Geophysical implications of the long-wavelength topography of Rhea

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[1] We use limb profiles to investigate the long-wavelength topography and topographic variance spectrum of Rhea. One-dimensional variance spectra show a break in slope at a wavelength of \( \approx 300 \) km; a similar effect is seen on the Moon and may be a signature of an elastic lithosphere having a thickness \( T_e \approx 10 \) km. The implied heat flux is \( \approx 15 \) mW m\(^{-2}\), much higher than can be explained by radiogenic heating. We use the 1-D spectral behavior to constrain our solution for the long-wavelength global topography of Rhea. The degree 3 topography is large enough, if uncompensated, to contaminate estimates of the degree 2 gravity using existing flyby data. Current models of Rhea internal structures which rely on these degree 2 estimates may thus be inaccurate, illustrating the need to acquire further Rhea gravity data.


1. Introduction

[2] The long-wavelength topography of satellites is important for at least three reasons. First, the spherical harmonic degree 2 topography may be used, subject to certain assumptions, to infer the moment of inertia and thus the internal structure of these bodies [e.g., Dermott, 1979; Iess et al., 2007; Anderson and Schubert, 2007]. Second, long-wavelength topography and its variation with wavelength may provide evidence of other physical processes occurring, in particular the effects of tidal heating [e.g., Ross et al., 1990; Nimmo et al., 2007], impact cratering [e.g., Thomas et al., 2002], flexure [Luttert and Sandwell, 2006] and/or relaxation processes [Malamud and Turcotte, 2001]. Finally, uncompensated long-wavelength topography can complicate the interpretation of degree 2 gravity anomalies and their implications for internal structures [Nimmo and Matsuyama, 2007; Mackenzie et al., 2008].

[3] Cassini optical images have yielded multiple topographic limb profiles of all the major icy satellites except Titan [Thomas et al., 2007], while Titan topographic profiles have been obtained using two different radar techniques [Zebker et al., 2009]. So far, these data have mainly been interpreted in the context of the first topic above, namely the internal structure of the satellites [Anderson and Schubert, 2007; Iess et al., 2007]. The aim of this paper is to investigate the second and third topics for Rhea. Both topics have specific applications to Rhea: as a rather tectonically inactive satellite [Jaumann et al., 2009], Rhea’s topography probably provides a record of ancient, rather than contemporary, processes; and there is currently some disagreement over the interpretation of the existing gravity data, and thus Rhea’s internal structure [Anderson and Schubert, 2007; Mackenzie et al., 2008].

[4] An alternative method of deriving topography is to use stereo images [e.g., Schenk and Moore, 1995; Giese et al., 1998]. This technique provides very different data. Stereo data are 2-D, rather than one dimensional profiles, and have a spatial resolution typically a few times worse than the image resolution. On the other hand, long-wavelength stereo topography typically has large errors (because of the small angles involved), while analysis of the frequency content of stereo data is greatly complicated by the use of smoothing and gap-filling techniques. Comparison of stereo-derived topography with subsequently acquired laser altimetry at Mars revealed errors in the former of several km in places [Bills and Nerem, 2001]. Similarly, comparison of photogrammetric-and laser altimeter-derived topography of the Moon revealed systematic differences at short wavelengths between the two data sets [Araki et al., 2009]. In section 3.4 we briefly compare our results with those of stereo topography.

[5] The rest of this paper is organized as follows. In section 2 we will investigate how topographic roughness varies as a function of length scale on Rhea; that is, we will determine the topographic variance spectrum. In order to do so, we will investigate the extent to which the limb profiling technique introduces systematic biases (section 2.1). In section 3, we will use the observed variance spectrum as a constraint to help determine the global long-wavelength topography of Rhea. Doing so is not straightforward, because of the restricted coverage and nonuniform distribution of the limb profiles. In section 4 we will discuss the geophysical
implications of our results. The key conclusions are that: Rhea’s variance spectrum shows a break in slope at \( \approx 300 \) km wavelength, in a somewhat similar fashion to the variance spectra of the Moon (sections 2.2 and 4.1); this break in slope may be the signature of Rhea’s elastic lithosphere (section 4.2); Rhea’s topography likely complicates interpretation of its degree 2 gravity (section 4.3).

We make extensive use of synthetic data sets to verify that our techniques are behaving properly. For those readers more interested in the results, these may be found in sections 2.2, 3.4 and 4. We have chosen here to restrict our analysis to Rhea, but identical techniques can be applied to other satellite data sets and will form the basis of future work.

2. Limb Profile Variance Spectra

A topographic variance spectrum is simply one measure of the roughness of the topography at different wavelengths [e.g., Shepard et al., 2001]. Although variance spectra can be derived from 1-D or 2-D data, here we will focus on the 1-D data (limb profiles). We discuss 2-D (spherical harmonic) variance spectra in section 3.1.

