An experimental and numerical study of surface tension-driven melt flow

R.A. Parsons a,⁎, F. Nimmo a, J.W. Hustoft b, B.K. Holtzman c, D.L. Kohlstedt b

a Department Earth and Planetary Sciences, University of California, 1156 High St., Santa Cruz, CA 95064, USA
b Department Geology and Geophysics, University of Minnesota, 108 Pillsbury Dr. SE, Minneapolis, MN 55455, USA
c Lamont-Doherty Earth Observatory, P.O. Box 1000, 61 Route 9W, Palisades, New York 10964, USA

Received 21 December 2006; received in revised form 20 November 2007; accepted 30 November 2007
Available online 23 December 2007
Editor: C.P. Jaupart

Abstract

To determine the role of surface tension-driven melt migration in planetary bodies, we investigated the effect of static annealing on the evolution of melt-rich bands in partially molten samples. In shear deformation experiments, deviatoric stress causes melt to segregate; when the stress is removed, surface tension causes the melt to relax back to a homogeneous distribution. Samples composed of 76 vol.% olivine + 20 vol.% chromite + 4 vol.% MORB were deformed to shear strains of ~3.5 at 1523 K, 300 MPa and shear stresses of 20 to 55 MPa. After deformation, the samples were statically annealed for 0, 10, or 100 h. During annealing, melt transport driven by surface tension occurs, but takes place much more slowly than flow driven by deviatoric stress. Finite difference numerical simulations were performed of surface tension-driven melt flow resisted by viscous deformation of the olivine matrix. These models best reproduce the distribution of melt in the annealed samples when the solid viscosity \( \eta_s = 1.7 \pm 0.5 \times 10^{12} \) Pa s with \( n = 2.4 \pm 0.3 \) and \( b = 9000 \pm 1900 \) in the expression for permeability \( \kappa = \phi n d^2 / b \) where \( d \) is grain size. The large value of \( b \) compared with estimates from geometrical models is probably due to clogging of the melt tubes by the secondary solid phase (chromite). Redistribution of melt by surface tension is likely to be the dominant process in small (~10 km radius) planetesimals in the absence of convection or impact-induced deformation. However, this redistribution process is sufficiently slow that large bodies of localized melt (magma chambers) are likely to develop.

© 2007 Elsevier B.V. All rights reserved.

Keywords: surface tension; melt migration; permeability; planetesimals

1. Introduction

The motion of melt through the mantles of planetary bodies has a fundamental influence on the mechanical and chemical properties of both melt and mantle. In the Earth, a combination of differential stress and buoyancy forces drives the porous flow of melt at a velocity of ~3 mm s⁻¹ under mid-ocean ridges, as recorded by U-series isotopes (McKenzie, 2000). However, over shorter length scales, or in cases where gravity (and thus buoyancy forces) is reduced, surface tension may be an important driver of melt motion (Stevenson, 1986). In this paper, we carry out a series of experiments to investigate the role of surface tension on melt transport, building on earlier work (Riley and Kohlstedt, 1991). We also use the results of our experiments to investigate redistribution of melt in a setting (small planetesimals) where buoyancy forces are less important than on Earth.

Simple-shear deformation of mantle-type rocks composed of olivine, chromite, and mid-ocean ridge basalt (MORB) at elevated temperature and pressures (1523 K, 300 MPa) produces anastomosing networks of melt-enriched (MORB) regions separated by melt-depleted lenses (Holtzman et al., 2003) as shown in Fig. 1a. During shear, the weak, melt-rich band regions orient antithetic to the shear direction and accommodate...
strain via viscous flow. The viscosity reduction in these melt-rich regions reduces the local pore pressure and results in an inward flux of MORB from the triple junctions along olivine and chromite grains in neighboring regions. In turn, addition of melt further weakens the melt-rich regions, resulting in an instability which segregates melt present in the deforming sample. This stress-driven flow is associated with local grain compaction in the melt-depleted regions and matrix decompression in the melt-rich bands. The primary force resisting melt segregation is surface tension (Holtzman et al., 2003). Under static conditions, experimental results indicate that surface tension dominates melt flow at short length scales, driving melt from regions of high concentration to regions of low concentration. The capillary force results from a gradient in melt fraction across an otherwise homogeneous two-phase material (Riley and Kohlstedt, 1991). Experimental studies by Riley and Kohlstedt (Riley and Kohlstedt, 1991; Riley et al., 1990), Watson (1982), and Daines and Kohlstedt (1993) address the role of surface tension in redistributing melt across a coupled melt source and melt sink. These studies indicate that compaction theory (McKenzie, 1984) together with a formalism for the capillary force (Riley and Kohlstedt, 1991) provides a good description of the melt redistribution observed in the experiments. Here, we provide further insight into the role of surface tension in laboratory samples by observing and modeling the redistribution of melt produced by static annealing of samples in which prior deformation had induced melt segregation. Our experiments differ from those done previously in two ways. First, our 100 h long annealed experiment is significantly longer than prior experiments (2 h in Riley and Kohlstedt (1991) and 32 h in Daines and Kohlstedt (1993)). Second, previous experiments either relied on analog materials (Wark et al., 2003) or used coupled discs of a MORB source and an olivine sink to do anneal experiments (Riley and Kohlstedt, 1991; Watson, 1982; Daines and Kohlstedt, 1993). Here we annealed samples in which deformation had previously segregated melt into melt-rich bands and melt-depleted lenses in a fashion we might expect to see in terrestrial bodies.

