Estimate the fraction of the total transported energy (in the form of gasoline) in the Trans-Alaska Pipeline that is consumed in pumping.

SOLUTION:

Let's first estimate the characteristics of the pipeline:

\[ L \sim 1000 \text{ km} \sim 10^6 \text{ m} \] : Probably similar to the length of California. [Actual value: 1300 km]

\[ d \sim 1 \text{ m} \] : From photo. [Actual value: 1.2 m]

\[ v \sim 1 \text{ m/s} \] : This is tough to estimate. What you'll find is that if you choose too large a value here, the losses become too large and thus it's not economical to pump it too quickly. But it shouldn't go too slow otherwise it won't provide enough throughput. [Actual value: 2 m/s]

This yields a volumetric flow of \[ \frac{\pi}{4} \cdot d^2 v \sim 1 \text{ m}^3/s \sim 10^5 \text{ m}^3/\text{day} \]

This lets us estimate the energy loss, assuming the pipeline builders were trying to make the pipe as smooth as possible to minimize losses (i.e. \( f \sim 0.01 \) rather than 0.1):

Energy per unit mass loss \[ \sim f \cdot \frac{L}{D} \cdot u^2 \sim 10^{-2} \cdot 10^6 \text{ m} \cdot (1 \text{ m/s})^2 / 1 \text{ m} \sim 10^4 \text{ J/kg} \]

The map clearly shows bends. Let's show that these are unlikely to matter.

Energy lost by bends: Find \( n \) such that \( nKu^2 \sim 10^4 \text{ J/kg} \). For \( K \sim 0.1 \) (assuming the engineers will build the bends as smoothly as possible), this yields \( n \sim 10^5 \). Seems unlikely that the number of bends is anywhere close to this (which is 1 bend per 10 m). So we can toss this out.

Now estimate the energy content of gasoline:

Many of you tried figuring out the energy requirement to move a car. This is tricky and requires knowing the efficiency of the engine and accounting for all the losses. This led to a wide range of numbers. The easiest way was to actually look at the list of useful numbers which had 500 kJ/mol_carbon for aerobic respiration and use that. This leads to an estimate of:

Energy content per unit mass: \[ 500 \text{ kJ/mol} / 0.01 \text{ kg/mol} \sim 5 \cdot 10^7 \text{ J/kg} \]

I thought the nicest estimate of this was to do what Andrew did, which is to remember the energy content of fat as 10 kcal/g (probably useful for all sorts of reasons). This leads to an estimate of 40 kJ/g or \( 4 \cdot 10^7 \text{ J/kg} \), basically exactly right. (FYI, it turns out that carbohydrates have values about half that of fat because they have more oxygens present, which boosts the mass without giving much energy [one can't burn water or carbon monoxide for energy]).

So putting it together, energy loss fraction is \[ \sim 10^4 \text{ J/kg} / 5 \cdot 10^7 \text{ J/kg} \sim 2 \cdot 10^{-4} \sim 0.02\% \]

Estimate the number of capillaries (the smallest blood vessels) in your circulatory system. Capillaries are just sightly bigger than a red blood cell, which are roughly 10 micrometers in size. If it helps, your blood pressure has units of mmHg, and the two values represent max and min BP. [Note: solution done for slightly different capillary size 10 µm rather than 6 µm, but you still get the right idea.]

First estimate the total length of capillaries required. This assumes that each cell needs a capillary adjacent to it to receive oxygen. If we assume a geometry like Fig. 1.

where a person’s cells are roughly the same diameter as the diameter of capillaries. This is probably not too bad since if they were much larger, then the oxygen couldn't be efficiently moved throughout the cell. Thus, we estimate that one-seventh of our body’s volume must be comprised of capillaries.
Figure 1: Blood in the center of the other cells

Total body volume: 70 kg/10^3 kg/m^3 \sim 7 \cdot 10^{-2} m^3 \text{ so total capillary volume must be } 10^{-2} m^3.

This yields a total length of capillaries $L \sim 10^{-2}m^3/(\pi/4 \cdot (10^{-5}m)^2) \sim 10^8 m \text{ (That's an impressively large value!) }$

Now, if this were one single capillary running throughout the body, the pressure needed to maintain the correct flow rate would be enormous. But we know roughly what our blood pressure is (120-over-80 mmHg). These two values represent the max and min blood pressures. So a characteristic $\Delta P \sim 40 mmHg \sim 40 mmHg/(800 mmHg/atm) \cdot 10^5 Pa/atm \sim 5 \cdot 10^3 Pa$. (If you didn’t know the conversion from mmHg to atmospheres, you can figure it out using an approximate density for Hg).

However, what we really need is the equivalent *constant* blood pressure drop. The heart doesn’t provide a constant pressure but pulses the pressure. So what we need is a *time-averaged* $\Delta P$, not the instantaneous max (because we’re using the time-averaged volumetric flow). Let’s say that time average is order $1/5$ of the max pressure (makes for a round number and seems not too bad). So the appropriate $\Delta P \sim 10^3 Pa$.

Flow rate of blood is something like the entire volume circulating about once per 10 s (say 0.5 L for heart volume times 60 beats per minute = 30 L/min or six times the total blood volume of 5 L per min). So volumetric flow is:

$\dot{V} \sim 5 \cdot 10^{-3} m^3/10 s \sim 5 \cdot 10^{-4} m^3/s$

Consider a network of capillaries as shown in Fig. 2.

So if we have $n$ capillaries, then each has a length $l \sim L/n$. The energy loss through these capillaries (we assume this is where most of the frictional loss occurs, which is perhaps reasonable since this is where the flow sees the most surface area of wall) is balanced by the pressure created by our hearts.

The total volumetric flow must also be satisfied as: $\dot{V} \sim n \cdot (\pi/4) \cdot d^2 u$ or $u \sim \dot{V}/(n \cdot (\pi/4) \cdot d^2)$

So using the Bernoulli equation, assuming that the closed circuit of blood flow primarily balances blood pressure against frictional losses:

$$\frac{\Delta p}{\rho} + \frac{\Delta (u^2)}{2} + g \Delta z = w - gh_L$$
becomes

\[ \frac{\Delta p}{\rho} \sim -ghL \sim -f \frac{l}{d} u^2 \]

so

\[ \frac{\Delta p}{\rho} \sim -f \cdot \frac{(L/n)}{d} \left( \frac{\dot{V}}{n \cdot (\pi/4) \cdot d^2} \right)^2 \]

Rearranging:

\[ n^3 \sim -2 \cdot f \cdot \frac{\rho \cdot L\dot{V}^2}{\Delta p \cdot d^5} \]

where \((\pi/4)^2 \sim 1/2\). Assume \(f \sim 0.1\) (seems like capillaries should be smooth, but maybe not relative to the diameter of the flow and we also need to throw in a correction for corners since they don’t seem like they should be straight lines) and substitute remaining values:

\[ n^3 \sim -2 \cdot 0.1 \cdot \frac{10^3 \text{ kg/m}^3 \cdot 10^8 m \cdot (5 \cdot 10^{-4} \text{ m}^3/s)^2}{-10^4 \text{ Pa} \cdot (10^{-5} \text{ m})^5} \sim 5 \cdot 10^{25} \rightarrow n \sim 4 \cdot 10^8 \]

This is actually a bit low by a factor of \(\sim 2\) to 3. The value we’re most sensitive to is \(d\), and \(n \sim d^{-2.5}\). Doing a bit more research shows capillaries are order 5 to 10 \(\mu m\) in diameter, so a small overestimation of \(d\) could account for the difference.