Supplementary Materials for

The Crust of the Moon as Seen by GRAIL


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Supporting Online Material for
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1 Density and porosity of the crust

The gravitational potential exterior to a planet can be expressed as

\[ U(r) = \frac{GM}{r} \sum_{l=0}^{\infty} \sum_{m=-l}^{l} \left( \frac{R_0}{r} \right)^l C_{lm} Y_{lm}(\theta, \varphi), \]  

(1)

where \( r \) is position, \( G \) is the gravitational constant, \( M \) is the mass of the planet, \( R_0 \) is the reference radius of the spherical harmonic coefficients \( C_{lm} \) of degree \( l \) and order \( m \), \( Y_{lm} \) are the \( 4\pi \)-normalized spherical harmonic functions, and \( \theta \) and \( \varphi \) are colatitude and longitude, respectively. In this harmonic form, eq. (1) is exact for all radii greater than the maximum topographic excursion of the planet. If relief \( h \) with respect to a spherical interface of radius \( D \) is expressed in spherical harmonics as

\[ h(\theta, \varphi) = \sum_{l=0}^{\infty} \sum_{m=-l}^{l} h_{lm} Y_{lm}(\theta, \varphi), \]  

(2)

the spherical harmonic coefficients of the gravitational potential resulting from a density contrast \( \rho(\theta, \varphi) \) can be calculated to arbitrary precision from the expression

\[ C_{lm} = \frac{4\pi D^3}{M(2l+1)} \sum_{n=1}^{N} \frac{(\rho h^n)_{lm} \prod_{j=1}^{n}(l+4-j)}{D^n n! (l+3)}, \]  

(3)

where the spherical harmonic coefficients of \( \rho h^n \) are calculated according to

\[ (\rho h^n)_{lm} = \frac{1}{4\pi} \int_{\Omega} [\rho(\theta, \varphi) h^n(\theta, \varphi)] Y_{lm}(\theta, \varphi) \, d\Omega, \]  

(4)

and where \( d\Omega = \sin \theta \, d\theta \, d\varphi \). We use \( N = 7 \) when evaluating eq. (3), which gives a relative precision better than \( 10^{-4} \) at harmonic degree 400 for the surface relief of the Moon. The radial component of the gravity field is obtained by taking the radial derivative of eq. (1), and using the sign convention that gravity is positive when directed downward, this is

\[ g(r) = \frac{GM}{r^2} \sum_{l=0}^{\infty} \sum_{m=-l}^{l} \left( \frac{R_0}{r} \right)^l (l+1) C_{lm} Y_{lm}(\theta, \varphi). \]  

(5)

The height of an equipotential surface above a reference radius \( R \) is, to first order,

\[ H(\theta, \varphi) \approx R \sum_{l=1}^{\infty} \sum_{m=-l}^{l} \left( \frac{R_0}{R} \right)^l C_{lm} Y_{lm}(\theta, \varphi). \]  

(6)

The degree-dependent relation between gravity \( g \) and topography \( h \) is quantified by the admittance

\[ Z(l) = \frac{S_{hg}(l)}{S_{hh}(l)}, \]  

(7)
and spectral correlation
\[ \gamma(l) = \frac{S_{hg}(l)}{\sqrt{S_{hh}(l)S_{gg}(l)}}, \]  
(8)

where the cross- and auto-power spectra \( S \) are
\[ S_{hg}(l) = \sum_{m=-l}^{l} h_{lm} g_{lm}, \]  
(9)
\[ S_{hh}(l) = \sum_{m=-l}^{l} h_{lm} h_{lm}, \]  
(10)
\[ S_{gg}(l) = \sum_{m=-l}^{l} g_{lm} g_{lm}. \]  
(11)

The observed gravity field of the Moon is a result of many factors, including surface topography, relief along the crust–mantle interface, relief along the core–mantle boundary, and heterogeneities in both the crust and mantle. Loads on the lithosphere, such as surface topography and magmatic intrusions, will cause the lithosphere to flex, giving rise to a signal from the crustal–mantle interface. Since gravitational signals are attenuated with increasing height above their depth of origin, and since the amount of flexure decreases with decreasing wavelength, the shortest-wavelength anomalies will be sensitive primarily to relief along the surface, and not the crust–mantle interface. We quantify the influence of lithospheric flexure on the observed gravity field with a lithospheric loading model that incorporates surface loads on a thin elastic shell (37). For this model, the admittance \( Z \) is simply the ratio of the gravity and topography coefficients in the spectral domain, and the spectral correlation \( \gamma \) is unity at all degrees.

