EART 265 Lecture Notes: Energy

1. Energy Production

85% of energy is generated using fossil fuels. Nuclear, biomass and hydroelectric make up most of the rest.

Fossil fuels

Fossil fuel energy is derived from the stability of the product molecules of combustion (CO$_2$ and H$_2$O) relative to the hydrocarbon and O$_2$. On a per mass basis, the energy contents tend to be fairly similar, around 50 MJ/kg. A good way to remember this is that fats that you eat have an energy content of 10 kcal/g. Energy content per volume, however, can vary widely since gases can not be easily compressed to liquid densities. On a per greenhouse warming potential basis, natural gas is better than other fuels because of the high hydrogen to carbon ratio.

dexample: Estimate total global carbon emissions (mass of carbon per year), both total and per person. What is this as percentage of the atmospheric reservoir of carbon?

Nuclear Energy

Represents 4% of US energy production

The binding energy (strong nuclear force) depends on the number of protons and neutrons (see Figure 1). At very small atomic numbers, the stability of the nucleus is higher as nucleons are added, as each additional nucleon creates bonds with all the other nucleons. For elements larger than Fe, there are so many nucleons that the additional nucleon's electrostatic repulsion more than offsets the nuclear bond energy, making them less and less stable for larger elements. Empirically, the bond energy change for small atomic number (i.e. nuclear fusion) is $\sim 1 \text{ (MeV/nucleon)/nucleon}$, whereas at large atomic number (nuclear fission), this slope is $\sim 0.01 \text{ (MeV/nucleon)/nucleon}$. The relative abundances of fissionable material to fusionable material, and the lifetime of radioactive waste, all make fusion more attractive than fission if it can be attained.

Fusion: In the Sun, the net fusion reaction is $4 \text{H}^1 \rightarrow \text{He}^4 + 2 \text{e}^+ + \text{neutrinos}$. He$^4$ has a binding energy of 7 MeV per nucleon, so the total binding energy is 30 MeV. Binding energy of H$^1$ is 0 MeV, so the net is 30 MeV per He$^4$ formed.

Fission: A typical fission reaction is $\text{U}^{235} + \text{n} \rightarrow \text{Kr}^{82} + \text{Ba}^{141} + 3 \text{n}$. Roughly, the 90 nucleons that end up in Kr have changed binding energy by $0.01 \text{ MeV/nucleon/nucleon} * (235 - 92 \text{ nucleons}) \sim 1 \text{ MeV/nucleon}$. The OOM estimate for Ba is about the same. So the 200 nucleons originally in U$^{235}$ have each become more stable by 1 MeV, for a net energy release of 200 MeV.

dexample: Calculate energy density of hydrogen fusion and uranium fission; activation barrier to fusion (what temperature must be achieved?); fusionable lithium reserves in the ocean (concentration of 200 ppb by mass in sea water).
Solar Energy

Solar input is $\sim 10^3$ W/m$^2$. Two main technologies for harnessing solar energy are photovoltaic (PV), which generates an electrical current from semiconductor material, and solar thermal systems, which convert solar energy to heat energy and subsequently to electricity. PV systems have been improving in efficiency; normal home systems have efficiencies between 10 and 20%, but numbers as high as 40% have been achieved in laboratory settings. Solar thermal systems can achieve efficiencies around 20 to 30% provided that the area considered is the solar-capturing (either panels or reflectors) area. When the entire plot of land required is considered, this efficiency drops by about an order of magnitude.

Q: Estimate the number of Arizonas that would have to be covered by solar panels to provide all US energy needs.

Q: What fraction of incident solar energy do we need to intercept to maintain the world's energy use?

Hydroelectric Energy

Currently represents 6% of the US energy production.

Q: What height would we have to trap all of our rainfall in order to generate all our energy from hydroelectricity?
Wind

Currently comprises 2% of the US energy production.

Typical turbines have 20 to 40 m long blades, are $\sim 100$ m off the ground (why?) and generate $\sim 20$ kW. Note that since we are a 10 kW society, we would need one turbine for every two people if we were to satisfy our energy requirements.

Power available from any turbine (tidal or wind) is given by:

$$P \sim \frac{\eta KE}{\tau} \sim \frac{\eta \rho A L}{u} \sim \eta \rho A u^3$$

where $A$ is cross-sectional area in the plane perpendicular to the flow, $L$ is a length scale in the direction of the flow, and $\eta$ is efficiency.

