard fudge factor of order unity that depends on the internal density structure. The internal energy is

\[ U_{\text{gas}} = \frac{3}{2} \int \frac{P}{\rho} \, dm = \frac{3}{2} \frac{R}{\mu} \int T \, dm = \frac{3}{2} \frac{R}{\mu} M \overline{T}, \]

where \( \overline{T} \) is the mean temperature. This is something of an approximation as far as the coefficient of \( 3/2 \). In general a diatomic molecule like \( \text{H}_2 \) should have \( 5/2 \) instead. The only reason we keep \( 3/2 \) is due to an odd quantum mechanical effect: the levels of \( \text{H}_2 \) are quantized, and it turns out that the lowest lying ones are not excited at temperatures as low as 10 K. Thus the gas acts to first approximation like it is monatomic. In any event, the exact value of the coefficient is not essential to our argument.

Plugging \( \Omega \) and \( U_{\text{gas}} \) into the virial theorem, we obtain

\[ \alpha \frac{G M^2}{2R} = \frac{3R}{2\mu} M \overline{T}. \]

It is convenient to rewrite this using density instead of radius as the variable, so we substitute in \( R = \left(\frac{3M}{4\pi \rho}\right)^{1/3} \). With this substitution and some rearrangement, the virial theorem implies that

\[ M = \frac{9}{2\sqrt{\pi} \alpha^3} \left( \frac{R}{\mu G} \right)^{3/2} \sqrt{\frac{\overline{T}^3}{\rho}}. \]

Thus far we have a result that looks very much like the one we derived for stars: there is a relationship between the mass, the mean temperature, and the radius or density. In fact, this equation applies equally well to stars and interstellar gas clouds. The trick comes in realizing that stars and cloud respond very differently if you perturb them.

Consider compressing a star or cloud, so that \( \rho \) increases slightly. The mass is fixed, so in a star the gas responds by heating up a little – \( T \) rises so that the term on the right hand side remains constant. A gas cloud tries to do the same thing, but it encounters a big problem: the molecules out of which it is made are very, very good at radiating energy, and they have a particular, low temperature they want to be. Unlike a star, where it takes the energy a long
time to get out because the gas is very opaque, gas clouds are transparent to the radio waves emitted by the molecules. Thus the cloud heats up slightly, but rapid radiation by molecules forces its temperature back down to their preferred equilibrium temperature immediately. One way of putting this is that for a star, $t_{\text{dyn}} \ll t_{\text{KH}}$, but for a molecular cloud exactly the opposite is true: $t_{\text{dyn}} \gg t_{\text{KH}}$.

This spells doom for the cloud, because now it cannot satisfy the virial theorem, and thus it cannot be in hydrostatic equilibrium. Instead, $U_{\text{gas}} \propto P/\rho$ is too small compared to $\Omega$. This means that the force of gravity compressing the cloud is stronger than the pressure force trying to hold it up. The cloud therefore collapses some. This further increases $\Omega$, while leaving $U_{\text{gas}}$ fixed because the molecules stubbornly keep $T$ the same. The cloud thus falls even further out of balance, and goes into a runaway collapse. This is the Jeans instability. The process ends only when the gas forms an opaque structure for which $t_{\text{dyn}} < t_{\text{KH}}$ – that is a newborn star.

As a result of this phenomenon, given the temperature at which the molecules like to remain, one can define a maximum mass cloud that can avoid collapsing due to Jeans instability. This is known as the Bonnor-Ebert mass, and its value is

$$M_{\text{BE}} = 1.18 \left( \frac{R}{\mu G} \right)^{3/2} \sqrt{\frac{T^3}{\rho}} = 4.03 \times 10^{34} \sqrt{\frac{T^3}{\rho}} \text{ cm}^3 \text{ cgs units.}$$

The factor of 1.18 comes from self-consistently solving for the structure of the cloud, thereby determining the coefficient $\alpha$. We also used $\rho = \mu m_{\text{H}} n$. An important property of $M_{\text{BE}}$ is that it is smallest in clouds with low temperature and high densities. In other words, regions that are dense and cold, like molecular clouds, have very small maximum masses that can be supported, while warmer, more diffuse regions have much larger masses.