To obtain the variance spectrum of a set of evenly spaced topographic observations \( h_i \) (with \( i = 0, \ldots, N-1 \)), we take the discrete Fourier transform [e.g., Press et al., 1992]:

\[
H_j = \sum_{n=0}^{N-1} h_n e^{2\pi i j n / N}
\]

where \( i, j \) are integers and the wave number \( k \) associated with \( H_j \) is given by \( k = 2\pi (j - 1)/L \), where \( L \) is the total length of the topographic profile. The variance at a particular wave number is then given by \( \frac{1}{N} |H_j|^2 \) with some suitable normalizing factor (we use 1 for simplicity). In practice, we interpolate to a constant spacing and then remove the mean and detrend the profiles prior to transforming them, and average the variance estimates within wave number bins to reduce the scatter.

2.1. Limb Profile Biases

A spacecraft image of the limb of a body can be used to determine the topography along that limb, with a vertical resolution that in ideal circumstances can be as small as one-tenth the pixel size [Dermott and Thomas, 1988]. The Cassini spacecraft has acquired numerous limb images of the Saturnian satellites, with the techniques and results described by Thomas et al. [2007].

For Rhea 22 images were used, resulting in a total of 15,764 data points. The distribution of limb profiles across the surface of Rhea is rather uneven (see Figure 6a), and the surface is quite rough (RMS roughness 1.08 km; see Figure 6c for individual profiles). The raw profiles have had the best fit ellipsoid of Thomas et al. [2007] subtracted, and thus would not be expected to contain much power in the longest wavelengths (spherical harmonic degree 2). Where appropriate, we compare results using raw profiles with results using profiles which have had the best fit ellipsoid added back in, and are thus referenced to a sphere.

Although they require less processing than stereo-derived or photoclinometrically derived topography, limb profiles have one significant disadvantage: they are biased toward topographic highs, because a depression may not be seen if there is a peak in front of or behind it. Thomas et al. [2007] estimated the mean bias to range from 0.15 to 0.7 km for the different Saturnian satellites, though in reality this bias will vary spatially.

We wish to understand the effect of this bias on the power spectral properties of the topography. To do so, we calculated synthetic topographic profiles having a variance spectrum similar to those actually measured (see below), and derived the corresponding synthetic limb profiles. The geometry of the situation is shown in Figure 1, where the bold line denotes the synthetic topography superimposed on a spherical satellite of radius \( R_0 \) viewed by a spacecraft at a distance \( d \) from the center of Rhea. As Figure 1 demonstrates, the apparent location of the limb may be displaced toward or away from the spacecraft relative to the actual location. The apparent limb amplitude \( R'' \) will equal or exceed the true limb amplitude \( R \). The apparent limb location is determined by finding the position at which the viewing angle \( \alpha \) is maximized, where \( \alpha \) is given by

\[
\tan \alpha = \frac{R'' \cos \theta}{d - R'' \sin \theta}
\]
and $R'(\theta)$ is the local radius at colatitude $\theta$. The resulting apparent limb amplitude is then simply $R'' = d \tan \alpha$. By carrying out a whole set of these calculations at different longitudes, an apparent limb profile may then be derived and compared with the (known) actual limb topography.

[13] Based on the Cassini ISS-NAC instrument characteristics [Porco et al., 2004], we assumed that $d/R_0 = 400$ for these calculations. Anticipating the results of section 2.2, we generated the synthetic topography by randomly picking spherical harmonic coefficients such that the global variance spectrum was flat out to spherical harmonic degree $l = 15$, and had a slope of $-2$ at shorter wavelengths (see section 3.1 for definitions). The synthetic topography was expanded out to $l = m = 120$.

[14] Figure 2a shows the apparent (black) and actual (red) limb profiles for our synthetic topography. Note that, as expected, the apparent limb amplitude always equals or exceeds the actual one. Figure 2b shows the 1-D topographic variance spectrum for the actual profile, calculated using equation (1), with crosses denoting the variance of individual wave numbers and the solid line giving a binned estimate. As intended, the spectrum is approximately flat out to degree 15 and follows a $-2$ slope thereafter. In comparison, Figure 2c shows the variance spectrum of the apparent limb profile. Importantly, the estimates, especially at the longest wavelengths, are essentially unaffected (compare Figure 2b). The main difference is that spurious higher-frequency power is introduced at $l > 120$ (not shown). We also checked that synthetic profiles in which negative topography has a systematically higher amplitude than positive topography (to mimic crater-dominated landscapes) produced the same outcome. Based on this analysis, we conclude that the positive biasing inherent to limb profiles is unlikely to have any effect on the long-wavelength variance spectral behavior.

