EART 265: Lecture Zero Notes

1. Introductions (syllabus; names + student interests)

2. Goals of the class:
   (a) Graduate students study problems with more detail, more rigor, longer time. But sometimes you need the opposite approach: less detail, less rigor, but FASTER! Useful for:
      i. sanity checks;
      ii. evaluating research ideas;
      iii. eliminating unimportant complications (esp. with your own work when someone says “have you thought about X” at your poster or talk).
   (b) Helps to teach you to identify what is most important/fundamental to a problem.

3. The ∼ sign. In general, we use ∼ signs instead of =.
   (a) Dimensions: usually we want the dimensions of a ∼ equation to have proper dimensions.
   (b) constants: ∼ will not know anything about dimensionless constants (such as the π in $A = \pi R^2$), and thus one main reason why we shouldn’t write =.
   (c) scaling: ∼ equations are useful for telling us how parameters scale
   (d) One significant figure: **All values in this class should be written as one significant figure.** Always. Always. Always.
   (e) Geometric mean and sigma: Because we are almost always multiplying and dividing (why?), usually the correct mean and standard deviation are geometric, not arithmetic. $\text{Geomean}(x, y) = (x \cdot y)^{1/2}$.

4. Important numbers:
   (a) Numbers to memorize: day in seconds; year in seconds; radii and density of Earth and Sun; solar constant.
   (b) Numbers to derive: mammal power consumption; per capita power consumption; energy content of hydrocarbons; per capita water consumption.

5. Important equations: See below.

6. After you get an answer. What now?
   (a) Big and small numbers: what is a big number? Depends on the context. $1 \text{ billion}$ is a lot of money compared to the average annual household income. It’s a normal amount of money for a jet fighter. It’s a small number compared to the US annual budget. Getting an answer is a good start, but convince yourself that it’s reasonable by comparing it to another number you know.
   (b) Cross-check: There is often more than one approach to solving a problem (good example is force balance methods vs. energy arguments). So go at the same problem in an entirely different way. The less in common with the first solution, the more independent your new estimate will be. If you get similar values from both approaches, then you’ll feel a lot better about your estimate!
7. Four techniques for solving problems:

(a) Use existing information. Example: derive energy usage of mammals (1 W/kg)

(b) Bracketing/limiting values. What’s definitely a number that’s too large? What’s definitely too small? Take the (geometric) mean!

(c) Divide and conquer. Example: Drake Equation \( N = R^* \cdot f_p \cdot n_e \cdot f_l \cdot f_i \cdot f_c \cdot L \) where

   i. \( N \) = observable civilizations
   ii. \( R^* \) = formation rate of stars in the Milky Way whose E-M signals we can detect;
   iii. \( f_p \) = the fraction of those stars with planetary systems;
   iv. \( n_e \) = number of planets, per solar system, with an environment suitable for life;
   v. \( f_l \) = fraction of suitable planets on which life actually appears;
   vi. \( f_i \) = fraction of life bearing planets on which intelligent life emerges.
   vii. \( f_c \) = The fraction of civilizations that develop a technology that releases detectable signs of their existence into space.
   viii. \( L \) = The length of time such civilizations release detectable signals into space.

(d) Dimensional analysis. Example: Stokes Law drag force on a moving object.

   i. Start with \( F \sim \mu ud \). Missing constant is 3\( \pi \), however, so we’ll be off by an order of magnitude. Nothing we can do about that as it turns out. If you care what the constant is, do an experiment! Note that if we’d used \( r \) instead of \( d \), we’d be off by even more.

   ii. Buckingham-Pi: Be aware of its existence, but it doesn’t tend to be too useful in practice.