Our knowledge of the interior structure and thermal history of planetesimals is poor. However, it appears that the asteroid Vesta (diameter of 520 km) is differentiated based on the howardite, eucrite, and diogenite (HED) families of meteorites thought to have originated there (Ruzicka et al., 1997; Drake, 2001). The likely heat source responsible for melting and differentiation of small bodies shortly after the formation of the solar system is the radioactive decay of $^{26}$Al (half-life of 0.72 Myr) or $^{60}$Fe (half-life of 1.5 Myr) (Baker et al., 2005; Greenwood et al., 2005; Ghosh and McSween, 1998). This hypothesis is supported by the detection of the decay products of $^{26}$Al and other comparably short-lived isotopes in some meteorites (e.g. Srinivasan et al., 1999; Bizzaro et al., 2005). Assuming a thermal diffusivity of $10^{-6}$ m$^2$ s$^{-1}$, the thermal diffusion timescale of an object with a radius of 20 km suggests a melt longevity of a few Myr. For such an object, redistribution of melt is likely to be driven by surface tension rather than buoyancy forces (see Section 2.1 below).

Scaling surface tension effects to conditions present in asteroid interiors is difficult given the range of possible characteristics affecting melt flow through asteroidal bodies. However, the diogenite, eucrite, and angrite classes of achondrite meteorites suggest sustained temperatures of $\approx 1100$ °C in a static environment (Hutchison, 2004). The minerals in these samples are generally dominated by Ca-rich plagioclase and pyroxene with grain sizes ranging from 0.5 to 4 mm together with trace amounts of olivine, nickel, and chromite. Although the magma present in these bodies evolved significantly during the crystallization process, the bulk composition of the HEDs and angrites suggests a basaltic magma with silica content between 40 and 50%. Meteoritic samples that are more “mantle-like” in composition are few in number, perhaps due to collisional disintegration during or after the disruption of planetesimal parent body/bodies (Burbine et al., 1996). These samples of olivine-rich (>75 vol.%)
achondrites (brachinites) suggest olivine grain sizes range from 0.7 to 1.6 mm in the mantles of planetesimals (Irving et al., 2005).

Vesta was large enough to sustain magma oceans in its interior for as long as 100 Myr (Bizzaro et al., 2005) following $^{26}$Al decay to $^{26}$Mg (Greenwood et al., 2005). Surface tension is unlikely to have redistributed melt over significant distances in Vesta due to its gravity and deformation induced by internal convection. However, the HEDs derived from Vesta and the angrites and brachinites derived from an unknown parent body provide the best samples of platonically cooled melt in bodies smaller than the Moon. We use the properties of these meteorites to scale our model to bodies with a radius that would provide the appropriate conditions for surface tension-driven melt migration to take place.

This paper is organized as follows: first, we describe the theory behind the dynamics of surface tension-driven flow through a porous medium and its application to planetesimals. We then discuss the experimental method and the results from annealing deformed samples of synthetic dunite. In the next section we implement the aforementioned theory in a numerical model to simulate the annealing experiments. We conclude by discussing implications for melt flow in the interiors of planetesimals.

2. Theory

When a liquid is in contact with another substance, be it solid, liquid, or gas, there is a free interfacial energy present between them. The interfacial energy arises from the difference between the inward attraction of the molecules in the interior of each phase and those at the surface of contact (Bear, 1972). If the surface energy of grain–grain contact is less than that of melt–grain contact, then a gradient in melt concentration results in an energy gradient that can drive melt flow (Stevenson, 1986). Melt migration in this case occurs from regions of high to regions of low melt fraction. A general expression for the redistribution of melt in the absence of buoyancy forces is given by Riley and Kohlstedt (1991):

$$\frac{\gamma_{sl}H(\theta)}{\sqrt{3d}} \phi^{-3/2} \frac{\partial \phi}{\partial z} + \kappa W = \left( \frac{\partial \eta_{pm} \partial W}{\partial z} + \eta_{pm} \frac{\partial^2 W}{\partial z^2} \right)$$

(1)

where $\gamma_{sl}$ is the solid–liquid interfacial surface energy, $d$ is the grain size, $\phi$ is the melt fraction, $z$ is distance, $\mu$ is the liquid viscosity, $\kappa$ is the permeability, $\eta_{pm}$ is the viscosity of the partially molten aggregate, and $W$ is the velocity of the solid grains.