### 2.2. One-Dimensional Variance Spectra

[15] At the long wavelengths of most interest to us, one might imagine that the best way of estimating a topographic variance spectrum would be to derive a spherical harmonic description of the topography from the limb profiles. This would facilitate comparison with other bodies for which the long-wavelength spherical harmonic topography is known [e.g., Bills and Lemoine, 1995]. Unfortunately, as will be explained in section 3, the limited and uneven coverage of the profiles makes such variance spectrum estimation very difficult. Instead, we will use equation (1) to derive 1-D variance spectra for the different limb profiles and then average the result. As long as the topography is isotropic, doing so should provide an unbiased estimate of the true variance spectrum, and we use synthetic profiles to demonstrate that this is the case.

[16] Figure 3a shows the variance spectrum derived by averaging the 1-D variance spectra of 17 of the longest Rhea limb profiles. The bold line and dots show the mean spectrum, the two thin lines are one standard deviation above and below. Each profile was detrended prior to the variance spectrum being taken, and estimates were made in 20 wave number bins spaced logarithmically. Two aspects of Figure 3a are particularly significant. First, at short wavelengths the variance spectral slope is roughly $-2$. Second, at long wavelengths ($\geq 300$ km, $l \leq 15$), the variance spectrum deviates from this slope, and is approximately flat. This second aspect is unexpected, and its geophysical implications will be discussed in further detail in section 4.

[17] To verify that the apparent break in slope of the variance spectrum at $\approx 300$ km is not an artefact, we generated two sets of synthetic limb profile data at the same locations as the real data. The first set had a global spherical harmonic variance spectral slope of $-2$ throughout; the second set had a flat spherical harmonic variance spectrum out to $l = m = 15$ and a slope of $-2$ at shorter wavelengths (similar to the observed behavior). Figure 3b shows the averaged variance spectrum of the first set, generated using exactly the same approach as the real data. A slope similar to $-2$ is clearly retained even at long wavelengths; the downturn at short wavelengths occurs because the synthetic
3. Deriving Global Topography

Given multiple limb profiles, it is desirable to determine the global long-wavelength topography of Rhea, expressed as spherical harmonics. Unfortunately, doing so is difficult because of the uneven coverage afforded by the profiles. The resolution along a profile track is typically 2 km, but there may be gaps hundreds of km wide between profiles. Thus, despite the good along-track resolution, only very long-wavelength features (small harmonic degree \(l\)) can be resolved.

A similar problem was faced by Smith and Zuber [1996] in their analysis of Mars occultation data sets, and Zebker et al. [1996] in their study of Titan altimetry profiles. The latter authors adopted an approach in which areas lacking data were constrained to have smooth topography. Here we will adopt a slightly different approach, and use the 1-D variance spectra (section 2) to provide a constraint on the amplitude of the global topography at different wavelengths. We describe our implementation of this constraint below; as with our 1-D variance spectra, we verified that our technique works by using synthetic topographic data sets.

3.1. Definitions

Topography \(h\) on a sphere can be expressed as a summation of spherical harmonic coefficients:

\[
h(\theta, \phi) = R_0 \sum_{l=0}^{N} \sum_{m=-l}^{l} P_{lm}[\cos(\theta)](C_{lm} \cos m\phi + S_{lm} \sin m\phi)
\]  

where \(\theta\) and \(\phi\) are colatitude and longitude, respectively, \(R_0\) is the mean satellite radius, \(S_{lm}\) and \(C_{lm}\) are the spherical harmonic coefficients of degree \(l\) and order \(m\) and \(P_{lm}[\cos(\theta)]\) are normalized associated Legendre functions. Here the resolution along a profile track is typically 2 km, but there may be gaps hundreds of km wide between profiles. Thus, despite the good along-track resolution, only very long-wavelength features (small harmonic degree \(l\)) can be resolved.

Figure 3. (a) Averaged 1-D variance spectrum of 17 Rhea limb profiles (numbers 5–17, 21–24 in Figure 6c). Dots and bold line are mean values; thin lines are ± one standard deviation. Dashed line has a (logarithmic) slope of −2. (b) As for Figure 3a, but using synthetic limb profile topography generated by assuming a global 2-D variance spectral slope of −2 and expanding out to \(l = m = 120\) (see section 3.1 for definitions). (c) As for Figure 3b, but using a broken variance spectrum which is flat at \(l = 15\) and has a slope of −2 at shorter wavelengths. Note the resemblance between Figures 3a and 3c.

topography is only calculated to \(l = m = 120\). Figure 3c shows the corresponding variance spectrum of the second set, and shows that our approach does capture the imposed break in variance spectral slope at \(l = m = 15\). We therefore conclude that Figure 3a provides an accurate estimation of the long-wavelength 1-D variance spectrum of Rhea’s topography.