   • Example 1: one dimensionless variable. Schwarzschild radius of a black hole. Ans:
     \( R \sim \frac{Gm}{c^2} \)
   • Example 2: drag on a moving object
   • Example 3: period of pendulum.
   • Example 4: energy released by nuclear bomb
Important Basic Equations for Order of Magnitude Estimation

Conservation Laws

Many equations are nothing more than the conservation of some quantity. Important conservation laws include:

- Mass (only valid for non-relativistic systems)
- Energy
- Linear momentum
- Angular momentum

Quantities

<table>
<thead>
<tr>
<th>Quantity</th>
<th>Formula</th>
</tr>
</thead>
<tbody>
<tr>
<td>Force</td>
<td>mass \times \text{acceleration (units of Newtons)}</td>
</tr>
<tr>
<td>Energy</td>
<td>force \times \text{distance (Joules)}</td>
</tr>
<tr>
<td>Power</td>
<td>\text{energy} / \text{time (Watts)}</td>
</tr>
<tr>
<td>Pressure</td>
<td>\text{force} / \text{area; energy} / \text{volume (Pascals)}</td>
</tr>
<tr>
<td>Density</td>
<td>\text{mass} / \text{volume}</td>
</tr>
<tr>
<td>Flux of quantity Z</td>
<td>Z / \text{area} / \text{time}</td>
</tr>
<tr>
<td>Concentration</td>
<td>(mass or moles) / \text{volume}</td>
</tr>
</tbody>
</table>

Geometry

<table>
<thead>
<tr>
<th>Geometry</th>
<th>Formula</th>
</tr>
</thead>
<tbody>
<tr>
<td>Circle: perimeter</td>
<td>2\pi r or \pi d</td>
</tr>
<tr>
<td>Circle: area</td>
<td>\pi r^2 or \frac{1}{4} \pi d^2</td>
</tr>
<tr>
<td>Sphere: surface area</td>
<td>4\pi r^2 or \pi d^2</td>
</tr>
<tr>
<td>Sphere: volume</td>
<td>\frac{4}{3} \pi r^3 or \frac{1}{6} \pi d^3</td>
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</tbody>
</table>

Forces

<table>
<thead>
<tr>
<th>Forces</th>
<th>Formula</th>
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</thead>
<tbody>
<tr>
<td>Newton’s Second Law</td>
<td>\Sigma F = ma</td>
</tr>
<tr>
<td>Force due to gravity</td>
<td>F_g = mg = \frac{Gm_1m_2}{r^2}</td>
</tr>
<tr>
<td>Drag force</td>
<td>F_d = \frac{1}{2} c_D \rho A v^2</td>
</tr>
<tr>
<td>Frictional force</td>
<td>F_f = c_f F_N</td>
</tr>
<tr>
<td>Buoyant force</td>
<td>F_b = \rho_{\text{fluid}} V g</td>
</tr>
<tr>
<td>Pressure</td>
<td>p = F / A = \rho gh</td>
</tr>
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</table>

Energy

<table>
<thead>
<tr>
<th>Energy</th>
<th>Formula</th>
</tr>
</thead>
<tbody>
<tr>
<td>General expression</td>
<td>E = \int F ds</td>
</tr>
<tr>
<td>Gravitational potential energy</td>
<td>\hat{E} = mgh</td>
</tr>
<tr>
<td>Kinetic energy</td>
<td>E = \frac{1}{2} m v^2</td>
</tr>
<tr>
<td>Sensible heat energy</td>
<td>E = \hat{m} c_p T</td>
</tr>
<tr>
<td>Latent heat energy</td>
<td>E = m L</td>
</tr>
</tbody>
</table>
Kinematics

Linear motion:
Velocity
\[ v = \frac{ds}{dt} \]

Acceleration
\[ a = \frac{dv}{dt} \]

Translation \((a = \text{constant})\)
\[ s = v_i t + \frac{1}{2} at^2 \]

Velocity \((a = \text{constant})\)
\[ v_f^2 = v_i^2 + 2as \]

Rotational motion:
Angular velocity
\[ \omega = \frac{d\theta}{dt} \]

Tangential velocity
\[ v = \omega r \]

Angular acceleration
\[ \alpha = \frac{d\omega}{dt} \]

Radial (centripetal) acceleration
\[ a = \omega^2 r \]

Solids

Strain
\[ \varepsilon = \frac{\Delta L}{L} \]

Stress
\[ \sigma = \mu \varepsilon \]

Work
\[ \frac{E}{V} = \frac{1}{2} \sigma \varepsilon \]

Kinematic viscosity
\[ \nu = \frac{\mu}{\rho} \]

Fluids

Ideal Gas Law
\[ pV = nRT \]

Universal Gas Constant
\[ R = N_{\text{Avogadro}} k_B \]

Shear stress
\[ \tau = \mu \frac{du}{dy} \]