For demonstration purposes, with only the first term on the right in eq. (3), we have calculated the admittance for several values of the lithosphere elastic thickness \( T_e \) and a crustal thickness of 45 km. As shown in fig. S1, the admittance depends strongly on the elastic thickness at low degrees, but at high degrees the admittance approaches an asymptotic value that is proportional to the crustal density. For elastic thicknesses greater than 5 km, the contribution to the gravity signal from lithospheric flexure is seen to be negligible at degrees greater than about 100. Even for an elastic thickness of zero, the admittance differs by less than 1% from that of a rigid shell at degrees greater than 170. Since it is unlikely that the elastic thickness would be zero for the shortest wavelengths, for which the gravity field is dominated by contributions from young impact craters, we have neglected the flexural signal by analyzing degrees greater than 150.

Even though the GRAIL gravity field has been developed to degree and order 420, the highest-degree terms are biased by noise and incomplete sampling between adjacent orbital tracks. In fig. S2, we plot the power spectrum of the gravity field, the measurement noise, the gravitational contribution from the surface topography (the Bouguer correction), and the gravitational signal that remains after removing the topographic contribution (the Bouguer anomaly).
Fig. S1. Gravitational admittance as a function of spherical harmonic degree for a lithospheric loading model that includes surface loads on a thin elastic spherical shell. For this global analysis, the crustal thickness was assumed to be 45 km, Young’s modulus was set to $10^{11}$ Pa, and the crustal and mantle densities were assumed to be 2800 and 3360 kg m$^{-3}$, respectively. Most of the highlands in Fig. 3 have crustal thicknesses greater than 45 km, which would act to reduce the magnitude of the compensating gravity signal from the crust-mantle interface.

Beyond degree 320, the power spectrum of the Bouguer anomaly increases in amplitude and follows the gravitational error. In order to minimize the consequences of noise from the gravity field in our analysis, we have used only degrees less than 310.

If the density of the crust $\rho_c$ were constant, the observed gravity $g$ at short wavelengths would be equal to the sum of the Bouguer correction $g^{BC}$ and the measurement noise $g^{\text{noise}}$. If the correct crustal density were chosen when calculating the Bouguer correction, the Bouguer anomaly $g^{\text{BA}}$ would simply be equal to the noise in the gravity model

$$g^{\text{BA}} (\rho_c) = g - g^{\text{BC}} (\rho_c) = g^{\text{noise}}. \quad (12)$$

Since the gravitational measurement noise should be uncorrelated with the surface topography, an unbiased estimate of the crustal density is obtained by finding the value that minimizes the correlation between the Bouguer anomaly and topography.

We calculated the correlation coefficient of the gravity and topography in the space domain using gridded Bouguer gravity and topography within a spherical cap of a specified diameter determined from gravity model JGGRAIL.420C1A and Lunar Reconnaissance Orbiter topography data (24). The gravity and topography coefficients were truncated below degree 150 and above degree 310, and the gravity was downward continued to the average radius of the analysis region. The gridded data were weighted by their associated areas when calculating the integrals of the products of Bouguer gravity and topography necessary for the correlation coefficient calculation. To neglect the gravitational contribution from mare basalts, which are more dense
than the highlands and which tended to pool at low elevations, we neglected data points that lay within the maria (38). To assure that each analysis is based on similar effective areas, we neglected those analyses for which more than 5% of the data were discarded. Analyses were centered on a 60-km (2° of latitude) equally spaced grid, and the crustal density was varied to find the value that minimizes the absolute value of the correlation coefficient.

Estimates of the uncertainties in our density determinations were obtained by Monte Carlo modeling. Using the known uncertainties in the global gravity field coefficients, noise in the Bouguer gravity field was simulated, and the probability distribution for the correlation coefficient between this noise and the topography was obtained. 1-σ error limits on the density were determined using the 68% confidence limits from the correlation coefficient probability distribution. To neglect regions with large uncertainties, a small number of analyses were discarded for which the minimum correlation coefficient was outside the 95% confidence limit.

Having calculated the bulk density from gravity and topography, we estimated the crustal porosity using independent estimates of the crustal grain density. An empirical correlation between grain density (estimated from a mineralogical norm) and the abundance of FeO and TiO$_2$ has been derived (15) from measurements over a large range of lunar rock compositions, including ferroan anorthosites, the Mg suite, the alkali suite, KREEP basalts, impact melts, granulitic breccias, mare basalts, and volcanic glasses. The mineralogical grain density estimates have been shown to be accurate to 20 kg m$^{-3}$ by direct measurements with helium pycnometry techniques (17), and the correlation with iron and titanium abundances has an intrinsic uncertainty of 45 kg m$^{-3}$ for the entire suite of lunar rocks. From iron and titanium abundances derived from Lunar Prospector gamma-ray spectrometer measurements (16), which are representative of the upper tens of cm of the surface, our global crustal grain density estimate is shown in
Crustal grain density estimated from Lunar Prospector iron and titanium abundances at 60-km resolution and a sample-based empirical correlation between grain density and composition. Thin lines outline the locations of the maria. Data are presented in two Lambert azimuthal equal-area projections centered over the near- (left) and farside (right) hemispheres, with each image covering 75% of the lunar surface, and with grid lines spaced every 30°.