**example:** How much total wind energy is available in the US?

**Betz’s Law**  
Betz’s Law prescribes the maximum efficiency of a turbine type system. At one limit, extracting all the kinetic energy from the wind and thus bringing the air to a standstill means that there’s no way for the air to drive a rotating machine, so no useful work can be extracted. A wind turbine that doesn’t reduce the wind speed at all can not generate useful work. Maximum efficiency lies somewhere between these two extremes. Betz’ Law sets the maximum efficiency of a wind turbine at $\sim 60\%$.

To derive Betz’s Law, one first demonstrates that the velocity of the air driving the turbine $\tilde{u}$ is the average of the air velocity far upstream ($u_1$) and far downstream ($u_2$). From there, the derivation is simple:

$$P \sim \frac{1}{2} \rho \tilde{u} (u_1^2 - u_2^2) \sim \frac{1}{2} \rho A \tilde{u} (u_1^2 - u_2^2) \sim \frac{1}{4} \rho A (u_1 + u_2) (u_1^2 - u_2^2)$$

With some manipulation, one can show that the max power occurs at $u_2/u_1 = 1/3$ which then yields:

$$P_{\text{max}} \sim \frac{16}{27} \cdot \frac{1}{2} \rho A u^3$$

where $16/27 \sim 60\%$ is the theoretical limit.

**example:** What is the minimum spacing for wind turbines?

Tides

Tides are derived from the gravitational interaction of the Earth and Moon. Tidal energy is dissipated primarily by the oceans through turbulence (and is a key process by which mixing of the ocean occurs). As turbulent drag of the oceans slows the Earth’s rotation, the Moon’s angular momentum must increase, causing its orbit to move further from the Earth. Total tidal dissipation is roughly 4 TW, but the areas where it is economically feasible to tap into this power are a very limited subset of the Earth’s surface.

Tidal energy can be captured by utilizing its kinetic energy (termed *tidal stream*) or potential energy (*barrage*). Tidal stream systems, the simplest of which are turbines, are lower cost and lower impact than barrages, which are essentially dams.

**example:** how much power from the Bay of Fundy ($\sim 50$ miles wide x 100 miles long x 10 m tidal range).
Geothermal

The geothermal heat flux was previously estimated to be about 0.1 W/m$^2$. There’s no chance it will be a large fraction of the global energy portfolio, although it can be locally (e.g. Iceland).

Biofuels

Trapping sunlight energy through photosynthesis is possible. Plants, however, contain a bunch of stuff that is hard to burn (water for one). Plus, depending on the biofuel, we could be displacing land that we currently use to produce food, which isn’t really going to work. The other problem is that plants aren’t just made of carbon dioxide, water and sunlight. The Redfield ratio is the approximate ratio of trace elements in plants and is OOM carbon:nitrogen:phosphorus∼100:10:1, by mole. If we grew all our energy from plants, how long could this be sustained? Remaining phosphorus reserves are estimated to be 0.2 to 1 GtP.

Growing hydrocarbon-rich algae has been proposed. The United States Department of Energy estimates that if algae fuel replaced all the petroleum fuel in the United States, it would require 40,000 km$^2$. Does this seem plausible? How much more fertilizer would we need to produce? How much water would we need?

2. Energy Storage

Because energy generation by many renewable resources does not match energy usage, having a way to store energy is important.

Hydrogen

Hydrogen production from electrolysis (H$_2$O $\rightarrow$ H$_2$ + O$_2$, with electrochemical potential of $\sim$1 V. How many electrons are transferred?) is one way of storing energy during the daytime when there is surplus wind or solar. Disadvantage is that hydrogen is small and diffuses quickly in many standard materials, such as stainless steel, rendering it brittle and prone to failure.

example: What volume of compressed hydrogen would we need if we had to maintain a storage of 3 days of the nation’s energy needs? [Our oil supply is many months’ worth, but this is mainly to dampen oil price shocks, not because it’s necessary as a backup.]

Batteries

Batteries store electrochemical energy. The basic components of a battery are an anode, cathode, a membrane to separate the two, and a medium through which ions can diffuse from one to the other.

Lithium ion batteries are the best batteries for energy-to-mass ratio. Energy density can be estimated rather accurately assuming an anode and cathode plus medium is 300 g/mol and a voltage of 3 V potential drop.
**Compressed Air**

Compressing air stores energy that can be released later. The amount of storage depends on the maximum pressure of the container, plus thermal losses due to adiabatic compression.

**Flywheels**

Not good for transportation energy storage! (Why?) The limit is the tensile strength of the material, which can be estimated as before.

How large a flywheel should each household have to pad their usage for 2 days? How fast would it have to rotate?