Tidal dissipation in the lunar magma ocean and its effect on the early evolution of the Earth–Moon system

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A B S T R A C T

The present-day inclination of the Moon reflects the entire history of its thermal and orbital evolution. The Moon likely possessed a global magma ocean following the Moon-forming impact. In this work, we develop a coupled thermal-orbital evolution model that takes into account obliquity tidal heating in the lunar magma ocean. Dissipation in the magma ocean is so effective that it results in rapid inclination damping at semi-major axes beyond about 20 Earth radii (R_E), because of the increase in lunar obliquity as the so-called Cassini state transition at ~30 R_E is approached. There is thus a “speed limit” on how fast the Moon can evolve outwards while maintaining its inclination: if it reaches 20 R_E before the magma ocean solidifies, any early lunar inclination cannot be maintained. We find that for magma ocean lifetimes of 10 Myr or more, the Earth’s tidal quality factor Q must have been ~300 to maintain primordial inclination, implying an early Earth 1–2 orders of magnitude less dissipative than at present. On the other hand, if tidal dissipation on the early Earth was stronger, our model implies rapid damping of the lunar inclination and requires subsequent late excitation of the lunar orbit after the crystallization of the lunar magma ocean.

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1. Introduction

The Earth–Moon system has had a complex history (e.g. Goldreich, 1966). The evolution of the lunar orbit involves the interplay of tides arising on both the Earth and the Moon and the subsequent transfer of energy from rotation and the lunar orbit into heat via viscous dissipation. This dissipation process is frequency- and time-dependent and can vary in magnitude for many different reasons, such as the internal thermal structure, the presence of a liquid magma ocean and for Earth specifically, the properties of the liquid water oceans (if present) and the atmosphere. Current observations of the present-day orbital and rotational state of both the Earth and the Moon are plentiful, and there are some geological constraints on lunar orbital evolution over the past 0.6 Gyr or possibly up to 2.5 Ga (Williams, 2000). In contrast the early evolution of the Earth–Moon system, in terms of the orbital parameters, the relevant excitation and damping mechanisms, and the dissipative properties of the two bodies, has few constraints.

Following the Moon-forming impact, the early Moon was likely covered by a global magma ocean (Canup, 2004; Canup and Asphaug, 2001; Smith et al., 1970; Wood et al., 1970). Based on geochronological techniques, the final stages of magma ocean crystallization may have taken up to 200 Myr (Borg et al., 1999; Carlson and Lugmair, 1988; Gaffney and Borg, 2014; Nemchin et al., 2006; Nyquist et al., 2010). This extended cooling timescale is difficult to reconcile with geophysical models of solidification (Elkins-Tanton et al., 2011; Solomon and Longhi, 1977). One possibility – which we will return to below – is that tidal dissipation primarily in the crust may have prolonged the lifetime of the underlying magma ocean (Meyer et al., 2010).

Tidal dissipation due to viscoelastic solid-body tides has been invoked as a heat source for many bodies in the Solar System (Meyer and Wisdom, 2007; Peale et al., 1979; Squyres et al., 1983; Tobie et al., 2005), the Moon included (Meyer et al., 2010; Peale and Cassen, 1978). For a global satellite ocean, Tyler (2008) suggested that a resonance between the obliquity tide and a Rossby–Haurwitz wave could generate significant ocean flow and that turbulent dissipation of the energy contained in this flow could also represent a different and, perhaps significant, tidal heat source. In certain instances, this ocean-related obliquity tidal dissipation can be larger than the corresponding solid-body tidal dissipation (Chen et al., 2014). The obliquity of the Moon is variable as the orbit...
expands. At a semi-major axis of about 30 Earth radii ($R_E$), there is a large obliquity excursion due to a transition between Cassini states 1 and 2 (Sieghler et al., 2011; Ward, 1975). Peale and Cassen (1978) specifically considered dissipation during large lunar obliquity excursions, but did not investigate the role of any magma ocean. Since it depends strongly on obliquity, tidal dissipation in the lunar magma ocean may have been a significant and hitherto unrecognized heat source when the Moon was still molten.

The goal of this paper is to present a model for the coupled thermal-orbital evolution of the early Moon, using an approach similar to Meyer et al. (2010) but including the effects of tidal dissipation in the lunar magma ocean. We find that if the lunar magma ocean is dissipative as a result of obliquity tides, then significant inclination damping will occur. The extent of this damping is related to the magnitude of the obliquity. Thus, to allow the Moon to maintain a primordial inclination, the magma ocean must have solidified prior to the large obliquity excursion associated with the Cassini state transition at around 30$R_E$. Since the timing of magma ocean solidification is approximately known from geochronological constraints (see summary in Elkins-Tanton et al. (2011)), this argument provides a “speed limit” on the rate at which the early Moon can evolve outwards while maintaining a primordial inclination. Dissipation in the Earth controls the rate of outward migration of the Moon. During the period when the lunar magma ocean exists, maintaining a primordial inclination requires that the tidal time lag of the Earth is less than ~10 s. This implies a much less dissipative state than that associated with the current value of ~600 s (Munk and MacDonald, 1960).

On the other hand, if the early Earth is characterized by stronger tidal dissipation, the Moon evolves outwards faster, violating the “speed limit”, and experiencing rapid inclination damping. In such a case, lunar magma ocean crystallization must be followed by a late excitation mechanism to reproduce the observed state of the Earth–Moon system (Pahlevan and Morbidelli, 2015). Hence, in principle determining the dissipative properties of the early Earth could permit discrimination between early and late mechanisms of excitation for the lunar inclination.

2. Methods

The evolution of the Earth–Moon system is a coupled thermal-orbital problem; the orbital parameters change due to tidal dissipation in the Earth and the Moon and the amount of dissipation depends on their internal thermal structures, which in turn is altered by the energy dissipated as heat. While many papers have looked at the evolution of the Earth–Moon system (Burns, 1986; Goldreich, 1966; Meyer et al., 2010; Mignard, 1979; 1980; 1981; Peale and Cassen, 1978; Touma and Wisdom, 1998; Ward and Canup, 2000), none of the preceding work has explicitly included tidal dissipation in a lunar magma ocean. In the case of a global ocean, it has been recently shown that the tide due to the obliquity can force a resonant response resulting in dissipation in the ocean that may be greater than that in the viscoelastic crust (Chen et al., 2014; Tyler, 2008). The Moon likely went through a period of very high obliquities (Ward, 1975). The tidal dissipation associated with this obliquity would have had significant, previously unconsidered, effects on the orbital evolution of the Earth–Moon system.