For each region in our gravity analyses, the average grain density was calculated, from which the porosity $\phi$ was determined from the relation

$$\phi = 1 - \rho_{\text{bulk}}/\rho_{\text{grain}},$$  \hspace{1cm} (13)

where the bulk density is the density obtained from the gravity analysis. Since impact craters excavate materials to depths as great as one-tenth of the crater diameter (39), the composition of the lunar surface is likely to be representative of the underlying crust for most regions of the Moon. Nevertheless, where the crustal composition changes laterally, such as between the highlands and South Pole-Aitken basin, or between the highlands and maria, ejecta from one compositional unit may overlie surface materials of a different composition.

Histograms of our results using 360-km-diameter analysis regions are shown in fig. S4 for the bulk density, bulk density uncertainty, and porosity. Our bulk density and porosity determinations are insensitive to variations in the filter applied to the gravity and topography by more than ±50 in harmonic degree, as well as to changes in the size of the analysis region by
a factor of two. Nevertheless, as the analysis region decreases in size, the uncertainties in the individual density determinations increase slightly in magnitude (from about 10 kg m$^{-3}$ from 540-km-diameter analyses to 30 kg m$^{-3}$ for 180-km-diameter analyses). Porosity maps derived with several different sizes for the analysis regions are shown in fig. S5.

We verified the accuracy of our bulk density determinations with an alternative analysis method that makes use of the relation between gravity and topography in the spectral domain. Since the gravity contribution from topography at high harmonic degree is a non-linear function of the topography, we modeled the observed gravity in terms of the gravity contribution from surface relief of unit density, $\hat{g}$,

$$g_{\ell m}^{\text{obs}} = \rho_c \hat{g}_{\ell m} + I_{\ell m},$$

where $I_{\ell m}$ is that portion of the signal not predicted by the model, assumed to be a random variable uncorrelated with $\hat{g}$. Multiplying both sides by $\hat{g}_{\ell m}$, summing over all degrees, and taking the expectation with respect to $I$, we obtain an unbiased estimate for the crustal density at each degree:

$$\rho_c(l) = \frac{S_{g^{\text{obs}}}(l)}{S_{\hat{g}\hat{g}}(l)}.$$  

This equation is analogous to the admittance between gravity and topography from eq. (7). In fig. S6, we plot the global density implied by the GRAIL gravity field as a function of spherical harmonic degree. The effective density is relatively constant from about degree 100 to 310, demonstrating why our space domain analyses are insensitive to the limits of the high- and low-pass filters within this range. By averaging the density spectrum between degrees 150 and 310, we find the average density to be $2550 \pm 18$ kg m$^{-3}$. This global approach neglects lateral variations in density and yields an identical average to that obtained from the local analyses, because there is little power in the gravity signal over the maria at short wavelengths.

We next spatially localized the free-air gravity and gravity from unit density topography using spherical Slepian functions (40–42) and then computed the localized degree-dependent admittance and correlation functions, eqs. (14) and (8). The spectral bandwidth $L$ of the window was chosen to ensure that more than 99% of its power was localized within a spherical cap of a specified diameter. Since each degree $l$ of the localized power spectrum has contributions from the global field between degrees $l - L$ and $l + L$, we calculated the average density for all degrees between $150 + L$ and $310 - L$. As the localized correlation should be nearly unity for uncompensated topography, those degrees with localized correlations less than 0.98 were discarded. If more than half of the degrees available for analysis were discarded, or if the analysis region contained more than 5% mare basalt, the analysis was discarded as well.

Our bulk density estimates using this spectral approach are shown in fig. S7 for a localization window with a diameter of 540 km and a spectral bandwidth of $L = 28$. The average density of the highlands is found to be identical to that obtained from our space domain analysis. Minor differences in the lateral variations exist between Fig. 1 and fig. S7 that are a result of slightly different aspects of the two analysis techniques: The spectral approach is not capable of removing small regions of maria in an analysis region; the spatio-spectral localization window
Fig. S4. Histograms of bulk density, bulk density uncertainty, and porosity for the lunar highlands obtained from Bouguer gravity and topography within 360-km-diameter circles.
Fig. S5. Porosity of the lunar crust from gravity, topography, and independent grain density estimates. Thin lines outline the maria, and circles denote regions of crustal thinning for prominent impact basins from Fig. 3. Bulk densities and porosities were calculated from data within circles with diameters of 180, 240, 300, 360, 420, and 480 km, which correspond to 6, 8, 10, 12, 14, and 16° of latitude, respectively. All images are shown in cylindrical projection centered over the farside hemisphere.
is not uniform in amplitude like the “box car” window used in the spatial domain analysis; and since the gravity signal is largest at the lowest degrees, the spatial domain analysis is somewhat more sensitive to the lowest degrees (cf. fig. S2), whereas the spectral approach weights each degree evenly.