3.2. Estimating Spherical Harmonic Coefficients

The standard linear least squares approach to estimating the set of coefficients [e.g., Bills and Ferrari, 1977] \(\mathbf{z} = \{C_{00}, C_{10}, C_{11}, S_{11}, C_{20}, C_{21}, S_{21}, C_{22}, \ldots, C_{NN}, S_{NN}\}\) from a set of observations \(\mathbf{\hat{z}} = \{\hat{h}_1, \hat{h}_2, \hat{h}_3, \ldots, \hat{h}_M\}\), where \(\hat{h}_i\) is the measured topography at position \((\theta_i, \phi_i)\), is given by the matrix equation

\[
\mathbf{\hat{z}} = (\mathbf{A}^T \cdot \mathbf{A})^{-1} \cdot \mathbf{A}^T \cdot \mathbf{\hat{z}}
\]  

where an underline denotes a matrix and \(A_{ij}\) is the partial derivative matrix

\[
A_{ij} = \frac{\partial \hat{z}_i}{\partial \hat{y}_j}
\]

each element of which can be obtained from equation (3), \(T\) denotes transpose and a hat denotes an estimated quantity. Equation (3) suggests that the diagonal elements of \(\mathbf{A}^T \cdot \mathbf{A}\) will be of the order \(MR_0^2\), where \(M\) is the number of observations. Since \(\mathbf{\hat{z}}\) is known (the observations) and \(\mathbf{A}\) can be calculated, in principle the spherical harmonic coefficients
can be recovered by application of equation (5). In practice, if coverage is too sparse or too uneven, the matrix $A^T \cdot A$ will be nearly singular and the inverse ill defined.

[24] If this problem occurs, then an additional constraint has to be applied. One possibility is to require that the topography not constrained by data be smooth [e.g., Zebker et al., 1995]. An alternative, which we will apply here, is to apply an a priori constraint on the variance spectrum of the topography [e.g., Bills and Ferrari, 1977]. The reason for doing so is that we can make good estimates of the 1-D variance spectra down to short wavelengths using individual limb profiles (see section 2.2); assuming that the topography is isotropic, these variance spectra can then be used to constrain the spherical harmonics.

[25] From section 2, the 1-D variance spectrum at long wavelengths ($l \leq 15$) is approximately flat, with a $-2$ slope at shorter wavelengths. It can be shown that the total variance $\sigma_n^2 = \sum_{l=1}^{\infty} V_n$ for such a spectrum is $2l_{\text{brk}}E$, where the degree at which the break in slope occurs is $l_{\text{brk}}$ the degree variance for $l \leq l_{\text{brk}}$ has a constant value of $E$ and we have approximated the slope $-2$ spectrum for $l > l_{\text{brk}}$ by $1/(l+1)$.

[26] To mimic the expected break in slope of the 2-D variance spectrum, we will therefore apply a constraint $b_l$ of the form

$$b_l = \begin{cases} \frac{\sigma_n^2}{2l_{\text{brk}}(2l+1)} & \text{if } l \leq l_{\text{brk}} \\ \frac{\sigma_n^2(l_{\text{brk}}+1)}{2(l+1)(2l+1)} & \text{if } l > l_{\text{brk}} \end{cases}$$

(7)

Here $\sigma_n^2$ is the total variance of the topography and we divide by $(2l+1)$ because there are that many degrees of freedom. In practice only the upper definition matters, since the maximum degree of our expansion $l_{\text{max}} < l_{\text{brk}}$. To apply this constraint we define the diagonal matrix $B$ where the values on the diagonal are given by $\{b_0, b_1, b_1, b_1, b_2, b_2, b_2, b_2, \ldots \}$. The form of equation (7) guarantees that the trace of the matrix $B$ if expanded out to sufficiently large $l$ equals the observed variance $\sigma_n^2$. Rather than equation (5), we will now impose an a priori constraint resulting in

$$\hat{\chi} = [A^T \cdot A + rMB]^{-1} \cdot A^T \cdot \chi$$

(8)

Here $r$ is an arbitrary dimensionless constant which allows the strength of the a priori constraint to be varied. The specific form of equation (8) is chosen so that the two matrices within the square brackets have diagonal elements of comparable sizes when $r = R_0^2/\sigma_n^2$. Values of $r$ in excess of this critical value will result in reduced variance of the estimated topography, while in the limit of $r \rightarrow 0$ equation (8) reduces to equation (5).

[27] The shape of our a priori constraint (equation (7)) is derived from 1-D variance spectra, but is applied in equation (8) to 2-D data. Our reason for doing so is that for an isotropic function, the covariance function will on average be the same irrespective of whether it is sampled in 1-D or 2-D [Bills and Lemoine, 1995]. Using our definitions of the 1-D and 2-D variance spectra, we verified that a synthetic global topography field having a 2-D slope of $-2$ (equation (4)) yielded an averaged 1-D slope of $-2$ when sampled by 36 evenly spaced, north-south profiles. We also demonstrate later (Figure 7a) that, using our definitions, the 1-D and 2-D variance spectra of Mars have approximately the same slope.