Let’s put some numbers on this. First think about a region of atomic gas. These typically have number densities of $n \sim 1 \text{ cm}^{-3}$, $\mu = 1.67$ (because the gas is not ionized), and temperatures of $T = 8000 \text{ K}$. Plugging in these numbers, we get a maximum mass $M = 5 \times 10^6 \ M_\odot$ – in other words, huge clouds can be held up by pressure. On the other hand, let’s try this for the interior of a molecular cloud, where the number density can be $n = 10^3 \text{ cm}^{-3}$ and the temperature $T = 10 \text{ K}$. These numbers give $M = 4 \ M_\odot$.

This leads to two conclusions. First, it explains why stars form in molecular clouds: they are much, much too massive to be stable against self-gravity given their temperatures and densities. They have no choice but to collapse, whereas lower density, warmer atomic regions won’t. Second, the characteristic mass scale set by this instability in the densest regions where stars form suggests an explanation for why the typical star is comparable to the Sun in mass, and not a million times more or less massive. The mass of the Sun is about the characteristic mass at which things are prone to going into collapse because they can no longer support themselves!
C. Cores

Let’s consider one of these collapsing blobs of gas and try to understand what will happen to it. Ordinarily these things come in clusters, but occasionally we can see one in isolation, and the most spectacular example is probably the object known as B68.

[Slide 6 – the core B68]

This particular object was first seen by William Herschel (the discoverer of Uranus) in the 1700s. When he saw it, in optical of course, he remarked “My God, there is a hole in the skies!” He attributed this to the inevitable decay of the cosmos caused by the Fall, and thought that it was a place where the stars had burned out. Today of course we know that this blob of gas is in fact the genesis of a new star, and that the only reason it appears dark is because the dust mixed with the gas is blocking out the background light.

We refer to objects like B68, which have masses of $\sim M_\odot$ and radii of $\sim 0.1$ pc, as cores. In the case of B68, we don’t see a star in the center when we look in infrared, which indicates that this core has not yet collapsed to form a star at its center. However, we can work out how objects like this collapse.

Suppose at first that we neglect pressure support, and ask how long it will take before the gas at the edge of an unstable core collapses into the center. Consider a spherical core in which the mass interior to a radius $r$ is $m$, and consider the shell of material of mass $dm$ that starts at rest at radius $r_0$.

Since the mass interior to $r_0$ is $m$, the initial gravitational potential energy of the shell is

$$E_{g,0} = -\frac{Gm dm}{r_0}.$$ 

If we come back and look some time later, when the shell has fallen inward to radius $r$, its new potential energy is

$$E_g = -\frac{Gm dm}{r}.$$ 

The kinetic energy of the shell is

$$E_k = \frac{1}{2} dm \left(\frac{dr}{dt}\right)^2$$

Since no work is being done on the shell other than by gravity (since we have neglected pressure forces), conservation of energy requires

$$-\frac{Gm dm}{r} + \frac{1}{2} dm \left(\frac{dr}{dt}\right)^2 = -\frac{Gm dm}{r_0}$$

$$\frac{dr}{dt} = -\sqrt{2Gm \left(\frac{r_0}{r} - 1\right)^{1/2}}$$
To figure out when a given shell reaches the protostar at $r = 0$, we integrate from the time $t' = 0$ when the shell is at $r_0$ to the time $t' = t$ when it reaches protostar at the center of the core:

$$- \int_0^t \sqrt{\frac{2Gm}{r_0}} \, dt' = \int_{r_0}^r \left( \frac{r_0}{r'} - 1 \right)^{-1/2} \, dr'$$