2.1. Coupled thermal-orbital evolution

Mignard (1979; 1980; 1981) derived equations describing the time-evolution of the lunar orbit. These equations are the averaged form of Lagrange’s planetary equations assuming that the disturbing function involves a constant time lag $\Delta \tau$ (Darwin, 1908; Mignard, 1979; 1980; 1981; Touma and Wisdom, 1994) and includes dissipation in both the Earth and the Moon. The assumption of a constant time lag is convenient, but can result in different behavior from a constant phase lag assumption (Efroimsky and Makarov, 2013; Efroimsky and Williams, 2009; Goldreich, 1966; Touma and Wisdom, 1994). We discuss this issue briefly in Section 4.3. We ignore evolution of the orbital eccentricity in this analysis, because tidal dissipation in the lunar magma ocean is likely to be dominated by obliquity tides (Chen et al., 2014).

However, the eccentricity has likely played an important role in many facets of the Moon’s evolution, including pumping of the orbital inclination during passage through the evaporation resonance (Touma and Wisdom, 1998), tidal heating of the plagioclase flotation crust (Meyer et al., 2010), and perhaps the determination of the Moon’s current shape (Cuk, 2011; Garrick-Bethell et al., 2014; 2006; Keane and Matsuyama, 2014).

Ignoring orbital eccentricity, the Mignard evolutionary equations are (Meyer et al., 2010; Mignard, 1981):

$$\frac{dX}{dt} = \frac{C_X}{X^2} \left[-\left(1 + A_X\right) + \left(UX^3/2 \cos i + A_X \cos \theta_0\right)\right]$$

$$\frac{di}{dt} = -\frac{C_i}{X^{17/2}} \left(U + A_i \right) \sin i$$

$$\frac{dU}{dt} = -\frac{C_{U,0}}{X^{17/2}} \left(U - \frac{n_0}{n_C}\right) + \frac{C_{U,1}}{X^{17/2}} \left(1 - UX^3/2 \cos i\right) \cos i$$

$$- \frac{C_{U,2}}{X^6} U \sin^2 i$$

$$\frac{d(n)}{dt} = -\frac{C_{n,0}}{X^6} \left(\frac{n}{n_C}\right)^2 \left(2 + \sin i\right)^2 + U^2 \left(3 \left(\cos i\right)^2 - 1\right)$$

$$+ 2 \frac{U}{X^{17/2}} \cos i + C_{n,1} \left(2 \frac{n_0}{n_C} U - \frac{\sin i}{n_C}\right)^2 - U^2$$

where $X = a/R_E$, the ratio of the semi-major axis $a$ to the radius of the Earth $R_E$, $i$ is the orbital inclination relative to the ecliptic, $\theta_0$ is the Moon’s obliquity, $\omega$ is the rotational frequency of the Earth, $n_C = \sqrt{GM/R_E^3}$ is the grazing mean motion, $\omega$ is the mean motion of the Earth around the Sun, and $U = \omega/n_C \cos i$ with the obliquity of the Earth $i$. The constants are defined as:

$$C_X = 6 \gamma m M / \mu$$

$$C_i = -\frac{3}{2} \gamma m M / \mu$$

$$C_{U,0} = 3 \gamma \alpha \left(\frac{M_\odot}{M}\right)^2 \left(\frac{R_E}{a_\odot}\right)^6$$

$$C_{U,1} = 3 \gamma \alpha \left(\frac{m}{M}\right)^2 \left(\frac{R_E}{a_\odot}\right)^{1/2}$$

$$C_{U,2} = \frac{3}{2} \gamma \alpha \left(\frac{m}{M}\right)^2 \left(\frac{R_E}{a_\odot}\right)$$

$$C_{n,0} = 3 \gamma \alpha \left(\frac{m}{M}\right)^2 \left(\frac{R_E}{a_\odot}\right)^2$$

$$C_{n,1} = \frac{3}{2} \gamma \alpha \left(\frac{M_\odot}{M}\right)^2 \left(\frac{m}{\mu}\right) \left(\frac{R_E}{a_\odot}\right)^6$$

with

$$\gamma = k_e n_C^2 \Delta \tau$$
where $m$ is the mass of the Moon, $M$ is the mass of the Earth, $M_\odot$ is the mass of the Sun, $\mu = (1/m + 1/M)^{-1}$ is the reduced mass, $\alpha$ is the dimensionless moment of inertia of the Earth, $a_0$ is the Earth–Sun distance, $k_{2E}$ is the tidal Love number of the Earth, and $\Delta t_\text{E}$ is the tidal time lag of the Earth. Note that we have corrected typographical errors for $\gamma$, $C_i$ and $C_{m,i}$ in Meyer et al. (2010). The values of the physical parameters are presented in Table 1. Our nominal model is initialized at a semi-major axis of $a = 6.5R_\oplus$, inclination of $i = 12^\circ$, and a rotation period of the Earth of 6 h. These are estimates of the conditions after passage through the ejection resonance (Tourma and Wisdom, 1998) and/or disk resonances (Ward and Canup, 2000). We explore the effect of different initial conditions in Section 4.

In Mignard (1981) and Meyer et al. (2010), the Mignard tidal parameter $A$ is defined as

$$A = \left( \frac{k_{2M}\Delta t_M}{k_{2E}\Delta t_\text{E}} \right) \left( \frac{M}{m} \right)^2 \left( \frac{R_M}{R_E} \right)^5$$  \hspace{1cm} (13)

where $k_{2M}$ is the tidal Love number of the Moon, and $\Delta t_M$ is the tidal time lag of the Moon and $R_M$ is the radius of the Moon. This tidal parameter relates the amount of energy dissipation in the Moon to that in the Earth assuming a constant time lag, viscoelastic dissipation model in both bodies. For viscoelastic solid-body tides, the Mignard parameter is a constant and in (1) and (2) above,

$$A_K = A_i = A.$$

Tides in the lunar magma ocean may also remove energy from and alter the orbit, but in a different manner: the tides force ocean flow and the kinetic energy of this flow is subsequently dissipated through turbulence (Tyler, 2008). We derive the effects of magma ocean tides on the orbit from considerations of conservation of energy and angular momentum. In order to preserve similarity to the Mignard equations, we define two Mignard-like parameters $A_K$ and $A_i$ for the effects of magma ocean tidal heating on the evolution of the semi-major axis and orbital inclination respectively. Unlike in the case of viscoelastic tides, these parameters are time-dependent.