### 3.3. Synthetic Data

[28] To verify our approach, we made use of our global synthetic topography having a spherical harmonic variance spectrum with a break in slope at $l = m = 15$ (compare section 2). We generated synthetic profiles at the same locations as the real profiles, and then applied equation (8) to these synthetic data for various values of $r$ to estimate the spherical harmonic coefficients. Based on the limb profile coverage, we set the maximum degree of our expansion $l_{\text{max}} = 8$; we examine the effect of varying $l_{\text{max}}$ in section 3.4.

[29] Figure 4a shows the original global synthetic topography, expanded out to $l = m = 120$, and Figure 4b shows the same topography expanded to $l = m = 8$. The latter has a global variance $\sigma_n^2 = 2.5 \text{ km}^2$. Figures 4c-4f show the global $l = m = 8$ topology derived by applying equation (8) to the synthetic limb profile data for various values of $r$. If $r = 0$, no a priori constraint is applied. The result (Figure 4c) is that the observed topography along the profiles is fit quite well, but that in areas where there are no observations, the topographic amplitudes are unrealistically large. As $r$ is increased, the global topographic amplitude is reduced. Comparison of Figures 4d-4f with the smoothed input topography (Figure 4b) suggests that $r$ in the range $3 \times 10^5$ provides a reasonable fit. This range of values agrees reasonably well with the expected critical value of $r$ ($R_0^2/\sigma_n^2 = 2.3 \times 10^5$).

[30] Figure 5 plots how two global quantities vary as $r$ varies. Figure 5a shows the global variance of the derived topography $\sigma_{\text{var}}$ compared with that of the smoothed input topography $\sigma_n$ (Figure 4b). As expected, for $r = 0$ the derived variance is unrealistically large, but decreases as $r$ increases. An amplitude comparable to the input value is obtained for $r \approx 3 \times 10^2$. Figure 5b plots the global RMS misfit between the input and the derived topography, $\sigma_{\text{rms}}$, compared with $\sigma_n$. The global misfit is large for small $r$, because of the large topographic oscillations in the areas between the limb profiles (compare Figure 4c). The minimum misfit occurs at $r = 1 \times 10^4$, with further increases in $r$ increasing the misfit because the derived topography is now over damped. This analysis confirms the impression derived by visual inspection of Figure 4 that $r = 3 \times 10^4$ provides the correct constraint.

### 3.4. Real Data

[31] As noted above, the along-track data have a very small spacing compared to the wavelengths for which we can actually hope to estimate global topography. We therefore analyzed different subsets of the data, each one using every tenth point from the full profiles. Doing so has two advantages. First, it avoids unnecessarily large matrices ($M = 15,764$). Second, it allows us to estimate the uncertainties in our results, because we generate ten separate topographic grids. Note, however, that these data subsets are not truly independent, and the uncertainties may thus be underestimates.

[32] Based on the synthetic results discussed above, we applied equation (8) to the decimated limb profile data with
Figure 5. (a) Ratio of output (σ_{out}) to input (σ_{in}) global topographic variance, both evaluated to \( l = m = 8 \). Input topography is synthetic data set expanded to \( l = m = 8 \) (Figure 4b). Output topography (Figures 4c–4f) is derived from synthetic limb profile topography (Figure 4a) using equation (8) and varying values of \( r \). Here \( σ_{in} = 1.58 \) km. (b) As for Figure 5a, but here the RMS misfit between the input and output topography evaluated to \( l = m = 8 \) is calculated as a function of \( r \), normalized to \( σ_{in} \).

Figure 4. (a) Synthetic topography, expanded out to \( l = m = 120 \). Spherical harmonic variance spectrum (equation (4)) is constrained to be flat out to \( l = m = 15 \) and have a slope of −2 at shorter wavelengths. Lines show the locations of the actual limb profiles. (b) As for Figure 4a, but expanded out to \( l = m = 8 \). (c) Calculated topography, obtained by expanding the spherical harmonic coefficients estimated from equation (8). The synthetic topographic observations \( h \) were located at the positions of the actual limb profiles and were calculated using the full synthetic topography (Figure 4a). Here the a priori constraint \( r = 0 \). (d) As for Figure 4c, but with \( r = 3 \times 10^5 \). (e) As for Figure 4c, but with \( r = 10^6 \). (f) As for Figure 4c, but with \( r = 3 \times 10^6 \). Here and elsewhere we take \( R_0 = 764.3 \) km [Thomas et al., 2007].
As shown in Figure 5, this choice should result in a small error and an approximately correct estimate of the true topographic variance.