The integral on the LHS is trivial, since $\sqrt{2Gm}$ doesn’t change with time, and the integral on the RHS can be done via the trigonometric substitution $r' = r_0 \cos^2 \xi$:

$$- \sqrt{\frac{2Gm}{r_0}} t = \int_{r_0}^r \left( \frac{r_0}{r'} - 1 \right)^{-1/2} \, dr'$$

$$= -2r_0 \int_0^{\pi/2} \left( \frac{1}{\cos^{-2} \xi - 1} \right)^{1/2} \cos \xi \sin \xi \, d\xi$$

$$= -2r_0 \int_0^{\pi/2} \cos^2 \xi \, d\xi$$

$$= -r_0 \left( \xi + \frac{1}{2} \sin 2\xi \right) \bigg|_0^{\pi/2}$$

$$= -r_0 \frac{\pi}{2}$$

Solving, we find that the time when a shell reaches the star is

$$t = \frac{\pi}{2} \sqrt{\frac{r_0^3}{2Gm}}$$

If the mean density interior to $r_0$ is $\rho$, then $m = (4/3)\pi r_0^3 \rho$, and we get

$$t = \sqrt{\frac{3\pi}{32G\rho}} \equiv t_{\text{ff}}$$

which defines the free-fall time $t_{\text{ff}}$, the time required for an object to collapse when it is affected only by its own gravity. Note that $t_{\text{ff}}$ is just the dynamical time multiplied by a constant of order unity.

For cores that form stars, typical densities are $n = 10^5 \text{ cm}^{-3}$ and mean molecular masses are $\mu = 2.33$; so $\rho = \mu m_H n = 4 \times 10^{-19} \text{ g cm}^{-3}$. Plugging this in gives $t_{\text{ff}} = 10^5$ yr – this is how long it would take a core to collapse if it were affected only by gravity. Of course there really is some pressure which opposes the collapse, and a more thorough analysis that includes the pressure shows that it increases the collapse time by a factor of a few. Nonetheless, what this shows is that, once a core forms, in a few hundred thousand years it must undergo collapse.

II. Protostars

Now that we have understood something about how the star formation process begins, let us turn our attention to the objects that are created by it: protostars. As we have
already mentioned, a protostar first appears when the gas gets sufficiently dense that it becomes opaque, and $t_{KH}$ becomes longer than $t_{dyn}$. At this point it becomes possible to satisfy the virial theorem, and a hydrostatic object forms, accumulating the gas that rains down on it from its parent collapsing core.

A. Accretion Luminosity

Protostars in this configuration can be extremely bright – not because they shine from nuclear fusion like main sequence stars, but because of the material raining down onto their surfaces. To get a sense of how this works, a protostar of mass $M$ and radius $R$, accreting at a rate $\dot{M}$.

The material falling onto the star started out a long distance away, which we can approximate as being infinitely far away. Its energy when it starts is zero, and conservation of energy dictates that, right before it hits the stellar surface, its kinetic and potential energy add up to zero. If we consider a blob of infalling material of mass $dm$, this means that

$$0 = \Omega + K = -\frac{GM}{R} \, dm + \frac{1}{2} \, dm \, v^2.$$ 

Thus its velocity right before it hits the stellar surface is

$$v_{ff} = \sqrt{\frac{2GM}{R}},$$

which is called the free-fall velocity. For $M = M_\odot$ and $R = R_\odot$, $v_{ff} = 620$ km s$^{-1}$ – the gas is moving fast!

When the gas hits the stellar surface, it comes to a stop, and its kinetic energy drops to zero. This energy must then go into other forms. Some of it goes into internal energy: the gas heats up and its chemical state changes from molecular to ionized. The rest goes into radiation that escapes from the star, and which we can observe.

We can fairly easily establish that the fraction of the energy that goes into dissociating the molecules and then ionizing the atoms can’t be very significant. Dissociating a hydrogen molecule requires 4.5 eV, and ionizing a hydrogen atom requires 13.6 eV, so for each hydrogen atom that falls onto the star,

$$\chi = 13.6 \text{ eV} + \frac{4.5 \text{ eV}}{2} = 15.9 \text{ eV}$$

go into dissociating and ionizing it. In contrast, the atom arrives at 620 km s$^{-1}$, so its kinetic energy is

$$K = \frac{1}{2} m_H v_{ff}^2 = 2.0 \text{ keV}.$$ 

Thus, less than 1% of the energy is used up in dissociating and ionizing the gas.
The rest goes into heat and radiation. Figuring out exactly how much goes into each is a complicated problem that wasn’t really solved until the 1980s and 1990s, but the answer turns out to be that it is about half and half. Thus, to good approximation, half the kinetic energy of the infalling gas comes out as radiation.