In a Keplerian orbit, the orbital energy is given by

$$E = -\frac{GMm}{2a}.$$  \hspace{1cm} (14)

and tidal dissipation causes changes in the semi-major axis of the orbit through the relation:

$$\dot{a} = \frac{GMm}{2a^2 \bar{a}}.$$  \hspace{1cm} (15)

where the dot designates a time derivative. For an inclined orbit with no eccentricity, the vertical component of the orbital angular momentum is (Chyba et al., 1989),

$$L = m(GM)^{1/2}a^{1/2} \cos i.$$  \hspace{1cm} (16)

In the absence of tides raised on the Earth, this quantity is conserved and we obtain the following relations after taking the time derivative of (16) and including the relation (15)

$$\frac{\dot{a}}{a} = 2 \tan \frac{d}{dt} = \frac{2a\dot{E}}{GMm}.$$  \hspace{1cm} (17)

These relations must hold true independent of the energy dissipation process. This approach assumes that dissipation in the primary is small, which in the case of the early Earth may be justified a posteriori (see Section 4.1).

In the Mignard equations, the Mignard tidal parameter conceals the relations (17). Ignoring dissipation in the Earth, the evolution of the semi-major axis from (1) and (17) is

$$\dot{a} = C_i \left( \frac{R_p}{a} \right)^8 a A_i (\cos \theta_0 - 1) = \frac{2a^2 \dot{E}_M}{GMm}$$  \hspace{1cm} (18)

where $\dot{E}_M$ represents dissipation in the Moon. Thus, the tidal parameter $A_K$ is related to $\dot{E}_M$ by

$$A_K = \frac{1}{37} \left( \frac{M}{m} \right) \left( \frac{a}{R_E} \right)^8 \frac{a \dot{E}_M}{GMm (\cos \theta_0 - 1)}.$$  \hspace{1cm} (19)

By substituting Eq. (19) into Eq. (13) it can be shown that the dissipation rate is given by

$$\dot{E}_M = -\frac{3}{2} k_{2M} \Delta t_M \frac{G^2 M^3 R_E^2}{\alpha^9} \sin^2 i$$  \hspace{1cm} (20)

which is the standard result for dissipation due to obliquity tides under the assumptions that $i \approx \theta_0$ and $i$ is small (Peale and Cassen, 1978; Wisdom, 2008).

Similarly, the evolution of the inclination from Eqs. (2) and (17) can be expressed as

$$\frac{di}{dt} = -C_i \left( \frac{R_p}{a} \right)^8 A_i \sin i = \frac{a \dot{E}_M \cos i}{GMm \sin i}$$  \hspace{1cm} (21)

and thus,

$$A_i = \frac{-2}{37} \left( \frac{M}{m} \right) \left( \frac{a}{R_p} \right)^8 \frac{a \dot{E}_M \cos i}{GMm (\sin i)^3}.$$  \hspace{1cm} (22)

which simplifies to (13) for the dissipation given by (20). Eqs. (19) and (22) thus allow the orbital evolution to be calculated via Eqs. (1)–(4) once the magma ocean dissipation rate $\dot{E}_M$ has been specified. We assume the only source of tidal heating $\dot{E}_M$ is from the magma ocean.

Table 1: Physical parameters utilized in the model for Earth–Moon system evolution.

<table>
<thead>
<tr>
<th>Symbol</th>
<th>Parameter</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>$G$</td>
<td>Gravitational constant</td>
<td>$6.674 \times 10^{-11}$ m$^3$ kg$^{-1}$ s$^{-2}$</td>
</tr>
<tr>
<td>$M$</td>
<td>Mass of the Earth</td>
<td>$5.97 \times 10^{24}$ kg</td>
</tr>
<tr>
<td>$R_0$</td>
<td>Radius of the Earth</td>
<td>6370 km</td>
</tr>
<tr>
<td>$m$</td>
<td>Mass of the Moon</td>
<td>$7.25 \times 10^{22}$ kg</td>
</tr>
<tr>
<td>$r_0$</td>
<td>Radius of the Moon</td>
<td>1737 km</td>
</tr>
<tr>
<td>$M_\odot$</td>
<td>Mass of the Sun</td>
<td>$2.0 \times 10^{30}$ kg</td>
</tr>
<tr>
<td>$a_i$</td>
<td>Semi-major axis</td>
<td>$149.6 \times 10^6$ km</td>
</tr>
<tr>
<td>$k_{2E}$</td>
<td>Love number of the Earth</td>
<td>0.97</td>
</tr>
<tr>
<td>$\alpha$</td>
<td>Normalized moment of inertia of the Earth</td>
<td>0.33</td>
</tr>
<tr>
<td>$g$</td>
<td>Surface gravity of the Moon</td>
<td>1.62 m$^2$ s$^{-1}$</td>
</tr>
<tr>
<td>$\rho$</td>
<td>Magma ocean density</td>
<td>3000 kg m$^{-3}$</td>
</tr>
<tr>
<td>$c_0$</td>
<td>Bottom drag coefficient</td>
<td>0.002</td>
</tr>
<tr>
<td>$c_i$</td>
<td>Normalized moment of inertia of the Moon</td>
<td>0.393</td>
</tr>
<tr>
<td>$L$</td>
<td>Latent heat of fusion</td>
<td>$5.0 \times 10^7$ J kg$^{-1}$</td>
</tr>
<tr>
<td>$T_m$</td>
<td>Temperature of magma ocean</td>
<td>1200 K</td>
</tr>
<tr>
<td>$T_0$</td>
<td>Temperature of lunar surface</td>
<td>280 K</td>
</tr>
<tr>
<td>$c_p$</td>
<td>Specific heat capacity</td>
<td>$1256$ J kg$^{-1}$ K$^{-1}$</td>
</tr>
</tbody>
</table>
2.2. Tidal dissipation in the lunar magma ocean

Tyler (2008; 2009) suggested that a global liquid ocean within a satellite, specifically Europa and Enceladus, would respond resonantly to obliquity-related tidal forcing and that frictional dissipation of this ocean flow could provide a significant heat source. For most of the outer Solar System satellites, the dominant heat source is unlikely to be ocean dissipation (see Chen et al. (2014)); this result is primarily due to the fact that these satellites formed locally in a disk (Canup and Ward, 2002), and thus their inclinations and the resultant obliquities are quite small. However, the Moon’s inclination of approximately 5° is currently much higher than many of the outer Solar System satellites (except for Iapetus and Triton), where regular satellites’ inclinations are less than 1° (Yoder, 1995), and the Moon likely underwent a Cassini state transition early in its history, where its obliquity could have been much larger than the orbital inclination (Ward, 1975). As a result, dissipation in the magma ocean may have contributed a significant amount of heat to the evolution of the Moon. Here we summarize a method of calculating tidal dissipation in this early magma ocean, and point the reader to Chen et al. (2014) for further details of the method.