2 Depth dependence of porosity

If the density of the lunar crust were constant with depth, eq. (3) could be used to calculate the gravity above the surface. This assumption was the basis of our bulk crustal density determinations described in Section 1. If the crustal density were instead a function of depth below the surface, the lateral variations in density along a spherical interface below the surface would give rise to an additional gravitational signal. These gravitational signals would be attenuated both with increasing depth and with increasing spherical harmonic degree. We use the wavelength dependence of this subsurface signal to investigate the subsurface density profile. Two models were investigated: one in which the porosity decreases exponentially with depth below the surface, and another in which a constant thickness layer with constant porosity overlies a non-porous basement. For a discrete profile of density versus depth, the expected gravitational signature can be calculated to arbitrary precision from eq. (3) for each layer. We calculated the synthetic admittance for each density profile, and then compared this admittance with observations.

Synthetic admittance functions are shown in fig. S8 for several assumed porosity structures of the crust, all for a grain density of 2900 kg m$^{-3}$. Two end-member models with zero and 12% porosity are seen to be related by a simple multiplicative constant involving the ratio of
**Fig. S7.** Bulk density of the lunar crust from a localized spectral analysis approach. At each point on a grid of 60-km spacing, the free-air gravity and predicted gravity from topography with unit density were multiplied by a localization window with a diameter of 540 km (18° of latitude). The bulk density was determined as the average of eq. (15). Image format is the same as for fig. S3.
the two bulk densities. The other models have density profiles and admittances bracketed by these end-members. For one set of models, we assume that a 5- or 30-km thick layer with 12% porosity overlies non-porous bedrock. At small degrees, the admittance approaches the value predicted for the zero porosity model, whereas at the largest degrees, the admittance approaches the value predicted for the constant-12%-porosity model. This behavior is easily understood as the shortest-wavelength signals become increasingly attenuated with increasing depth of the source region below the surface. At the shortest wavelengths, the gravity field is simply a result of the density of the surface relief. Models that utilize an exponential decrease in porosity with depth yield very similar results if the depth of the porous layer $D$ is replaced by the $e$-folding depth $\lambda$, and if the porosity of the layer $\phi$ is replaced by the surface porosity $\phi_0$.

For illustrative purposes, we calculated the global effective density from eq. (15) for a layer of thickness $D$ and porosity $\phi$ under the assumption that the grain density is 2915 kg m$^{-3}$. This function gives the effective bulk density at each degree that would be obtained if it were assumed that the density of the crust is uniform. The root mean square (RMS) misfit between this effective density and that obtained from the observed gravity field is shown in fig. S9 for degrees between 150 and 310. A clear tradeoff is found between layer thickness and porosity, but a best fit is found for a layer thickness of 28 km and a porosity of 13%. Given that the gravity signal for the shortest wavelengths is attenuated with increasing depth of the source, the admittance is largely insensitive to layer thickness for values in excess of about 30 km. Similar results are found for a model in which the porosity decreases exponentially with depth.

We next used a spatio-spectral localization technique to calculate localized admittances, and then used Monte Carlo methods to estimate the 1-$\sigma$ upper and lower bounds for the layer thickness and porosity. The procedure used was similar to that described in Section 1, except that here we used a localization window with a diameter of 900 km (corresponding to 30° of latitude). By employing a larger window size, the spectral bandwidth of the window is reduced to $L = 17$, and this lower bandwidth gives us a higher spectral resolution and a greater number of localized admittances to analyze. In fig. S10, we plot the best-fit layer thickness and porosity, as well as the 1-$\sigma$ upper and lower bounds. The porosity is found to be rather well constrained and is consistent with our analyses in Section 1. In contrast, the layer depth is not well constrained. For most of the highlands, the 1-$\sigma$ upper bound is simply greater than about 40 km. Nevertheless, the 1-$\sigma$ lower bound varies from about zero to 31 km, implying that some regions of the Moon possess a porous layer that is tens of kilometers thick.

3 Density and porosity of lunar samples

Measurements of the density and porosity of lunar feldspathic rocks, including lunar meteorites and samples collected during the Apollo program, provide a context for interpreting the results derived in this study. Twenty-four feldspathic samples that contain 22–35 wt.% $\text{Al}_2\text{O}_3$, corresponding to 60–95% anorthite, have been analyzed using modern techniques (17, 43). The samples include a moderate abundance of mafic minerals, characterized by $\text{FeO} + \text{MgO}$ of 0.5–
Fig. S8. Synthetic admittances predicted by several assumed density profiles of the highlands crust. Black and gray curves represent end-member cases with a constant bulk density of 2900 and 2552 kg m$^{-3}$, respectively. Red and magenta curves are for models for which a layer of thickness $D$ and porosity $\phi$ overlies non-porous bedrock. Blue and green curves are for models in which the porosity decreases exponentially with depth with $e$-folding depth $\lambda$ and surface porosity $\phi_0$. 
Fig. S9. RMS misfit for a model with a layer of constant thickness and porosity overlying non-porous bedrock. The misfit is the rms difference between the effective density $\rho_c(l)$ calculated from eq. (15) for the model and that calculated from observed gravity between degrees 150 and 310.