### 3.4.1. Gridded Topography

Figure 6a shows the resulting topography averaged from our analysis of the ten data subsets, expanded out to \( l = m = 8 \) (wavelength \( \approx 650 \) km). Here the raw profiles are referenced to the best fitting ellipsoid of Thomas et al. [2007], so the degree 2 component of the topography (which would otherwise dominate) is not evident. The global variance is 0.77 km and the total relief exceeds 6 km. Several prominent topographic or geographic features are labeled.

Figure 6b shows an image mosaic of Rhea, highlighting two large impact basins (features A and B). Basin A (Tirawa) is crossed by several profiles, and shows up in the topography as a faint depression roughly 0.5 km deep. Basin B, despite being crossed by a similar number of profiles, does not show up in the long-wavelength topography.

Three other prominent lows (C, D, E) are crossed by at least one profile; inspection of profiles 11 and 15 in Figure 6c suggest that C and E at least are real features. The topographic feature F is only skirted by profiles and may be an artefact. None of these four topographic lows has an obvious expression in the image mosaic. Likewise, feature G is a pronounced topographic high that inspection of profile 22 suggests is a real feature, but it has no obvious surface expression.

Comparison of the results shown in Figure 6a with those derived from stereo techniques is not straightforward. Examination of stereo topography for Rhea [Jaumann et al., 2009, Figure 20.6] reveals only a few common features, such as Tirawa (label A) and features E (low) and G (high). This is mostly due to the fact that the stereo topography contains short-wavelength components (total relief 10 km) which obscure the longer-wavelength pattern. Furthermore, the stereo topography of Jaumann et al. [2009] is referenced to a biaxial ellipsoid (P. M. Schenk, personal communication, 2010) which results in long-wavelength highs around 0° and 180° longitude not present in Figure 6a.

### 3.4.2. Spherical Harmonic Coefficients

The spherical harmonic coefficients \( C_{lm}, S_{lm} \) used to derive the topography shown in Figure 6a are also important in their own right. Table 1 lists the values of \( C_{lm}, S_{lm} \).
together with their standard deviations, obtained by analyzing the ten subsets of the limb profile data. Here we have used the data referenced to a sphere, because in this case the degree 2 coefficients have a physical significance.

[38] Using the $l = m = 2$ coefficients given in Table 1, we determine the three axes $a$, $b$, and $c$ when $r = 3 \times 10^5$. We find that $a - R_0 = 2.45 \pm 0.08$ km, $b - R_0 = -1.39 \pm 0.04$ km and $c - R_0 = -1.06 \pm 0.10$ km when $r = 3 \times 10^5$, where the error bars are one standard deviation calculated from the ten observation subsets. When $r$ is reduced to $3 \times 10^4$, the spherical harmonic coefficients are typically 20% larger and the corresponding axes are $2.78 \pm 0.13$ km, $-1.54 \pm 0.05$ km and $-1.25 \pm 0.14$ km. These two sets of values may be compared with the corresponding values from Thomas et al. [2007]: $2.9 \pm 2.2$ km, $-1.8 \pm 0.8$ km and $-1.2 \pm 1.1$ km, respectively. Our results thus agree within error with those of Thomas et al. [2007]; increasing $r$ further results in a larger disagreement, because of the damping effect of the \textit{a priori} constraint on topographic amplitudes.

[39] For a hydrostatic body undergoing present-day tidal and rotational distortion, the only nonzero coefficients should be $C_{20}$ and $C_{22}$ and they should be in the ratio 10:3 [e.g., Dermott, 1979]. Our results (Table 1) are in agreement with the conclusions of Thomas et al. [2007] that Rhea’s topography is not hydrostatic: $C_{22}$ exceeds $C_{20}$ and although $C_{20}$ and $C_{22}$ are the largest degree 2 coefficient, $C_{21}$ is significantly greater than zero, and $S_{31}$, $S_{41}$ and $C_{43}$ are also large. However, a larger set of Rhea limb profiles results in a flattening that is consistent with hydrostatic topography [Thomas, 2010]; thus, the extent to which Rhea’s topography is hydrostatic is currently unclear.

[40] We also examined the effect of expanding the topography out to different maximum degrees $l_{max}$. Shorter-wavelength topography is more affected by the data gaps; higher-degree expansions are thus more likely to yield spurious topography of the kind seen in Figure 5c. Table 2 compares the degree variance $V_l$ for different values of $l_{max}$. All three expansions show a peak in power at $l = 2$, as expected. The $l_{max} = 8$ and $l_{max} = 10$ expansions in particular differ by no more than a few tens of percent, and suggest that the long-wavelength topography derived is not greatly affected by the value of $l_{max}$ chosen.