The behavior of a global magma ocean can be described through the linear shallow-water equations on a sphere:

$$\frac{\partial \mathbf{u}}{\partial t} + 2\Omega \cos \theta \mathbf{\hat{r}} \times \mathbf{u} = -g \nabla \eta - \nabla U + \nu \nabla^2 \mathbf{u}$$

(23)

$$\frac{\partial (\xi \eta)}{\partial t} + h \nabla \cdot \mathbf{u} = 0$$

(24)

where \(\mathbf{u}\) is the radially-averaged, horizontal velocity vector, \(\Omega\) is the constant rotation rate, which is the same as the orbital mean motion for a synchronous satellite, \(\mathbf{\hat{r}}\) is the unit vector in the radial direction, \(g\) is the surface gravity, \(\eta\) is the vertical displacement of the surface, \(h\) is the constant ocean depth and \(\nu\) represents an effective horizontal viscous diffusivity.

To approximate the effect of a rigid lid on the ocean tidal response (Kamata et al., 2015, cf.), we define a damping factor \(\xi\), where

$$\xi = k_2 - h_2$$

(25)

and \(\xi \in [0, 1]\). If \(\xi = 1\), this describes the response in the limit of a weak or absent lid, while \(\xi \approx 0\) in the case of a very rigid lid. The models we present below are for \(\xi = 1\). This damping factor is expected to be in the range 0.3–1 for likely magma ocean parameters. Including its effect results in small modifications to the final results (shorter ocean lifetimes, higher final inclinations) but not enough to change any of our conclusions.

\(U\) represents the radially-averaged forcing potential due to tides, here for the obliquity tide:

$$U = -\frac{3}{2} \Omega^2 h^2 \rho \Omega^2 \sin \theta \cos(\phi - \Omega t) + \cos(\phi + \Omega t)$$

(26)

Here \(\theta\) is colatitude and \(\phi\) is longitude. For synchronously rotating satellites, the forcing potential due to obliquity is resonant with a Rossby–Haurwitz wave (Tyler, 2008), and thus flow in a global ocean can be significant.

The energy dissipation associated with the obliquity tide can be written as:

$$\dot{E}_{\text{diss}} = \frac{108\pi}{25} \frac{\rho h v \Omega^2 \xi^2 \rho \Omega^2 \rho^2}{\xi^2 + \frac{1044 \nu \rho h}{\xi^2}}$$

(27)

where \(\rho\) is the magma ocean density and \(v\) is a turbulent viscous diffusivity given by the numerically-derived scaling law (Chen et al., 2014)

$$v = \frac{\nu}{\xi^2} \left( \frac{9}{25} \frac{\Omega^2 \rho^2}{\mu_m} + \left( \frac{9}{25} \frac{\Omega^2 \rho^2}{\mu_m} \right)^2 + \frac{4}{144} \left( 0.4 \frac{c_0}{\xi^2 \rho^2} \Omega^2 \rho^2 \theta_0 \right) \right)^{1/2}$$

(28)

with an associated bottom drag \(c_p\). We adopt a bottom drag formulation rather than assuming a dissipation factor (\(Q\)) for the ocean because for the latter it is unclear a priori what value should be adopted. In contrast, for the terrestrial oceans \(c_p \approx 0.002\) (Egbert and Ray, 2001; Jayne and St. Laurent, 2001; Sagan and Dermott, 1982) and the dependence on Reynolds number is weak (Sohl et al., 1995). Ultrafast lavas have viscosities comparable to water (~10\(^{-3}\) Pa s; (Lumb and Aldridge, 1991; Rubie et al., 2003)) and the high-Ti basalts associated with late-stage crystallization of the magma ocean also have low (~<100 Pa s) calculated viscosities (Williams et al., 2000). We initially adopt an Earth-like value of \(c_p\) and discuss this issue further in Section 4.3. Also note that this mechanism for dissipation only applies to synchronous bodies; dissipation in a deep terrestrial magma ocean is expected to be small.

The energy dissipation in the ocean evolves in time as the Moon’s rotation period \(\Omega\), obliquity \(\theta_0\) and ocean depth \(h\) evolve. Assuming the Moon has remained synchronous, the rotational frequency is equal to the orbital mean motion, which changes with the orbital semi-major axis (cf. Eq. (1)).

2.3. Cassini state obliquity

While the current lunar obliquity can be measured, the obliquity of the Moon in the past is uncertain. For this work, we assume that throughout its evolution, the Moon has occupied a Cassini state, where its spin axis, the orbit normal and an invariable pole remain coplanar as they precess (Colombo, 1966; Peale, 1969). The obliquity is given by the angle between the orbit normal and the Moon’s spin axis.

For coplanar precession to occur, the obliquity angle \(\theta_0\) must satisfy the relation (Bills and Nimmo, 2008; Peale, 1969)

$$\frac{3}{2} \left| (J_2 + C_{22}) \cos \theta_0 + C_{22} \rho \sin \theta_0 = \frac{c}{\sin (i - \theta_0)} \right|$$

(29)

where \(J_2\) and \(C_{22}\) are the degree-2 gravity coefficients and \(c\) is the normalized polar moment of inertia of the Moon. The ratio \(p\) of the orbital motion to the orbit plane precession is

$$p = \frac{\Omega}{d \Omega_{\text{orb}} / dt}$$

(30)

where \(\Omega_{\text{orb}}\) is the longitude of the ascending node.