To see what this implies about the luminosity, consider that, for an accretion rate $\dot{M}$, an amount of mass $dm = \dot{M} dt$ must arrive over a time $dt$. In this amount of time, the amount of energy radiated is

$$dE = \frac{1}{2} \left( \frac{1}{2} v^2 \right) dm = \frac{GM}{2R} dm,$$

where we have assumed that exactly half the energy comes out as radiation, and we have neglected the 1% correction due to energy lost to ionization and dissociation. To get the luminosity, we divide both sides by the time $dt$ over which the energy is emitted, which gives

$$L = \frac{dE}{dt} = \frac{GM\dot{M}}{2R}.$$

On your homework you will use this result to do a somewhat more sophisticated calculation of what sort of luminosity something like the proto-Sun should put out, but we can make a simple estimate now. Recall that we said that the collapse of a protostellar core takes a few hundred thousand years. To accumulate the mass of the Sun in this time, the accretion rate must be roughly $\dot{M} \sim 10^{-5} M_\odot \, yr^{-1}$. Plugging this in, along with $M = M_\odot$ and $R = 2 R_\odot$ (since the radius of a protostar is generally bigger than that of a pre-main sequence star), gives $L = 100 L_\odot$. Thus a proto-Sun would be roughly 100 times as bright as the same star on the main sequence.

B. Hayashi Contraction

In addition to the radiation emitted by infalling material as it strikes the stellar surface, the star itself also radiates. However, since the protostar is initially not hot enough to burn hydrogen, it has no internal source of nuclear energy to balance out this radiation, and it is forced to contract on a Kelvin-Helmholtz timescale. (It can burn deuterium, but this all gets used up on a timescale well under the KH timescale.)

This contracting state represents the “initial condition” for a calculation of stellar evolution. In terms of the $(\log T, \log \rho)$ plane describing the center of the star, we already know what this configuration looks like: the star lies somewhere on the low $T$, low $\rho$ side of its mass track, and it moves toward the hydrogen burning line on a KH timescale. We would also like to know what it looks like on the HR diagram, since this is what we can actually observe. Therefore we want to understand the movement of the star in the $(\log T_{\text{eff}}, \log L)$ plane.

To figure this out, we can approximate the protostellar interior as a polytrope with

$$P = K_P \rho^{(n+1)/n},$$
or
\[ \log P = \log K_P + \left( \frac{n+1}{n} \right) \log \rho. \]

Recalling way back to the discussion of polytropes, the polytropic constant \( K_P \) is related to the mass and radius of the star by
\[ K_P \propto M^{(n-1)/n} R^{(3-n)/n} \implies \log K_P = \left( \frac{n-1}{n} \right) \log M + \left( \frac{3-n}{n} \right) \log R + \text{constant}, \]
so we have
\[ \log P = \left( \frac{n-1}{n} \right) \log M + \left( \frac{3-n}{n} \right) \log R + \left( \frac{n+1}{n} \right) \log \rho + \text{constant}. \]

Now consider the photosphere of the star, at radius \( R \), where it radiates away its energy into space. If the density at the photosphere is \( \rho_R \), then hydrostatic balance requires that
\[ \frac{dP}{dr} = -\rho_R \frac{GM}{R^2} \implies P_R = \frac{GM}{R^2} \int_R^\infty \rho \, dr, \]
where \( P_R \) is the pressure at the photosphere and we have assumed that \( GM/R^2 \) is constant across the photosphere, which is a reasonable approximation since the photosphere is a very thin layer. The photosphere is the place where the optical depth \( \tau \) drops to a value below \( \sim 1 \). Thus we know that at the photosphere
\[ \kappa \int_R^\infty \rho \, dr \approx 1, \]
where we are also approximating that \( \kappa \) is constant at the photosphere. Putting this together, we have
\[ P_R \approx \frac{GM}{R^2 \kappa} \implies \log P_R = \log M - 2 \log R - \log \kappa + \text{constant}. \]