18.9 wt.% and a range of magnesium numbers [the molar ratio Mg/(Mg+Fe)] from 0.44 to 0.80. By the inclusion of lunar meteorites, the data set is likely to be more representative of the Moon’s global crust than would be the case for measurements of Apollo samples alone (44). The FeO abundances and Mg numbers for these samples are similar to the ranges inferred for the lunar highlands from remote sensing data (16, 45). The samples measured in this study include Apollo samples 15418, 179, 60025, 174, and 61016, 484, and lunar meteorites Dhofar 081, 908, 910, 911, 1084, 1085, and 1443, Kalahari 008, Northwest Africa 482, 2998, 3163, 4932, 5000, 5406, 5744, 6355, 6470, 6570, and 6578, and Shisr 162 and 166 (46–57). By igneous nomenclature, these rocks are anorthosites, noritic anorthosites, and anorthositic norites. However, all of these rocks have experienced considerable post-igneous processing, such as brecciation, shock melting, and sometimes thermal annealing, and thus are usually classified as fragmental breccias, impact melt breccias, and granulitic breccias. The measured suite of feldspathic rocks is assumed to be representative of the range of impact processing experienced by rocks in the upper few kilometers of the lunar crust.

The bulk density of the rocks is based on the total volume, which includes the effects of void space from porosity. This density is appropriate for use in gravity modeling. Bulk density was measured by immersion of the sample in glass beads (18), and the grain density is based on the volume of solid material in the rock, excluding the effects of porosity, as measured using helium pycnometry methods (58). The small atomic radius of helium allows it to diffuse through even very small cracks, and it has been shown that there is no unmeasured pore space in these samples (17). From the bulk and grain densities, one can calculate the porosity from
Fig. S10. Best-fit layer thickness and porosity, as well as the 1-σ upper and lower bounds as determined from Monte Carlo modeling. For this analysis, localized inversions were performed using a window of 900 km diameter (corresponding to 30° of latitude).
eq. (13). Measurement uncertainties are typically 30–70 kg m$^{-3}$ for the bulk density and 1–3% for porosity. Fig. S11 shows that the measured bulk densities have a broad, flat-topped histogram, with a mean density of 2580±170 kg m$^{-3}$, and that the porosities also have a broad distribution, with a mean of 8.6±5.3%.

An earlier review of Apollo samples estimated a mean breccia porosity of 17±10% (59), which overlaps our estimate. Many of the data points used in the earlier porosity compilation, particularly of samples with high reported porosity, were based on measurements of aluminum foil shape models of Apollo samples, a method that typically overestimates porosity and that has large errors (∼10%) (60). The lunar meteorites included in the present study were all collected in various deserts and experienced varying amounts of alteration due to terrestrial weathering, as assessed both by thin section examination and from the abundance of elements such as Ba and Sr that are mobile during weathering. It is possible that deposition of minerals such as calcite or gypsum has modified the porosity of these samples. In most cases, published petrographic descriptions and chemical analyses suggest that such modification has only a limited effect on the observed porosity. For example, NWA 482 has experienced little compositional alteration by weathering (50, 52, 57). Dhofar 908, 911, and 1085 have been estimated to contain about 2% terrestrial calcite (53). Terrestrial weathering fractures in Kalahari 008 are filled with calcite, with possibly little net change in porosity, and only weak chemical alteration is observed in Sr and Ba (54, 56). On the other hand, Dhofar 1084 may contain 8-10% terrestrial calcite or gypsum (55).

4 Viscous closure of pore space

Rocks at elevated temperatures will flow if subjected to stress, and this process can result in thermal annealing of porosity over long timescales. To calculate the time evolution of lunar porosity, we follow the approach outlined in Appendix B of Nimmo et al. (22), which is based
on the approaches of Fowler (61) and Eluszkiewicz (62). The porosity $\phi$ evolves according

$$\frac{d\phi}{dt} = \phi \rho g z \eta,$$

(16)

where $\rho$ is the crustal density, $g$ is the acceleration due to gravity, $z$ is depth below the surface, and $\eta$ is the viscosity. The overburden pressure $P$ is $\rho g z$. In general, viscosity is a function of temperature, grain size, and stress, and here we calculate $\eta$ according to

$$\eta = \frac{P^{1-n}}{A} d^p \exp \left( \frac{Q}{RT} \right),$$

(17)

where $A$, $n$, and $p$ are rheological parameters, $Q$ is the activation energy, $R$ is the gas constant, $d$ is the grain size, and $T$ is temperature.