It is well-established that the observed gravity coefficients of the Moon are inconsistent with a presently hydrostatic, tidally-distorted body (Garrick-Bethell et al., 2006; Jeffrey, 1915; Konopliv et al., 2001; 1998). Because we are concerned with the Moon during a period where it was warm and likely to have behaved like a fluid, we make the hydrostatic assumption; modifying this assumption does not greatly change the position at which the Cassini state transition occurs (Siegel et al., 2011). For hydrostatic bodies, the normalized polar moment of inertia \(c\) can be related to the displacement Love number \(h_2\) via the Radau–Darwin relation (Hubbard and Anderson, 1978)

$$c = \frac{2}{3} \left( 1 - \frac{2}{5} \frac{5}{h_2 - 1} \right)^{1/2}$$

(31)

For a synchronous, slowly-rotating, hydrostatic satellite, the degree-2 gravity coefficients can be related to \(h_2\) by

$$J_2 = \frac{5}{6} \left( \frac{R_M^2 \Omega^2}{\gamma M} \right) (h_2 - 1)$$

(32)
and

\[ C_{22} = \frac{3}{10} J_2 . \]  

(33)

The precession of the orbit arises due to torques from the rotational flattening of the Earth and solar tides. The total precessional frequency is the sum of the two

\[ \frac{d\Omega_{2\text{orb}}}{dt} = -\frac{3}{2} \eta_{2\text{E}} \left( \frac{R_E}{a} \right)^2 - \frac{3}{4} \frac{M_\oplus}{(M + m)} \left( \frac{a}{a_\oplus} \right)^3 \]  

(34)

with orbital mean motion \( n \), which for synchronous satellites is equal to the rotation rate \( \Omega \), and degree-2 gravity coefficient of the Earth \( J_2 \). The effect of the Earth’s flattening is important when the Moon is close to the Earth, i.e. \( a \approx 17R_E \) (Goldreich, 1966). For a hydrostatic planet, \( J_{2\text{E}} \) can be expressed as

\[ J_{2\text{E}} = \frac{1}{3} \left( \frac{R_E^3}{GM} \right) R_E^2 \]  

(35)

where \( \omega \) is the Earth’s rotation rate and \( k_{2\text{E}} \) is the secular Love number which we here assume is equal to the tidal Love number, owing to the fluid state of the early Earth. The physical constants used for these calculations are tabulated in Table 1.

2.4. Solidification of the lunar magma ocean

The solidification of the lunar magma ocean is a complex problem, involving fractional crystallization from a chemically- and thermally-evolving reservoir. Our main interest with respect to the solidification process is to determine the ocean depth \( h \) as a function of time because the tidal dissipation is dependent on this quantity. Unlike other models, we do not seek to explain the geochemistry and geochronology of the Moon’s surface (Borg et al., 1999; Carlson and Lugmair, 1988; Gaffney and Borg, 2014; Nemchin et al., 2006; Nyquist et al., 2010) and therefore, we choose to model the solidification of the magma ocean as a Stefan problem (Turcotte and Schubert, 2002) with bottom heating to account for the effect of tidal dissipation. The evolution of the radial position of the solid-liquid interface \( r_i \), in this case between the solid crust and the liquid magma, is then

\[ \rho L \frac{d(R_M - r_i)}{dt} = -k \frac{\partial T}{\partial r} \Bigg|_{r=r_i} - H \]  

(36)

where \( L \) is the latent heat of fusion, \( k \) is the thermal conductivity of the crust, and \( H \) is the heat flux into the base of the crust. This heat flux is assumed to be solely due to tidal dissipation in the magma ocean such that

\[ H \approx \frac{\dot{E}_M}{4\pi R_M^2} \]  

(37)

Assuming a conductive thermal profile in the solid crust, the temperature gradient at the interface is given by (Turcotte and Schubert, 2002)

\[ \frac{\partial T}{\partial r} \Bigg|_{r=r_i} = \frac{-(T_m - T_0)}{(R_m - r_i)} - \frac{2}{2 \sqrt{\pi}} e^{-\lambda_1^2} \text{erf}(\lambda_1) \]  

(38)

where \( T_m \) is the magma ocean temperature, \( T_0 \) is the surface temperature, \( c_p \) is the specific heat capacity of the crust, and \( \lambda_1 \) is determined by

\[ e^{-\lambda_1^2} \frac{\lambda_1}{\text{erf}(\lambda_1)} = \frac{L}{c_p (T_m - T_0)} \]  

(39)

The approach adopted here is very simple, in that it neglects effects such as secular cooling, heat production (either tidal or radiogenic) in the crust, advection of heat via volcanism, and so on.

To take into account these factors, and to also reflect the uncertainties in the actual duration of the magma ocean (Elkins-Tanton et al., 2011; Solomon and Longhi, 1977), we adopt the simple expedient of allowing the thermal conductivity \( k \) to vary. By doing so, we can generate magma ocean solidification timescales in the absence of tidal heating ranging from 4 to 110 Myr, in agreement with the more sophisticated models of magma ocean solidification. The general conclusions drawn from our orbital model only depend on the timing of magma ocean solidification and not the details of the process.

2.5. Summary and numerical details

Our model may be summarized as follows. The orbital evolution of the Moon is updated based on Eqs. (1)–(4), while the lunar obliquity is calculated based on the inclination assuming it occupies a Cassini state (Section 2.3). Given the obliquity, the dissipation rate in the magma ocean is then calculated using the approach of Section 2.2, and the magma ocean thickness updated according to Section 2.4. Finally, the dissipation rate is fed back into the orbital evolution equations for the next time step. We numerically integrate equations (1)–(4) using a second-order Adams–Bashforth method with a maximum time step of 3000 years.

3. Results

Our nominal model is initialized at a semi-major axis of \( a = 6.5R_E \), inclination of \( i = 12^\circ \), and a rotation period of the Earth of 6 hours. These are estimates of the conditions after passage through the ejection resonance (Touma and Wisdom, 1994; 1998) but are also roughly consistent with the results of interactions with a remnant proto-lunar disk (Ward and Canup, 2000). The initial magma ocean thickness is taken to be 100 km, since we are concerned with the final, slow stages of solidification after the plagioclase flotation crust was established (Elkins-Tanton, 2012; Elkins-Tanton et al., 2011; Shearer et al., 2006). Fig. 1 shows results for a model with tidal dissipation only in the Earth as well as fully coupled models incorporating lunar tidal heating.