Eq. (1) describes how the matrix velocity $W$ evolves under the influence of surface tension, matrix compaction and redistribution of the melt phase. The first term on the left side of Eq. (1) is the capillary force per unit volume. The right side is the viscous force and the remaining term relates the net force density to the deformation of the solid matrix. This term is, in fact, a pressure gradient as illustrated in Riley and Kohlstedt (1991) Eqs. (14), (15), and (19). Values of the parameters in Eq. (1) are given in Table 1. $H(\theta)$ is a dimensionless geometric constant which depends on the dihedral angle, $\theta$, as defined in Riley and Kohlstedt (1991). $H(\theta) \approx 3$ for all $\theta$ considered here.

<table>
<thead>
<tr>
<th>Description</th>
<th>Symbol</th>
<th>Value(s)</th>
<th>Reference</th>
</tr>
</thead>
<tbody>
<tr>
<td>Interfacial energy</td>
<td>$\gamma_{sl}$</td>
<td>0.5 J m$^{-2}$</td>
<td>Cooper and Kohlstedt (1982)</td>
</tr>
<tr>
<td>Melt viscosity</td>
<td>$\mu$</td>
<td>10 Pa s</td>
<td>Shaw (1969)</td>
</tr>
<tr>
<td>Dry olivine viscosity*</td>
<td>$\eta_{sl}$</td>
<td>$4 \times 10^{12}$ Pa s</td>
<td>Mei et al. (2002), Scott and Kohlstedt (2006), Zhao et al. (2005)</td>
</tr>
<tr>
<td>Grain size</td>
<td>$d$</td>
<td>8.8 µm</td>
<td>This study</td>
</tr>
<tr>
<td>Dihedral angle</td>
<td>$\theta$</td>
<td>17±5°</td>
<td>This study</td>
</tr>
<tr>
<td>Permeability exponent*</td>
<td>$n$</td>
<td>2 (tubes)–3 (sheets)</td>
<td>Turcotte and Schubert (2002)</td>
</tr>
<tr>
<td>Geometric term*</td>
<td>$b$</td>
<td>$10^4$</td>
<td>Holtzman et al. (2003)</td>
</tr>
</tbody>
</table>

* Note: these parameters are treated as variables in our simulations.

Work by Cooper and Kohlstedt (1982) gives an interfacial energy of $\gamma_{sl}=0.5$ J m$^{-2}$.

The dihedral angle influences the shape of the channels. Values reported for $\theta$ vary greatly in the literature. Recent works by Renner et al. (2003) and Holness (2006) suggest values of 25° and 29±13°, respectively, but angles as low as 0°–10° (Cmíral et al., 1998) have also been reported for the olivine–basalt system. Riley and Kohlstedt (1991) measured a mean value of 42°. We measured an angle of 17±5° in our samples (Section 3.3).

The permeability is given by

$$\kappa = \frac{d^2 \phi^n}{b}$$

(2)

(e.g. McKenzie 1989) where $d$ is the olivine grain diameter, $n$ is the melt fraction exponent, and $b$ is a geometric term. Channel geometry is influenced by the geometric term in Eq. (2) and the dihedral angle. In our model, we use a value of $d=8.8$ µm measured from the experiments. In Eq. (2), $n$ governs the sensitivity of permeability to changes in melt fraction. Fluid flow through a network of interconnected tubes gives a value of $n=2$ whereas $n=3$ is associated with flow through sheets (Turcotte and Schubert, 2002). Work by Wark and Watson (1998) suggests $n=3$ in monomineral analog experiments with fluid fractions larger than 0.6%. However, their monomineralic experiments are not representative of our samples containing both a secondary phase (chromite) and a spatially variable fluid concentration. We treat $n$ as a variable in our model of melt migration due to uncertainties in the flow regime operating in our samples. The geometric term, $b$, relates the shape of channels in the network to the sample permeability. In essence, $b$ is determined by the tortuosity of the flow through the sample (Holtzman et al., 2003). For a cubical network of cylindrical tubes $b$ is $72\pi$ (Turcotte and Schubert, 2002) ($\approx 226$), but $b$ can be as large as $10^4$ if tubes in the shape of a triple junction are obstructed by a secondary phase (Holtzman et al., 2003). We treat $b$ as a variable in our simulations.