For simplicity we will approximate \( \kappa \) as a powerlaw of the form \( \kappa = \kappa_0 \rho_R T_{\text{eff}}^b \), where \( T_{\text{eff}} \) is the star’s effective temperature, i.e. the temperature at its photosphere. Free-free opacity is \( b = -3.5 \). Plugging this approximation in gives
\[ \log P_R = \log M - 2 \log R - \log \rho_R - b \log T_{\text{eff}} + \text{constant}. \]

Finally, we know that the ideal gas law applies at the stellar photosphere, so we have
\[ \log P_R = \log \rho_R + \log T_{\text{eff}} + \text{constant}, \]
and we have the standard relationship between luminosity and temperature
\[ \log L = 4 \log T_{\text{eff}} + 2 \log R + \text{constant} \]
We now have four equations

\[
\log P_R = \left(\frac{n-1}{n}\right) \log M + \left(\frac{3-n}{n}\right) \log R + \left(\frac{n+1}{n}\right) \log \rho_R + \text{constant}
\]

\[
\log P_R = \log M - 2 \log R - \log \rho_R - b \log T_{\text{eff}} + \text{constant}
\]

\[
\log P_R = \log \rho_R + \log T_{\text{eff}} + \text{constant}
\]

\[
\log L = 4 \log T_{\text{eff}} + 2 \log R + \text{constant}
\]

in the four unknowns \(\log T_{\text{eff}}, \log L, \log \rho_R, \) and \(\log P_R\). Solving these equations (and skipping over the tedious algebra), we obtain

\[
\log L = \left(\frac{9 - 2n + b}{2 - n}\right) \log T_{\text{eff}} - \left(\frac{2n - 1}{2 - n}\right) \log M + \text{constant}.
\]

Thus to figure out the slope of a young star’s track in the HR diagram, we need only specify \(n\) and \(b\). Many young stars are fully convective due to their high opacities, so \(n = 1.5\) is usually a good approximation, so that just leaves \(b\). For free-free opacity \(b = -3.5\), but we must recall that a young star is initially quite cold, about 4000 K. This makes its opacity very different from that of main sequence stars. In main sequence stars, the opacity is mostly free-free or, at high temperatures, electron scattering. At the low temperatures of protostars, however, there are too few free electrons for either of this to be significant, and instead the main opacity source is bound-bound. One species in particular dominates: \(\text{H}^-\), that is hydrogen with two electrons rather than one.

The \(\text{H}^-\) opacity is very different than the opacities we’re used to, in that it strongly increases rather than decreases, with temperature. That is because higher temperatures produce more free electrons via the ionization of metal atoms with low ionization potentials, which in turn can combine with hydrogen to make more \(\text{H}^-\). Once the temperature passes several thousand K, \(\text{H}^-\) ions start falling apart and the opacity decreases again, but in the crucial temperature regime where protostars find themselves, opacity increases extremely strongly with temperature: \(\kappa_{\text{H}^-} \propto \rho T^4\) is a reasonable approximation, giving \(b = 4\).

Plugging in \(n = 1.5\) and \(b = 4\), we get

\[
\log L = 20 \log T_{\text{eff}} - 4 \log M + \text{constant}.
\]

Thus the slope is 20, extremely large. Stars in this phase of contraction therefore make a nearly vertical track in the HR diagram. This is called the Hayashi track. Stars of different masses have Hayashi tracks that are slightly offset from one another due to the \(4 \log M\) term, but they are all vertical.

Contraction along the Hayashi track ends once the star contracts and heats up enough for \(\text{H}^-\) opacity not to dominate, so that \(b\) is no longer a large positive number. Once \(b\) becomes 0 or smaller, as the opacity changes over to other sources, the track flattens, and the star contracts toward the main sequence at roughly fixed luminosity but increasing temperature. This is known as a Heyney track.
Only stars with masses $\sim M_\odot$ or less have Hayashi phases. More massive stars are “born” hot enough so that they are already too warm to be dominated by $H^-$.