### 4. Geophysical Implications

#### 4.1. Comparison With Other Bodies

[41] Figure 7 replots the averaged 1-D Rhea variance spectrum shown in Figure 3a, and compares it with an averaged 1-D variance spectrum for Mars calculated using the same approach. The 2-D variance spectrum of Mars follows a power law with an approximately constant slope of $-2$ at wavelengths longer than a few km [e.g., Bills and Lemoine, 1995; Aharonson et al., 2001]. This 2-D slope is not significantly different from the slope derived from the 1-D profiles (Figure 7a), justifying the approach adopted in section 3.2.

Of the terrestrial bodies, the Moon shows a drop in power at wavelengths longer than $\approx 100$ km, perhaps indicating the wavelength at which flexural support becomes important [Araki et al., 2009]. We observe a similar effect for Rhea (Figure 7b) and will examine the possibility of flexural support below.

#### 4.2. Elastic Thickness

[42] The elastic portion of Rhea’s ice shell will support short-wavelength, but not long-wavelength, topography. A similar argument using crater topography was made for the Galilean satellites by Luttrell and Sandwell [2006]. A related observation was made by Malamud and Turcotte [2001], who showed that the variance spectrum of ice cap topography on both Earth and Mars shows a break in slope at a wavelength similar to the thickness of the cap.

[43] For a relatively small body like Rhea, both bending and membrane stresses are likely to be important in supporting topography [Turcotte et al., 1981]. The fraction $F$ of topography which is supported by these stresses is given by

$$F = 1 - C_l$$

where $C_l$ is the compensation factor defined by of Turcotte et al. [1981, equation (27)] and which depends on the

### Table 1. Selected Spherical Harmonic Coefficients Derived From Limb Profiles Referenced to Sphere Using $r = 3 \times 10^5$ and Expanded to $l = m = 8$

<table>
<thead>
<tr>
<th>$l$</th>
<th>$m=0$</th>
<th>$m=1$</th>
<th>$m=2$</th>
<th>$m=3$</th>
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<td>$S_{l2}$</td>
</tr>
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<td>0.190(28)</td>
<td>0.053(28)</td>
<td>1.402(26)</td>
<td>0.070(37)</td>
</tr>
<tr>
<td>3</td>
<td>0.063(36)</td>
<td>-0.076(37)</td>
<td>0.244(34)</td>
<td>-0.089(36)</td>
<td>-0.056(31)</td>
</tr>
<tr>
<td>4</td>
<td>-0.030(32)</td>
<td>-0.155(46)</td>
<td>0.214(25)</td>
<td>-0.005(43)</td>
<td>-0.007(32)</td>
</tr>
</tbody>
</table>

*All values are in km. Values in brackets give the standard deviation of the last two digits; thus 0.094(20) should be read as 0.094 ± 0.020.

### Table 2. Degree Variance $V_l$ (Equation (4)) for Topographic Expansions up to Different Maximum Degrees $l_{max}$ All Using $r = 3 \times 10^5$ and the Profile Data Referenced to a Sphere