The viscosity of the partially molten aggregate is

$$\eta_{pm} = \eta_s \exp (-\alpha \phi)$$

(3)
where \( \eta_s \) is the viscosity of olivine (Mei et al., 2002). The viscosity of the solid is treated as a variable in our model due to uncertainties in the degree of olivine hydration. Mei et al. (2002) suggest \( \eta_s = 4 \times 10^{12} \) Pa s for dry olivine and \( \eta_s = 1.5 \times 10^{12} \) Pa s for hydrated olivine. For the melt phase, we used a viscosity of 10 Pa s (Shaw, 1969) in our model, corresponding to molten tholeiitic basalt at 1523 K. Works by Scott and Kohlstedt (2006) and Mei et al. (2002) quantify the effect of melt on the viscosity of partially molten aggregates. Their results substantiate the model described by Eq. (3) where \( \alpha = 25 \). As will be discussed in Section 3.3, this equation agrees with the deformation behavior observed in our three experiments.

We can relate the velocity of the solid grains to the melt velocity using the conservation of volume relation:

\[
w = \left( 1 - \frac{1}{\phi} \right) W \tag{4}
\]

where \( w \) is the melt velocity (Riley and Kohlstedt, 1991). In the next section we compare the surface tension and buoyancy forces to put an upper bound on the size of planetesimals in which fluid flow will be dominated by surface tension. A lower bound on planetesimal size is determined by the smallest body that can sustain supersolidus conditions in its interior for ~Myr timescales.

2.1. Scaling arguments

Under conditions relevant to km-scale melt transport on the Earth, shear (Bruhn et al., 2000) and buoyancy forces greatly outweigh the effect of surface tension. However, at smaller length scales, or under reduced gravity, surface tension becomes more important. The buoyancy force term is given by \((1 - \phi) g \Delta \rho R \theta \Delta R \) where \( g \) is gravity, \( \Delta \rho \) is the density contrast, \( R \) is the planetesimal radius, \( \rho \) is its density and \( G \) is the gravitational constant (Riley and Kohlstedt, 1991).

To determine the length scale over which surface tension and buoyancy forces are comparable, we make use of Eq. (1) and balance these two terms to derive

\[
R^2 \approx \frac{\gamma_d H(0) \rho_o^{-1/2}}{d \rho G A \rho} \tag{5}
\]

where we have assumed conservation of volume. We have approximated the melt fraction gradient as \( \phi_o / R \), where \( \phi_o \) is the maximum melt fraction, and use an olivine grain size of 1 mm based on measurements made by Irving et al. (2005) in a Brachinite (a dacitic achondrite) meteorite sample. For \( \gamma = 0.5 \) J m\(^{-2}\), \( H(\theta) \) is \( \approx 3 \) for \( \theta \) between 0 and 60°, \( \phi_o = 0.1 \), \( d = 1 \) mm (Irving et al., 2005), \( \Delta \rho \) of 100 kg m\(^{-3}\), and a density of 2700 kg m\(^{-3}\) this critical length scale is 20 km. Thus, for bodies with a diameter smaller than \( \approx 40 \) km, melt redistribution is likely to be controlled by surface tension rather than buoyancy forces.

Surface tension-driven melt flow is a likely melt transport mechanism on small silicate bodies such as asteroids. These bodies would have to be large enough to maintain supersolidus conditions in their interiors during radioactive decay (primarily of \( ^{26} \)Al), and/or after large impacts. To retain melt over a million year period, a time span significantly greater than the half-life of \( ^{26} \)Al, requires a thermal diffusion length scale of about 10 km. Balancing the capillary and buoyancy forces suggests that surface tension will dominate melt flow in objects with a radius smaller than 20 km. These two scales provide a range of planetesimal sizes in which melt would be present and flow would be dominated by capillary forces.

3. Experimental method and results

3.1. Sample preparation

All samples were prepared by mechanically mixing powders of San Carlos olivine (8 µm), Oman chromite (2 µm), and mid-ocean ridge basalt (8 µm) in a 19:5:1 volumetric ratio. The powders were then isostatically hot-pressed at 1523 K and 300 MPa in a gas-pressure medium apparatus for 4 h to obtain a dry, fully dense starting material.

3.2. Simple-shear deformation and static annealing of samples

Discs cut from the hot-pressed samples were deformed in simple shear as described by Holtzman et al. (2003) and Hustoft and Kohlstedt (2006). One sample was deformed at shear stresses of 22–55 MPa and strain rates of 1.1–6.4×10\(^{-4}\) s\(^{-1}\), whereas the remaining two samples were deformed at a stress and strain rate ranges of 26–55 MPa and 1.8–7.1×10\(^{-4}\) s\(^{-1}\), respectively. The final shear strain was 3.5 for all three samples.