In the absence of lunar dissipation (black curves), the inclination damps slowly as the semi-major axis increases. The obliquity rises to a high value as the Cassini state transition is approached; during this period, magma ocean solidification is completed. Because dissipation in the Earth is modest, the outwards evolution of the Moon is quite slow; at the time of magma ocean solidification (65 Myr), \( a \approx 24R_E \), which is, prior to the Cassini state transition at \( = 30R_E \). The lunar inclination remains comfortably above the present-day value of 5.16°. Since increase of the inclination via orbital resonances is not thought to have occurred beyond the ejection resonance (Touma and Wisdom, 1998), any early excitation model must meet this minimum inclination criterion.

The most obvious effect of including lunar magma ocean dissipation is the rapid damping of the orbital inclination (Fig. 1a). The extent to which inclination is damped depends primarily on the total amount of energy dissipated in the ocean before it solidifies, and thus on the obliquity evolution and ocean lifetime. Different lines represent different dissipation rates in the Earth, and thus different rates of outwards evolution. Low dissipation in the Earth (blue curves) results in slow outwards motion and gives the magma ocean time to solidify prior to the onset of the Cassini state transition. As a result, the amount of inclination damping is modest. In contrast, higher dissipation in the Earth (red curve) results in rapid outwards motion and a magma ocean that is still present during the approach to the Cassini state transition. Even at relatively small values for obliquity (1–2°), the effect of inclination damping is significant. In short, models that delay the evolution through the Cassini state transition until after the magma ocean
The models begin with the same initial conditions. The only variations are the tidal time lag of the Earth and whether the model accounts for ocean tidal heating; the plots are colored as follows: (black) $\Delta t = 10^3$ s without magma ocean tidal heating, (black-dashed) $\Delta t = 1000$ s without magma ocean tidal heating, (red) $\Delta t = 10$ s with magma ocean tidal heating, (blue) $\Delta t = 1$ s with magma ocean tidal heating, (blue-dashed) $\Delta t = 0.1$ s with magma ocean tidal heating, and (blue-dash dot) $\Delta t = 0.01$ s with magma ocean tidal heating. The thermal diffusivity for these models is $1.0 \times 10^{-6}$ m$^2$ s$^{-1}$ which translates to a magma ocean solidification time of $11$ Myr without tidal heating. These models are circled in the parameter space in Fig. 2. In (a), the current orbital inclination is indicated by the black-dashed line. The evolution of the inclination with respect to semi-major axis for models without magma ocean heating are the same and are indicated by the black curve. This evolution differs in time (see (e)). In (e), the evolution of the semi-major axis with respect to time is primarily controlled by dissipation in the Earth. The black stars plots the evolution of the model without tidal heating with $\Delta t = 10$ s; this curve collapses on top of the evolution curve for that with tidal heating. (For interpretation of the references to color in this figure legend, the reader is referred to the web version of this article).

The lunar magma ocean must have solidified prior to passage through the Cassini state transition (at around 30 $R_E$). In this case, the early Earth’s effective tidal time lag was likely less than $\sim 10$ s, a value that is orders of magnitude less than the present value of $\sim 10$ minutes (Munk and MacDonald, 1960), i.e. the Earth was significantly less dissipative in its early history when compared to its present state.

The primordial inclination of the Moon was damped via obliquity tides, but the lunar inclination was subsequently re-excited following magma ocean crystallization, for example, via collisionless encounters with remnant planetesimals (Pahlevan and Morbidelli, 2015). It has been long recognized that the Moon has anomalously high inclination, the “mutual inclination problem” (Burns, 1986; Touma and Wisdom, 1998), and has likely undergone a period where the obliquity was extremely high, the Cassini state...
transition (Ward, 1975). Using a solid-body dissipation model, Peale and Cassen (1978) looked at the Moon and concluded that a high obliquity excursion did not significantly affect the Moon’s long-term thermal evolution. We do not debate this conclusion; however, it was only recently recognized that resonant flow behavior in the ocean due to the obliquity tide (Tyler, 2008) can result in tidal dissipation that is orders of magnitude larger than that predicted for solid bodies even at small obliquities (less than 1°) (Chen et al., 2014).

4.1. Inclination damping

The major result of our coupled model is that dissipation in the lunar magma ocean causes significant damping in the orbital inclination (Fig. 1). Inclination damping due to satellite tidal dissipation is generally not emphasized because inclinations for regular satellites of the Solar System are small and inclination tidal dissipation is significantly less important in thermal evolution than eccentricity tidal dissipation.

The ratio of the inclination damping rate due to dissipation in the Moon to that in the Earth can be extracted from Eqs. (2) and (17):

\[
\left( \frac{\dot{\sin i}}{\dot{\sin i}} \right)_{\text{Moon}} = \frac{X_{\text{RM}} E_{\text{M}}}{C_{\text{UX}}} \sin i = \frac{X_{\text{RM}}^2 R_E E M_{\text{M}}}{C_{\text{UX}} C_{\text{UX}} \sin i} \approx \frac{X_{\text{RM}}^2 R_E E M_{\text{M}}}{C_{\text{UX}} C_{\text{UX}} \sin i} \quad (40)
\]

It is not obvious from this ratio that the inclination damping should be dominated by dissipation in the Moon. Substituting values for the constants and appropriate values from the evolution models found in Fig. 1, the ratio is

\[
\left( \frac{\dot{\sin i}}{\dot{\sin i}} \right)_{\text{Moon}} = 1.75 \left( \frac{E_{\text{M}}}{5 \, \text{TW}} \right) \left( \frac{X}{10} \right)^2 \left( \frac{10}{7} \right)^2 \left( \frac{0.1 \, \text{s}}{\Delta t_{\text{E}}} \right) \left( \frac{P_E}{6 \, \text{hr}} \right) \quad (41)
\]

While a factor of 1.75 may seem relatively insignificant, this ratio clearly changes with time. Ocean tidal dissipation decreases with time; however, the ratio actually increases because of expansion of the orbit and decrease in inclination. Later in the evolution, the value of this ratio trends between 3 and 5 depending on model parameters. This increase in damping rate is illustrated in the steep decrease in inclination between our orbit evolution models with (blue and red curves) and without (black curves) lunar magma ocean dissipation in Fig. 1a.