Anorthite deformation is dominated by dislocation creep, which is independent of grain size at the relevant temperatures and strain rates. Its viscosity changes by an order of magnitude for every 15 K temperature change, which is equivalent to 1-2 km change in depth. Thus, the transition from porous to pore-free material should be sharp, and the thickness of the porous layer is insensitive to uncertainties in the amount of stress concentration.

Given a temperature profile, eqs. (16) and (17) can be solved numerically to determine the porosity evolution as a function of time. Fig. S12 shows sets of normalized porosity profiles after 4 billion years of evolution for two different rheologies and different representative surface heat fluxes. As expected, initial near-surface porosity is unchanged because the viscosities are too high for flow to occur, whereas porosity at greater depth is reduced to zero. As expected, higher heat fluxes result in a thinner porous layer, and the transition interval occurs over a narrow depth range because of the strong temperature-dependence of viscosity. Olivine (mantle) is less deformable than plagioclase (crust), and as a result, the olivine rheology has a porous layer extending to greater depth. This approach is predicated on the assumption that porosity is initially present and is not subsequently regenerated; such an assumption is approximately in line with the steeply declining impactor flux on the Moon.

5 Thickness of the lunar crust

We constructed a global model for the thickness of the lunar crust that satisfies the observed gravity field and seismic constraints. For this model, the observed gravity was assumed to be a result of relief along the surface, relief along the crust–mantle interface, and lateral variations in density of the crust:

$$C_{lm}^{obs} = C_{lm}^{topo} + C_{lm}^{c_m} + C_{lm}^{p}.$$  

(18)

If the density of the crust depends upon position, but not depth, the first two terms can be calculated from eq. (3). For the first contribution, the surface relief has a mean radius $R$ and possesses a density contrast $\rho(\theta, \varphi)$, whereas for the second contribution, the relief of the crust–mantle interface has a mean radius $D$ and possesses a density contrast $\rho_m - \rho(\theta, \varphi)$. The
**Fig. S12.** Normalized porosity profiles after 4 Gy of evolution for (a) a dry plagioclase rheology (63) and (b) a dry olivine rheology (64). The temperature structure is assumed constant with time and is calculated for surface heat fluxes $F$ of 20, 30 and 40 mW m$^{-2}$. To produce these plots, we assumed a constant specified heat flux $F$, a surface temperature of 250 K, $\rho = 2800$ kg m$^{-3}$, and $g = 1.6$ m s$^{-2}$. A constant thermal conductivity of 2.3 W m$^{-1}$ K$^{-1}$ was assumed to account for the effect of porosity in reducing conductivity (65).

The gravitational contribution from lateral variations in density of the shell between the two radii $R$ and $D$ is calculated from the potential coefficients (referenced to radius $R$)

$$C_{lm}^\rho = \frac{4\pi R^3}{M(2l+1)(l+3)} \left(1 - (D/R)^{l+3}\right) \rho_{lm},$$

where $\rho_{lm}$ are the spherical harmonic coefficients of $\rho(\theta, \varphi)$. In our model, we do not attempt to fit the total mass of the Moon (i.e., the term $C_{00}$), as this depends on the unknown density profile of the mantle and the size and composition of the core.

Following Wieczorek and Phillips (27), we solve eq. (18) for the spherical harmonic coefficients of the first-order term of the crust–mantle boundary relief, here denoted as $h$, multiplied by the density contrast. The resulting non-linear equation is then solved in an iterative manner: Given the $i$th estimate of the crust–mantle boundary relief, the spherical harmonic coefficients of the next estimate multiplied by the density contrast are calculated from

$$\left(\left(i+1\right)h^i \rho\right)_{lm} = w_l \left[\left(C_{lm}^{\text{obs}} - C_{lm}^{\text{topo}} - C_{lm}^\rho\right) M \left(2l+1\right) \left(\frac{R}{D}\right)^i - D \sum_{n=2}^{N} \frac{\left(i\right)^n \rho_{lm} \prod_{j=1}^{n} \left(l + 4 - j\right)}{D^n n! \left(l + 3\right)}\right],$$

for all $l > 0$, where $w_l$ is a filter that removes high-frequency signals resulting from physical processes not accounted for in our model. Since we do not explicitly model the degree-0 potential coefficient, the degree-0 term of $\left(i+1\right)h^i \rho$ is calculated separately as the average of $\left(i\right)h^i \rho$. 

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(If the density were constant, this term would be zero.) The function $h \rho$ is then expanded on a grid, and the relief $h(\theta, \phi)$ is obtained after dividing by the density contrast $\rho(\theta, \phi)$. To stabilize oscillations between successive iterations, and to speed convergence, the following iterative scheme is used:

$$
(i+3) h = \left( (i+2) h + (i+1) h \right) / 2, \quad (21)
$$

$$
(i+4) h = f \left( (i+3) h \right), \quad (22)
$$

where $f$ represents schematically the above-described calculation of $(i+1) h$. The iteration is initialized using the first-order term of eq. (20).