After deformation had generated an initial heterogeneity, samples were statically annealed at elevated temperature and pressure for 0, 10, or 100 h (see Table 2). Confining pressure was maintained at 300 MPa, but in the 100 h anneal, temperature was allowed to drift down from 1523 to 1473 K during the first hour of the anneal; the latter temperature was maintained for the remainder of the anneal. Sections of the samples annealed for 0, 10, and 100 h are shown in Fig. 1a,b, and c, respectively.

The microstructures of the samples were examined using reflected light microscopy at a resolution of 0.28 µm/pixel. The melt concentration profiles were extracted from the images using the technique shown in Fig. 2. First, from the original greyscale image (Fig. 2a), we produced a binary image of the MORB distribution (Fig. 2b), which was then smoothed to remove grain-scale melt variations (Fig. 2c). Two images from each of the experimental samples shown in Fig. 1 resulted in a
total of six melt concentration profiles (Fig. 2d). The profiles were taken perpendicular to the melt-rich bands and generally included two melt-rich regions. We isolate each peak in the experimental data to compare with the numerical simulations.

3.3. Experimental results

The microstructures of all three samples, presented in Fig. 1, demonstrate that melt-rich bands are present in all three samples. However, melt-rich regions in the annealed samples are more diffuse than those in the quenched sample, suggesting that melt migration has occurred. Analysis of the melt distribution in these images gave melt concentration profiles across each of the three samples. Fig. 3 shows the melt concentration profiles taken from the locations in the three experiments indicated by boxes in Fig. 1. In order to quantify the differences between these melt concentration profiles, Gaussian curves were fitted to the melt-rich peak regions of each profile as shown in Fig. 4. The properties of these Gaussian curves are listed in Table 3. The raw profiles (Fig. 3) and Gaussian fits both show a significant decrease in the amplitude of the melt concentration peaks between the quenched and the 100 h annealed sample, coupled (in most cases) with an increase in the baseline melt fraction. The width of the melt concentration peak, however, remains relatively constant during the anneal. These observations imply that lateral flow of melt out of the high melt fraction zones is occurring during the static anneal.

Table 3 summarizes the baseline, amplitude, and width measurements for all of the Gaussian fits.

Fig. 2. Melt concentration profiles across images of samples from the three experiments were made using the following method: (a) first, an image was taken perpendicular to a pair of melt-rich bands; (b) second, only the MORB pixels were selected from the image; (c) third, the image was smoothed over a 20 µm radius to remove grain-scale melt variations, and contoured; (d) finally, three cross-sections along the length of the image were averaged together to give the melt concentration profile, which plots melt concentration versus distance across the sample.

Fig. 3. Melt profiles extracted from images of (a) the quenched sample, (b) the sample annealed for 10 h, and (c) the sample annealed for 100 h.

Fig. 4. Example of Gaussian curves (thin lines) fit to two melt concentration peaks (thick line) obtained from the sample annealed for 10 h. The solid thin line indicates the section of the experimental profile to which each Gaussian was fit.
The images used to generate the melt concentration profiles indicate that an insignificant amount of olivine grain growth occurred during the anneal. Olivine grain size measurements made for the quench, 10 h and 100 h anneal samples are 9.0±3.2 µm, 8.8±2.8 µm, and 9.2±3.2 µm, respectively. We used another set of reflected light optical microscope images taken at a resolution of 29.5 pixels/µm in order to measure dihedral angles. Angles were measured at 112 locations over the three samples using image processing software (Scion Image). Our measurements gave a median angle of 17±5°.

The effective viscosity data for all three samples were similar. The deformation conditions for the samples that were subsequently annealed for 10 and 100 h were slightly different than the pre-quench sample deformation (see previous section). The pre-anneal samples experienced greater shear stresses resulting in larger shear strain rates than those of the pre-quench sample. These slightly different deformation conditions resulted in the differences in viscosity shown in Fig. 5. The concave down shape of these data indicates that each sample experienced an initial stage of strain-hardening followed by a strain-softening stage. During the strain-hardening stage a nearly homogeneous melt fraction of 4% resulted in a high sample viscosity and limited deformation. As deformation induced melt flow into the melt-rich regions of the sample, the sample viscosity decreased resulting in the strain-softening stage of deformation. This observation suggests that similar sets of melt-rich band networks developed in all three of the samples during shear deformation.

The viscosity data from the deformation phase of the experiments provide a constraint on the viscosity of the solid olivine grains contained in the samples. Based on Eq. (3), for an average melt fraction of 4%, the viscosity of the partially molten aggregate is a factor of ε lower than the solid phase. Therefore, the experimental data (Fig. 5) suggest a solid viscosity of $\eta_s \approx 6 \times 10^{11}$ Pa s.