4.2. Dissipation in the early Earth

The amount of tidal dissipation in the early Earth is unknown. The current tidal quality factor of the Earth is small, Q_E \sim 12, due mainly to dissipation in the oceans (Egbert and Ray, 2000; Munk, 1997; Munk and MacDonald, 1960), which is equivalent to a tidal time lag of \Delta t_{\text{E}} \approx 1/(2 \Omega Q_E) \approx 10 \, \text{min} (Goldreich and Soter, 1966; Munk and MacDonald, 1960). Integrating back in time with this present value of Q_E results in an Earth–Moon system age of \sim 1.6 \, \text{Gyr} (Bills and Ray, 1999; Murray and Dermott, 1999); therefore, it is clear that the Q_E of the Earth has evolved over time, and must have had a period where it was significantly higher (less dissipative). Dissipation in the Earth is geologically unconstrained prior to 2.5 Ga (Williams, 2000).

From our model, we find that the tidal time lag of the early Earth, specifically when the magma ocean was still present in the Moon, must have been less than \sim 10 \, \text{s} to preserve a primordial, early-acquired lunar inclination. The time lag controls the rate of outsides semi-major axis evolution of the Moon. As the Moon moves outsides from the location of the eviction resonance (6 – 7 R_E), its obliquity grows. The magma ocean dissipation scales with the square of the obliquity (cf. Eq. (27)) and thus dissipation also increases as the Moon moves outsides. As dissipation occurs, energy is drawn from the orbit and the orbital inclination is damped. Therefore, to retain a primordial inclination, the high obliquity excursion related to the Cassini state transition must not occur while a magma ocean is present. Since the duration of the magma ocean is (approximately) known, we therefore have an upper bound on the inclination damping to occur, with an upper bound on Q_E to retain a primordial inclination.

As discussed below, these conclusions are likely to be robust to most uncertainties. Indeed, the main conclusions can be derived from a simpler analytical model, which avoids having to specify many imperfectly-known parameter values. Fig. 3 presents the semi-major axis evolution as a function of time for a model in which the constant Q of the Earth is specified and dissipation in the Moon is neglected (Murray and Dermott, 1999). For a magma ocean that remains liquid for 10–100 \, \text{Myr} (Elkins-Tanton et al., 2011; Solomon and Longhi, 1977), a reasonable estimate for Q_E such that the semi-major axis does not exceed 20 \, R_E is approximately \sim 300. This value is equivalent to an upper bound on \Delta t_{\text{E}} of about 10 \, \text{s}, compatible with the more sophisticated calculations described above. It is also consistent with the results of Zahnle et al. (2007; 2015), who suggested that an atmosphere on the early Earth could reduce the tidal dissipation of the Earth such that Q_E \sim 300. However, the lifetime of this atmosphere was probably quite short, likely to be around 2 \, \text{Myr}, but no more than 10 \, \text{Myr} (Abe, 1997; Abe and Matsui, 1988; Zahnle et al., 2007; 1988), and tidal dissipation in the solid Earth may have reduced Q_E subsequently to lower values.

4.3. Caveats

Through our coupled thermal-orbital model of the early Earth–Moon system, we have suggested that the current orbital inclination of the system provides a constraint on the dissipative behavior
of the early Earth and the lifetime of the lunar magma ocean. This result is related to the fact that dissipation in a global liquid ocean subject to obliquity tides is significantly larger than solid-body tidal dissipation and therefore, ocean tidal dissipation can be very effective in damping the inclination (cf. Eq. (41)). This result can be interpreted in two ways. In the absence of late lunar inclination excitation, it constrains the tidal time lag of the early Earth and the threshold value of the semi-major axis where the lunar magma ocean is likely to have been solidified. Alternatively, if later inclination excitation occurred, this must have happened after the lunar magma ocean crystallized.

Our conclusions are based on the assumption of a liquid, low-viscosity lunar magma ocean. If future work suggests that the Q of the early Earth was in fact low, one possibility is that our assumed lunar magma ocean rheology was incorrect. It could be that the magma ocean formed a mushy layer, rather than a true liquid (Elkins-Tanton, 2012; Solomatov, 2000). Because of the low viscosities of high-Ti basalts (Williams et al., 2000) and the monomineralic nature of the anorthosite highlands, we think it likely that separation of solid from liquid was inefficient, and that a significant mushy layer did not develop. Nonetheless, this possibility should be borne in mind. Accepting the assumption that a lunar magma ocean behaved as a low-viscosity fluid for some period of time, we now discuss the likeliest sources of uncertainty in our models.

One large uncertainty is in the calculation of $\dot{E}$ of the magma ocean. The total amount of energy dissipation scales with the value of the effective diffusivity $\kappa$, which is poorly constrained (see discussion in Chen et al. (2014)). For the magma ocean, we have adopted a drag coefficient value of $C_D = 0.002$, commonly used in models for bottom friction in terrestrial oceans (Egbert and Ray, 2001; Jayne and St. Laurent, 2001), and mapped this value to a viscous diffusivity through Eq. (28). Because of its dependence on Reynolds number (Sohl et al., 1995), this value of $C_D$ also implies molecular viscosities similar to those of water and ultramafic lavas (Lumb and Aldridge, 1991; Rubie et al., 2003). The lunar magma ocean viscosity likely evolves in time as composition and temperatures change. Our numerically-derived scalings are valid for relatively low effective diffusivities (cf. Eq. 29 in Chen et al., 2014), i.e. less than $4.5 \times 10^6$ m$^2$s$^{-1}$ at a semi-major axis of 6.5 RE. This converts to a maximum model $C_D$ of 0.01 and a maximum molecular viscosity of 800 Pa s (Sohl et al., 1995). This limit is larger than values for the viscosity of high-Ti magmas estimated from lava channels on the Moon (Williams et al., 2000), suggesting that our scaling is valid. In the case where the actual magma viscosities are higher than our modeled viscosities (but less than the scaling limit), our results would be considered conservative estimates.

On the other hand, if the drag coefficient of the magma ocean is in reality smaller than the value used here, the tidal time lag in the Earth can be larger. Figs. 4 and 5 illustrate the effects of reducing the drag coefficient. The drag coefficient controls the amount of tidal dissipation. With reduced dissipation, the solidification times of the magma ocean are closer to a case without tidal heating (compare Fig. 4a–c). Reducing $C_D$ slightly expands the parameter space of successful models. Nonetheless, even for $C_D = 10^{-5}$, Earth time lags $> 30$ s are not consistent with maintaining an early inclination for magma oceans that survive 10 My or longer. The reason is that as the Cassini state transition is approached, the obliquity becomes so high that dissipation dampens the inclination. As illustrated in Fig. 5, a reduced drag coefficient allows for further outwards migration, but in no case do successful models include the Cassini state transition itself, because of the resulting high dissipation. Passage through the Cassini state transition while a magma ocean is present would require subsequent re-excitation of inclination.