As an estimate for the grain density of the lunar crust, we make use of Lunar Prospector elemental abundances and an empirical correlation between grain density and composition (see section 1). Given that the most prominent lateral variations in density are associated with broad compositional units, such as the highlands and South Pole-Aitken basin, we used $5^\circ$ gridded Lunar Prospector data (16). The crust beneath the mare basalts is thought to be broadly anorthositic in composition, and we discard all pixels that containing mare, as well as a few pixels that yield grain densities larger than about 3050 kg m$^{-3}$. A global grid for the crustal grain density was then obtained by interpolating among the remaining data (fig. S13). The average grain density of the crust is 2927 kg m$^{-3}$, and the bulk density was obtained by multiplying each pixel by $(1 - \phi)$. This density map was then used in the calculation of $C_{\text{topo}}$, $C_{\rho}$, and eq. (20).

To obtain a unique crustal thickness model, we varied the average thickness of the crust $(R - D)$ and the mantle density $\rho_m$ in order to find the solution that fits the seismic constraints at the Apollo 12 and 14 sites. We further constrained the minimum crustal thickness to be less than 1 km, given that at least one of the impact basins on the Moon should have excavated through the entire crust (14, 26). As justification, we note that a vertical impact forming the Crisium basin (with a diameter close to 600 km) should have excavated materials to depths of about 60 km (39), which greatly exceeds the Apollo seismic crustal thickness estimates. Furthermore, several large impact basins, such as Apollo, are known to have formed within the previously thinned crust associated with the South Pole-Aitken basin. Increasing the minimum crustal thickness constraint by 1 km would increase the average crustal thickness by 2 km.

The mantle density obtained from our inversion should be considered as representative only to the greatest depth of the crust–mantle interface, which is less than 80 km. At greater depths, our constant density mantle does not generate a gravitational signature. The degree $\lambda_{1/2}$ at which the filter $w_1$ achieves a value of 0.5 was chosen to be close to 80 in order to minimize high-frequency signals that are not included in our model and that would otherwise be mapped to crustal thickness variations. Unmodeled signals include lateral variations in crustal density with spatial scales that are finer than those in fig. S13. Filtering also stabilizes the crustal thickness inversion, because noise in the gravity field is amplified when downward continued to the average depth of the crust-mantle interface $D$ in eq. 20. To minimize the consequences of noise in the gravity field, the gravity coefficients were truncated above degree and order 310. All calculations were performed with $N = 7$, and to avoid aliases when computing the spherical
Fig. S13. Grain density of the lunar crust used for constructing a global crustal thickness model. Grain densities exterior to the maria (indicated by the symbol +) were estimated from a 5° grid of Lunar Prospector compositional data along with an empirical correlation between composition and density. After discarding a few pixels with densities greater than about 3050 kg m$^{-3}$, a global map was determined by interpolating among the remaining data. Image format is the same as for fig. S3.
Tests show that if lateral variations in crustal density were neglected, long-wavelength errors with amplitudes of \(\pm 4\) km would arise. By including lateral variations in density, the thickness of the crust within the South Pole-Aiken basin is about 4 km greater, and the thickest portion of the farside highlands crust is about 4 km thinner, than would be obtained from a uniform density model. Crustal models constructed from GRAIL bulk density maps (Fig. 1) give nearly identical average crustal thicknesses to those derived from the grain density map of fig. S13 and yield lateral variations in thickness that are very similar to those of Fig. 3. Since we do not yet have good constraints on how crustal porosity varies with depth, use of the GRAIL bulk densities for the entire crust is not justified at the present time. The neglect of the gravitational signal arising from the dense mare basalts should have only minor consequences on our crustal thickness model. These deposits are in general less than 1 km thick (14), and with a density close to that of the mantle, the neglect of these deposits would bias the thickness of the crust downward only by an amount approximately equal to the thickness of these deposits.

Analyses of the Apollo seismic data initially suggested that the crust was about 60 km thick in the region of the Apollo 12 and 14 landing sites (67), and this value was used as a constraint in several global crustal thickness models following the Clementine mission (27,68,69). After the Lunar Prospector mission, reanalyses of the Apollo seismic data by two groups using different data sets showed that the crust in this region is thinner than once thought. Khan et al. (70) initially suggested a thickness of 45 \(\pm\) 5 km in this region, and they later revised their value to 38 \(\pm\) 3 km (25). An independent analysis by Lognonné et al. (23) and Gagnepain-Beyneix et al. (71) obtained a value of 30 \(\pm\) 2.5 km for the same region, with a later analysis suggesting values of 33 \(\pm\) 5 and 31 \(\pm\) 7 km for the Apollo 12 and 14 landing sites (72), respectively. Most subsequent crustal thickness models were not able to fit the revised seismic constraints, and in retrospect, this was because these studies employed a crustal density between 2800 and 2900 kg m\(^{-3}\). Because of this problem, along with initial skepticism to the revised seismic estimates, most subsequent studies (14,28,73) either employed the 45-km constraint of Khan et al. (70) or ignored the Apollo seismic data altogether. Chenet et al. (72) were successful in reconciling the seismic and crustal thickness modeling, but their study did not consider crustal densities below 2600 kg m\(^{-3}\) and was based on gravity models from Lunar Prospector observations that possessed poor resolution over the farside hemisphere.