<table>
<thead>
<tr>
<th>$l_{max}$</th>
<th>$V_1$</th>
<th>$V_2$</th>
<th>$V_3$</th>
<th>$V_4$</th>
<th>$V_5$</th>
<th>$V_6$</th>
<th>$V_7$</th>
<th>$V_8$</th>
<th>$V_9$</th>
<th>$V_{10}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>6</td>
<td>0.004</td>
<td>2.371</td>
<td>0.041</td>
<td>0.126</td>
<td>0.109</td>
<td>0.209</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>8</td>
<td>0.006</td>
<td>2.233</td>
<td>0.103</td>
<td>0.219</td>
<td>0.201</td>
<td>0.425</td>
<td>0.248</td>
<td>0.164</td>
<td></td>
<td></td>
</tr>
<tr>
<td>10</td>
<td>0.029</td>
<td>1.783</td>
<td>0.100</td>
<td>0.256</td>
<td>0.345</td>
<td>0.460</td>
<td>0.276</td>
<td>0.164</td>
<td>0.140</td>
<td>0.161</td>
</tr>
</tbody>
</table>
and the elastic thickness for likely strain rates. Rhea 10 km. Nimmo et al. and the entire topography is flexurally supported. ≈ Dermott (a) Average 1 flyby has returned gravity data, so determination of → Anderson and Schubert E10008–[2003] for Tethys and Europa, ≈ and rigidity of ≈ Nimmo and Matsuyama implies that its silicate mass fraction is ≈ þ[2007] pointed out that spherical degree l and the elastic thickness Te and rigidity of the ice shell. For short-wavelength loads (l → ∞) we have P ≈ 1 and the entire topography is flexurally supported. [44] The dashed lines in Figure 7b show how a topographic variance spectrum with an initial slope of −2 would be modified (equation (9)) if the elastic thickness were 5 km, 10 km and 20 km, respectively. As expected, short-wavelength topography is not affected, while long-wavelength topography is reduced in power. This reduction in power occurs at the point when bending stresses no longer contribute significantly to support, and occurs at longer wavelengths for higher values of Te. Comparison of the theoretical curves with the observations suggests that Te ≈ 10 km. [45] The inferred elastic thickness of 10 km is somewhat higher than the values of ≈6 km inferred by Giese et al. [2007] and Nimmo et al. [2003] for Tethys and Europa, respectively, but lower than the range of 50–100 km inferred for Iapetus [Giese et al., 2008]. This result is consistent with the level of past tectonic activity inferred for these bodies: Iapetus has suffered essentially no tectonic activity postdating the formation of its equatorial ridge [Jaumann et al., 2009]; Rhea shows signs of minor tectonic activity [Jaumann et al., 2009]; and Europa and Tethys have both undergone significant tectonic deformation, the latter possibly as result of an ancient tidal heating episode [Chen and Nimmo, 2008]. [46] Based on Giese et al. [2007, Figure 4], an elastic thickness of 10 km suggests a heat flux in the range 13–17 mW m−2 for likely strain rates. Rhea’s bulk density of 1233 kg m−3 implies that its silicate mass fraction is ≈30% [Castillo-Rogez, 2006; Hussmann et al., 2006]. This in turn suggests the present-day chondritic heat flux should be roughly 0.4 mW m−2 [Hussmann et al., 2006]. Four billion years ago, this heat flux would probably have been higher by a factor of 3–4, still insufficient to explain the inferred heat flux. One possible explanation for the discrepancy is an ancient tidal heating episode; since the accretionary heating of Rhea is expected to be minor, another possibility is the decay of short-lived radioisotopes such as 26Al [Castillo-Rogez et al., 2007]. [47] Although we favor a flexural explanation for the variance spectral behavior observed, there are other possibilities. One potential explanation is crater relaxation. For instance, the largest basins on Rhea (e.g., Tirawa, at 450 km diameter) appear somewhat relaxed when compared with similar basins on Iapetus [Schenk and Moore, 2007]. This observation suggests that topographic relaxation is occurring at a wavelength comparable to the wavelength at which the variance spectral break in slope occurs. Thus, an alternative explanation for the observed shape of the variance spectrum is that viscous relaxation is reducing the topographic amplitude at long wavelengths. This behavior is expected for ice shells in which the viscosity decreases with depth [e.g., Parmentier and Head, 1981]. Further modeling of this effect is beyond the scope of the current paper, but could be investigated in future work.

4.3. Gravity

[48] Finally, we wish to consider the effect that long-wavelength uncompensated topography might have on Rhea’s gravity. For a hydrostatic synchronous satellite, the degree 2 coefficients may be used to determine the internal structure [e.g., Dermott, 1979]. However, for Rhea to date only a single Cassini flyby has returned gravity data, so determination of even the degree 2 gravity coefficients is challenging [less et al., 2007]. Furthermore, it is not yet clear whether the gravity data support the assumption of hydrostatic equilibrium [Mackenzie et al., 2008].

[49] Nimmo and Matsuyama [2007] pointed out that uncompensated basins might affect the degree 2 gravity coefficients measured; and Mackenzie et al. [2008] argued that shorter-wavelength gravity harmonics could be aliased into a degree 2 signal. Since the degree 2 gravity coefficients have been used to infer Rhea’s internal structure [less et al., 2007; Anderson and Schubert, 2007], errors in these coeffi-
cients are likely to cause corresponding errors in the internal structure deduced.

[50] From McKenzie and Nimmo [1997], the gravity coefficients $C_{lm}^{G} S_{lm}$ may be derived from the dimensional topography coefficients $C_{lm} S_{lm}$ as follows:

$$
(C_{lm}^{G} S_{lm}) = \frac{Z_{l}}{l + 1} (C_{lm} S_{lm})
$$

(10)

where $Z_{l}$ is the ratio of the gravity to topography at a particular wavelength, known as the admittance, and is given by $Z_{l} = 4\pi \rho_{p} (l - 1)/(2l + 1)$ for uncompensated topography of density $\rho_{p}$.

[51] Mackenzie et al. [2008] found that normalized degree 3 gravity coefficients ($C_{33}^{G}, S_{33}^{G}$) larger than $2.8 \times 10^{-5}$ would degrade the estimation of the degree 2 gravity coefficients, equivalent to $5.6 \text{ m}^2 \text{s}^{-2}$. Table 1 shows that the existing topography (i.e., after partial relaxation) results in $S_{33}^{G} = 0.151 \text{ km}$; when $r = 3 \times 10^{4}$ this value increases to 0.208 km. For uncompensated ice, $Z_{l} \approx 23 \text{ mGal/km}$ (or $2.3 \times 10^{3}$ deg/m). The authors are grateful for the comments of two anonymous reviewers. This work supported by NASA-OPR and NASA-PGG.

References