Another uncertainty is the initial inclination of the orbit. Ward and Canup (2000) suggest that through resonances the initial inclination may have exceeded 16°. Figs. 6 and 7 show the successful parameter space for different values of the initial inclination. The models that begin with inclinations of 18° are similar to those with initial inclinations of 12°; in particular, solutions with Earth time lags $> 30$ s result in inclination damping. Minor differences arise because the additional energy in the inclined orbit tends to prolong the lifetime of the magma ocean, despite the higher rate of energy dissipation associated with the higher inclination values. Lastly, it is worth noting that all models with initial inclination of 6° are inconsistent with the current orbital configuration unless later inclination excitation occurs. This is consistent with models of inclination damping due to Earth tides alone (Goldreich, 1966; Touma and Wisdom, 1998).

The evolution of the Moon’s eccentricity is still debated (e.g. Garrick-Bethell et al., 2006; Meyer et al., 2010; Čuk, 2011), but here we have neglected it entirely. The main reason for doing so is to clarify the role of the magma ocean, which is the most novel aspect of our work. Eccentricity tidal heating in a deep magma ocean is minimal, while obliquity tidal heating in the same ocean can vastly exceed eccentricity tidal heating in the solid Moon (Chen et al., 2014). We have therefore chosen here to focus on obliquity and inclination, rather than eccentricity, but it would clearly be of interest to include the latter effect in future work.

A final major assumption in our models is that the Moon did not suffer significant inclination excitation during or following the period under consideration (a $\lesssim 30$ RE). Simulations of the Moon-forming impact suggest that the initial inclination of the orbit was likely $< 1°$ directly following lunar accretion (Ida et al., 1997). Proposed mechanisms to excite the lunar inclination (e.g.
Touma and Wisdom, 1998; Ward and Canup, 2000; Ćuk and Stewart, 2012) occur very early in lunar history with the inclination subsequently decreasing monotonically over time due to tides (Peale and Cassen, 1978). Later resonances may have occurred (e.g. Ćuk, 2007) but do not appear likely to have excited the primordial lunar inclination. The most likely source of late inclination excitation is close encounters with planetesimals not yet incorporated into the terrestrial planets (Pahlevan and Morbidelli, 2015). Such encounters occur with decreasing probability as time proceeds, but may have been common during the epoch of interest. Because of its potential importance, we discuss the potential consequences of late inclination excitation further below.

5. Implications and conclusions

In our model for the coupled thermal-orbital evolution of the Moon that includes dissipation produced by the lunar magma ocean, we find that dissipation in the ocean has a strong tendency to damp the orbital inclination. The dissipation is related to the Moon’s obliquity. Because of the occurrence of the Cassini state transition at \( \sim 30 \, R_E \), the obliquity increases rapidly near a semi-major axis of \( \sim 20 \, R_E \) and it is difficult to maintain a high orbital inclination in the presence of enhanced dissipation due to the increasing obliquity. Therefore, one solution is that the magma ocean solidified prior to \( \sim 20 \, R_E \) (with the exact distance being sensitive to the initial conditions and details of the thermal-orbital model), in order for the inclination to not be damped to a value lower than that which is currently observed, \( i = 5.16^\circ \).

The evolution of the semi-major axis of the Moon’s orbit is mainly controlled in our model by dissipation in the Earth. For this solution, we find that the time lag of the early Earth is required to be quite small (less than \( \sim 10 \) s) for a lunar magma ocean that solidifies prior to a semi-major axis of \( \sim 20R_E \). This small time lag loosely translates to a constant tidal Q for the early Earth of approximately 300, roughly comparable to the present-day value for the solid Earth (Ray et al., 2001). This conclusion is robust to likely uncertainties in both initial inclination and the drag coefficient assumed. Our result can be compared to models without lunar magma ocean dissipation, where the time lag can be larger by many orders of magnitude, even up to \( \sim 1000 \) s, while still yielding a higher inclination than present.

It may appear paradoxical to argue that dissipation in the lunar magma ocean is an important source of inclination damping, while at the same time requiring dissipation in the Earth – which presumably also possessed a magma ocean – to be small. The resolution of this paradox is simple: the fluid dynamical resonance driving dissipation on the Moon is only applicable to bodies in synchronous rotation, and is thus not relevant to the Earth. Deep magma oceans, in the absence of the kind of the resonance proposed for the Moon, are not expected to be highly dissipative (Sohl et al., 1995).

A larger challenge may be to obtain high values for Q of the Earth after crystallization of the terrestrial magma ocean. Zahnle et al. (2007; 2015) suggested that as the Earth cooled between the liquidus and solidus, it became significantly more dissipative (Q_E \( < 10 \)) due to viscous dissipation in the mantle. The cooling rate would be limited by the rate at which an early thick atmosphere could transmit heat; however, this atmosphere probably had a
Fig. 7. Similar to Fig. 6 with variation of the initial inclination in the fully coupled model. Successful cases are plotted against the orbital semi-major axis. Once again, successful cases are not present beyond the Cassini state transition (a ≈ 30R_E). The initial inclination must be at least 6° to be consistent with the current orbital configuration.

lifetime of less than 10 Myr (Abe, 1997; Abe and Matsui, 1988; Zahle et al., 2007; 1988). Additionally, surficial liquid water was probably present during the Hadean (Mojzsis et al., 2001; Wilde et al., 2001). To avoid high dissipation in the early Earth, either this water was not global in extent or early oceans could not have contributed significantly to tidal dissipation as they do at present.

An alternative solution to requiring low dissipation in the early Earth is a scenario in which the lunar inclination was excited via collisionless encounters with remnant bodies at a later stage, as has been recently proposed (Pahlevan and Morbidelli, 2015). In this case, strong damping of the primordial orbital inclination could have occurred, but lunar magma ocean crystallization must still have been complete prior to the subsequent excitation. In contrast with the first solution, this second solution is actually more effective with strong tides on the early Earth and rapid outwards lunar motion. Hence, with our model, determining the dissipative properties of the early Earth could in principle distinguish between the various solutions to the longstanding lunar inclination problem.

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References