The parameters of several crustal thickness models are summarized in Table S1. Two pre-GRAIL models predict an average crustal thickness of 53 km (14,28) and crustal thicknesses near 45 km at the Apollo 12 and 14 landing sites. Using the grain density map of fig. S13 with a 12% porosity, and for a 30-km crustal thickness at the Apollo 12 and 14 sites, the average crustal thickness and mantle density are predicted to be 34 km and 3220 kg m\(^{-3}\), respectively. For an assumed porosity of 7%, the average crustal thickness barely changes, but the mantle density increases to 3360 kg m\(^{-3}\). This behavior is easily understood: the largest crustal thickness variations are a result of relief along the crust–mantle interface, and the amplitude of this relief is controlled by the density contrast between the crust and mantle. A change in crustal density
simply trades off to a change in mantle density. For a crustal thickness constraint of 38 km at the Apollo 12 and 14 sites, the average crustal thickness increases to 43 km, and the mantle density decreases by about 60 kg m$^{-3}$.

A final crustal thickness model was constructed under the assumption that porosity exists in a layer of constant thickness confined between the surface and depth $z$. Mathematically, the porosity in this layer is modeled as having a negative density $-\phi \rho_c(\theta, \varphi)$, where $\rho_c$ is the grain density, and the gravitational contribution from this void space is added to the other contributions in eq. (18). A gravity signal is generated with a density contrast $-\phi \rho_c(\theta, \varphi)$ at the surface, and also from an equal and opposite density contrast $\phi \rho_c(\theta, \varphi)$ with the same relief, but at depth $z$ km below the surface. The density contrast should in fact be $-\phi \rho_m$ wherever the crust is thinner than $z$, but since this condition holds only beneath the largest impact basins where temperatures could have exceeded the solidus, it is reasonable to use a density contrast with a smaller amplitude in these regions. As with our previous models, the largest variations in crustal thickness are a result of relief along the crust–mantle interface. With the porosity contribution already having been included, the density contrast at the base of the crust is $\rho_m - \rho_c(\theta, \varphi)$, which is smaller by $\phi \rho_c$ than those used in our previous models. This density contrast is similar to that used in the pre-GRAIL models, and it is not possible to construct a global crustal thickness model that satisfies both the Apollo seismic and minimum crustal thickness constraints, regardless of the values chosen for $\phi$ and $z$. If the seismic constraint is ignored, and if the mantle density is set to 3400 kg m$^{-3}$ to maximize the density contrast, an average crustal thickness of $\sim$65 km is obtained, with a thickness of about 57 km at the Apollo 12 and 14 sites.

### Table S1. Summary of crustal thickness models. Those models that include crustal porosity make use of the crustal grain density map in fig. S13.

<table>
<thead>
<tr>
<th>Model</th>
<th>Average thickness, km</th>
<th>Minimum thickness, km</th>
<th>Apollo 12/14 thickness, km</th>
<th>$\rho_c$ kg m$^{-3}$</th>
<th>$\phi$ %</th>
<th>$\rho_m$ kg m$^{-3}$</th>
<th>$\lambda_{1/2}$</th>
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<tr>
<td>Lunar Prospector gravity, Clementine topography</td>
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<td></td>
<td></td>
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<td></td>
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<td>Wieczorek et al. (2006)</td>
<td>53.4</td>
<td>0</td>
<td>45</td>
<td>2900</td>
<td>—</td>
<td>3320</td>
<td>30</td>
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<td>Kaguya gravity and topography</td>
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<td></td>
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<tr>
<td>Ishihara et al. (2009)</td>
<td>53</td>
<td>1.2</td>
<td>47.5</td>
<td>2800</td>
<td>—</td>
<td>3360</td>
<td>50</td>
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<tr>
<td>GRAIL gravity and LRO topography</td>
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<td></td>
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<td>Model 1</td>
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<td>29.9</td>
<td>—</td>
<td>12</td>
<td>3220</td>
<td>80</td>
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<td>30.8</td>
<td>—</td>
<td>7</td>
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<td>80</td>
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<td>38.1</td>
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<td>12</td>
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<td>38.0</td>
<td>—</td>
<td>7</td>
<td>3300</td>
<td>70</td>
</tr>
</tbody>
</table>
References and Notes


11. M. T. Zuber et al., Gravity field of the Moon from the Gravity Recovery and Interior Laboratory (GRAIL) mission. Science (2012); published online 5 December 2012; 10.1126/science.1231507

12. Methods and additional materials are available as supplementary materials on Science Online.