In summary, static annealing causes a progressive decrease in the amplitude of the peaks in the melt concentration profiles. This effect is presumably caused by surface tension, which drives lateral flow of melt away from regions of high concentration. The dependence of surface tension-driven melt migration on sample permeability, fluid viscosity, and channel geometry is explored in the next section in which we carry out a numerical investigation of the experimental observations.

4. Numerical modeling

4.1. Numerical scheme

We used the non-dimensionalized form of Eq. (1) governing the response of the solid matrix to a gradient in melt fraction:

$$H(\theta)\phi'^{-3/2}\frac{d\phi'}{dz'} = \frac{d}{dz'} \left( e^{-z'\phi} \frac{dW'}{dz'} \right) - \frac{W'}{\phi'^n}. \quad (6)$$

As described by Riley and Kohlstedt (1991), $\phi'$ is the melt concentration at a particular location divided by the maximum concentration in the profile, and $z'$ is the length of the melt concentration profile divided by the compaction length. The characteristic solid velocity, $W_o$, is given by

$$W_o = \frac{\gamma \phi_o}{d} \left( \frac{\kappa_o}{\eta_s \mu} \right)^{1/2}. \quad (7)$$

where $\kappa_o$ is the maximum permeability in the profile and is given by Eq. (2). $W'$ is calculated by dividing the dimensional solid velocity, $W$, by $W_o$.

The non-dimensionalized momentum formula given in Eq. (6) forms the basis for our model by allowing us to solve for the solid velocity at every point along the profile. Given an initial melt concentration profile $\phi(z)$, a permeability expression (Eq. (2)), a

![Fig. 5. Deformation behavior of the three samples of olivine + chromite + MORB plotted as sample viscosity versus shear strain.](image-url)
grain size, an interfacial energy, and a dihedral angle, the solid velocity can be solved implicitly in Eq. (6).

Once the solid velocity has been defined at every point in the profile, we can determine the spatial evolution of the melt fraction using the conservation of volume expression from Riley and Kohlstedt (1991):

\[
\frac{\partial \phi}{\partial t} = \frac{\partial}{\partial z} \left[ (1 - \phi) W \right]
\]

(8)

As shown in Eq. (8), a decrease in solid velocity across a unit cell results in a decrease in melt fraction due to compaction of the solid matrix. Similarly, an increase in solid velocity across a cell tends to separate solid grains and is associated with an influx of melt.

The finite difference model uses 600 pixels over a length scale of 0.2 mm to simulate surface tension-driven melt flow resisted by viscous sample deformation. We choose 0.2 mm as the total profile length based on the average spacing between the melt-rich peaks in the experiments. One shortcoming of using a fixed-length model is that the total amount of melt in the Gaussian fit to the quench peak (the initial condition) does not necessarily match the total melt in the anneal profile we wish to reproduce. Because melt is conserved in our simulations, matching profiles in which significant melt loss has occurred is difficult, leading us to discard the most depleted profile #11 (see below).

4.2. Results

Gaussian fits to the melt-rich peaks in the quenched samples were allowed to evolve for 100 h according to Eq. (6). In order to match the profiles from the sample annealed for 100 h, the numerical model must transport melt out of the melt-rich bands and into the melt-depleted regions in a way that significantly reduces the amplitude of the peak, has little effect on the peak width, and increases the baseline melt concentration (see Table 3).

Values of \( n, b, \) and \( \eta_s \) were varied to obtain several hundred simulated 100 h anneal profiles, one for each combination of \( n, b, \) and \( \eta_s \). The goal of modeling the experiments was to determine the values of the flow parameters that resulted in the best reproduction of the profiles from the annealed sample (peaks 9, 10, 12 in Fig. 3). The parameters that give the best reproduction of the profiles from the annealed samples are \( \eta_s = 1.7 \pm 0.5 \times 10^{12} \) Pa s, \( b = 9000 \pm 1900 \), and \( n = 2.4 \pm 0.3 \). These results are tabulated in Table 4. These values assume a fluid viscosity of 10 Pa s, a dihedral angle of 17°, and an interfacial energy of 0.5 J m\(^{-2}\).

The viscosity model for partially molten aggregates (equation 3) proposed by Mei et al. (2002) gives a viscosity that is less sensitive to melt fraction than the Riley and Kohlstedt (1991) model. The higher effective viscosity in Mei et al.’s model resists significant differential motion of the olivine–basalt aggregate resulting in a broad, smoothly varying solid velocity profile. Because the solid velocity changes gradually outside the melt-rich band, the matrix undergoes nearly uniform extension. As a result, the melt flux into the melt-depleted region is, spatially, nearly constant. It is this deformation behavior that causes the width of the melt-rich bands to remain nearly constant during the anneal.

Fig. 6a shows the evolution of Gaussian peak number 2 for a 100 h simulated anneal. The amplitude of the melt concentration peak decays exponentially with time as the melt flows into the melt-depleted regions. The RMS misfit between the model output and the experiment (jagged line in 6a) was calculated for various values of the flow parameters. The misfit between the model output and the observed melt profile for various values of \( n \) and \( b \) is shown in Fig. 6b. This plot shows that the best-fit value for this set of simulations occurs at \( n = 2.4 \) and \( b = 29,000 \). We reproduced the profiles from the sample annealed for 100 h by varying \( \eta_s \) as well as \( n \) and \( b \) to get the best model results.

Images of the sample annealed for 100 h reveal that melt has pooled at the ends of the sample in the region where the tungsten pistons do not overlap (see Fig. 2 in Hustoft and Kohlstedt (2006)). This migration occurs during deformation (Hustoft and Kohlstedt, 2006). This phenomenon results in a depletion in melt content of the interior regions of the sample in which the melt profiles for the 100 h were taken. Attempting to model the most depleted of these profiles (#11) is difficult because of the effect of melt depletion. We exclude flow parameter data for simulations involving profile #11 from Table 4 for this reason. Despite this complexity in the experimental data, numerical simulations closely reproduce the annealing experiments using values for the flow parameters that are in agreement with previous work. Based on fitting the model outputs to profiles 9, 10, and 12 the best-fit value of \( n, b, \) and \( \eta_s \) are 2.4, 9000, and 1.7 \times 10^{12} \) Pa s, respectively.

4.3. Application to planetesimals

To estimate the timescale of surface tension-driven flow in asteroids, we compare the surface tension, porous flow, and viscous terms in Eq. (1). Over spatial scales of ~km, the surface tension force will be balanced by the porous flow term as suggested by

<table>
<thead>
<tr>
<th>Quench peak</th>
<th>Anneal peak ( (\times 10^{-3}) )</th>
<th>Misfit ( (\times 10^{12} ) Pa s)</th>
<th>( \eta_s )</th>
<th>( n )</th>
<th>( b )</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>9</td>
<td>0.75</td>
<td>2.5 \pm 0.08</td>
<td>2.2 \pm 0.25</td>
<td>10,000 \pm 7000</td>
</tr>
<tr>
<td>2</td>
<td>9</td>
<td>0.62</td>
<td>2.0 \pm 0.06</td>
<td>2.4 \pm 0.3</td>
<td>8300 \pm 3500</td>
</tr>
<tr>
<td>3</td>
<td>9</td>
<td>2.84</td>
<td>1.3 \pm 0.07</td>
<td>2.1 \pm 0.24</td>
<td>8800 \pm 6700</td>
</tr>
<tr>
<td>4</td>
<td>9</td>
<td>4.5</td>
<td>1.1 \pm 0.08</td>
<td>2.2 \pm 0.24</td>
<td>8000 \pm 3500</td>
</tr>
<tr>
<td>1</td>
<td>10</td>
<td>1.03</td>
<td>2.3 \pm 0.05</td>
<td>2.9 \pm 0.15</td>
<td>13,900 \pm 5800</td>
</tr>
<tr>
<td>2</td>
<td>10</td>
<td>0.82</td>
<td>1.7 \pm 0.0</td>
<td>2.9 \pm 0.12</td>
<td>10,600 \pm 3500</td>
</tr>
<tr>
<td>3</td>
<td>10</td>
<td>1.79</td>
<td>0.9 \pm 0.05</td>
<td>2.3 \pm 0.26</td>
<td>8100 \pm 3500</td>
</tr>
<tr>
<td>4</td>
<td>10</td>
<td>2.42</td>
<td>0.8 \pm 0.04</td>
<td>2.4 \pm 0.4</td>
<td>8200 \pm 3500</td>
</tr>
<tr>
<td>1</td>
<td>12</td>
<td>1.97</td>
<td>1.3 \pm 0.07</td>
<td>2.2 \pm 0.26</td>
<td>8000 \pm 3500</td>
</tr>
<tr>
<td>2</td>
<td>12</td>
<td>1.45</td>
<td>1.4 \pm 0.05</td>
<td>2.1 \pm 0.17</td>
<td>7800 \pm 3400</td>
</tr>
<tr>
<td>3</td>
<td>12</td>
<td>5.0</td>
<td>0.7 \pm 0.02</td>
<td>2.5 \pm 0.2</td>
<td>8500 \pm 3600</td>
</tr>
<tr>
<td>4</td>
<td>12</td>
<td>7.5</td>
<td>14 \pm 4.5</td>
<td>2.3 \pm 0.14</td>
<td>8000 \pm 3200</td>
</tr>
<tr>
<td>Global mean</td>
<td>1.7 \pm 0.5</td>
<td>2.4 \pm 0.3</td>
<td>9000 \pm 1900</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Error values represent simulations with misfits falling within 1% of the best fit. The misfit value is the minimum RMS difference between the modeled and observed profiles. The error values for the global means are standard deviations.
several of these simulations with different values for the best match of the results from the simulated anneal with those from the experimental anneal (black circle). The contour labels give the RMS misfit value.

We find evidence for melt transport from melt-rich regions to melt-poor depleted regions of the olivine + chromite + MORB sample. This series of annealing experiments provides an empirical means of analyzing melt flow driven by surface tension.

The best-fit value of $\eta$ from the numerical simulations is $1.7 \pm 0.5 \times 10^{12}$ Pa s, in agreement with Mei et al.’s viscosity for hydrated olivine. The homogenizing effect of surface tension in these experiments is consistent with current theory regarding grain boundary flow at low melt fractions (Hier-Majumder et al., 2006). The value of $n = 2.4$ suggests a flow geometry intermediate between tube and sheet flow, perhaps as a result of varying melt fractions (Renner et al., 2003).

The value of $b$ obtained ($9000 \pm 1900$) is significantly greater than the theoretical value of $72\pi \approx 226$. Holtzman et al. (2003),
who suggested that the presence of chromite grains restricted flow through the tubes, used an analytical approach to infer a value of $b=10^3$. This theoretical value, which agrees surprisingly well with our experimentally derived value, demonstrates the substantial effect that even a relatively minor phase can have on the overall permeability of the system. Physically, an increase in $b$ reduces both the compaction length scale and the fluid velocity.

6. Conclusions

For small bodies, melt redistribution is likely to be controlled by surface tension rather than buoyancy forces (Section 2.1). Our experimental investigations of surface tension-driven melt flow indicate that the presence of secondary phases can have an important effect, reducing the overall permeability by clogging melt tubes (Sections 4 and 5). For our experimentally determined parameters, we conclude that melt redistribution via surface tension is likely to be a slow process ($10^8$–$10^{11}$ yrs) within small planetesimals (Section 4.3). In the absence of other effects (such as shear), this slow rate will allow the development of localized areas of high melt fraction, perhaps leading to magma chambers or volcanic activity (Greenwood et al., 2005).

Best-fit numerical model simulations of two melt concentration profiles in annealed samples suggest values for $n$ and $b$ of 2.4±0.3 and 9000±1900, respectively. These values result in a sample permeability ranging between $10^{-16}$ and $10^{-12}$ m² for $\phi=0.004$ and 0.2, respectively, for a grain size of 1 mm. These relatively low permeabilities compared to monomineralic studies by, for instance, Wark and Watson (1998) and Riley and Kohlstedt (1991) are due primarily to the clogging effect of chromite. Secondary phases create tortuosity in pore volumes between primary mineral grains, reducing the effective permeability of the sample (Holtzman et al., 2003).

In the model simulations, the redistribution of melt by capillary flow results in deformation of the solid matrix in order to conserve mass. Due to the high sample viscosity used in these simulations ($9 \times 10^{11}$–$8 \times 10^{12}$ Pa s), this deformation extends along the entire length of the melt concentration profile (0.2 mm) even though the melt concentration varies over a 0.05 mm length scale. This viscosity-dominated flow results in a nearly monotonic addition of melt to the melt-depleted regions of the sample as the aggregate deforms. Matching simulated melt concentration profiles produced by static annealing with profiles observed in annealed samples yields a mean solid viscosity of 1.7±0.5×10^{12} Pa s. This value lies between the calculated melt-free viscosity of our samples and the viscosity of dry olivine suggested by Mei et al. (2002). This value also matches the viscosity of hydrated olivine as measured by Mei et al.

Direct measurements of 112 dihedral angles taken from optical images in the experimental samples gives a median angle of 17±5°. This value is somewhat smaller than those reported for most olivine–basalt samples.

Surface tension is likely to be the dominant process controlling melt migration in small (~15 km radius) planetesimals, in the absence of shear deformation (Section 2.1). However, based on the flow parameters derived in our study, the timescales for melt redistribution under likely conditions are $\sim 10^5$–$10^{11}$ yr (Section 4.3). These timescales are much longer than the timescale over which melt, generated by the decay of short-lived radio isotopes, is likely to persist. The slow rate of melt redistribution due to surface tension therefore suggests that large, localized pockets of melt are likely to have developed on small planetesimals.

Acknowledgements

We would like to acknowledge the National Science Foundation (OCE-0648020) and NASA (PG&G NNX07AP68G) for funding the experimental portion of this research project. Funding for the modeling portion was provided by NASA (MFR NNG06GH13G).

References


