Rotational dynamics and internal structure of Titan

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\textbf{ABSTRACT}

Estimates of the moments of inertia of Titan, as separately deduced from its gravitational field and spin pole orientation, are quite different. This discrepancy can be resolved if Titan is either not precessing as a rigid body (e.g. if the shell is decoupled from the interior by an ocean), or if the spin pole is not fully damped (e.g. due to atmospheric excitation). By the end of the Cassini mission, continued monitoring of the changing spin pole orientation, by Cassini radar observations, will determine which effect dominates.

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1. Introduction

The gravitational field and obliquity, or angular separation between spin and orbit poles, of a tidally damped body both provide information about the internal structure of the body. For Titan, the Cassini mission Doppler tracking data has provided information pertinent to the low-degree gravity field (less et al., 2010), and the radar data has been used to determine the obliquity (Stiles et al., 2008, 2010).

Estimates of the moments of inertia of Titan obtained from these two approaches are quite different, and this suggests that the outer shell is mechanically decoupled from the deeper interior (e.g. by an ocean), or that the spin pole is not fully damped, or some combination of those two effects.

We will argue in this paper that the best current estimate of Titan’s spin pole orientation (Stiles et al., 2008, 2010) can be fit exactly by any member of a one-parameter family of dynamical models in which there is a presently unresolved trade-off between the spin pole precession rate parameter and the amplitude of an un-damped free precession mode.

However, we also show that, as the spin pole continues to precess, accurate monitoring of the precessional motion will allow resolution of this ambiguity. As additional passes of radar data are collected on Titan, our ability to resolve this issue will improve dramatically. The improvement accrues both from a rapidly increasing number of overlap regions in the radar swath coverage, which allows many more tie-points, and from the increasing time elapsed between the most widely separated overlapping passes, which yields increasing sensitivity to rotation rate and direction.

The low-degree gravitational field of Titan, as determined from the Doppler shift in the tracking data, is consistent with the pattern expected for a body in hydrostatic equilibrium, when subjected to rotational and tidal potentials (Hubbard and Anderson, 1978). In that case, the size of the degree two gravitational potential coefficients indicates the polar moment of inertia of the body. The inferred polar moment value for Titan is

\[ c = C/(M R^2) = 0.342 \pm 0.001 \]

(less et al., 2010), where \( M \) is mass and \( R \) is mean radius. This is smaller than the homogeneous value of 2/3, and thus implies some degree of differentiation and central condensation, though less than had been anticipated in most theoretical models (Grasset et al., 2000; Sohl et al., 2003; Tobie et al., 2005).

For a body in a tidally damped rotation state, the obliquity is also diagnostic of the moments of inertia (Bills and Nimmo, 2008). At the time that Bills and Nimmo (2008) was published, no constraints on the degree-two gravity coefficients of Titan were available. As we discuss below, when the recently-determined degree-two gravity coefficients are combined with the radar-derived obliquity of 0.32 degree, the implied dimensionless moment of inertia is \( c = 0.45 \pm 0.02 \), which is considerably above the homogeneous spherical value. This suggests that one or more of the assumptions leading to that inference is wrong. It could be that Titan’s spin pole is not fully damped, or that it is not precessing as a rigid body. Either of these results would be interesting and somewhat unexpected. We now describe these background studies in more detail.

2. Gravity constraints

For a body which is in hydrostatic equilibrium and synchronous rotation, the imposed tidal and rotational potentials together induce changes in the mass distribution which are mainly manifest as degree two spherical harmonic coefficients in the gravitational potential (Hubbard and Anderson, 1978):

\[ \frac{J_2}{C_{22}} = \frac{k_f}{4 \pi} \left[ \frac{10}{3} \right] \]

(2)

where \( q \) is the ratio of centrifugal and gravitational accelerations on the equator

\[ q = \frac{\alpha^2 R^2}{GM} = 1.315 \times 10^{-6} \]

(3)

\( G \) is the gravitational constant and \( k_f \) is a fluid Love number (Munk and MacDonald, 1960), or scale factor, which relates the imposed potential to the induced potential. Observed values of the gravitational coefficients are \( J_2/(C_{22}) \approx (32.3 \pm 0.3) \times 10^{-6} \) and \( C_{22} = (9.99 \pm 0.03) \times 10^{-6} \). If the corresponding values of fluid Love numbers are then used in the Radau–Darwin relation (Radau, 1885; Darwin, 1900; Bourda and Capitaine, 2004) which relates \( k_f \) to \( C_{22} \), we then obtain a value for the dimensionless polar moment of inertia \( c = 0.342 \pm 0.001 \) (less et al., 2010).

A requirement, for the application of this relation, is the assumption of hydrostatic equilibrium. In contrast to the situation for the Galilean satellites (Anderson et al., 1996a,b, 1998a,b), the Titan gravity solution of less et al. (2010) did not involve any a priori constraints. Despite that, the inferred ratio of \( J_2/C_{22} \) is very close to the hydrostatic value of 10/3. This value thus likely reflects the actual moment of inertia of Titan, and suggests a reasonable degree of central condensation.
3. Spin pole constraints

We now briefly review relevant background material on precessional dynamics, and then present a series of three successive approximations to the spin pole state of Titan. The first two cases involve damped spin poles, with steady and non-steady orbit precession, and the third case allows a non-damped spin pole.

3.1. Background

The classical means of determining the moment of inertia of a planet, without hydrostatic assumptions, is via measurement of the rate of spin pole precession, together with $J_{2}$ and $C_{22}$. Together, these three observations are sufficient to determine the three principal moments of inertia ($a$, $b$, and $c$). This is the means by which the moments of inertia of Earth (Williams, 1994) and Mars (Folkner et al., 1997) are known.

A disadvantage, for application of this strategy to a body like Titan, is that the expected spin pole precession rate is quite slow. A better approach, in such cases, is available if the spin pole is fully damped, since then the angular separation between spin and orbit poles is itself diagnostic of the precession rate, and hence of the moments of inertia. All that is required then is an accurate determination of the spin pole direction, rather than a determination of its rate of change.

3.2. Uniform orbit precession

If the orbit pole precession rate is uniform, the damped spin pole will maintain a constant obliquity, or angular separation from the orbit pole, and will remain coplanar with the orbit pole and the invariable pole, about which the orbit pole is precessing. Such a configuration is known as a Cassini state (Colombo, 1966; Peale, 1969), in honor of G.D. Cassini who realized in 1693 that the Moon behaves that way.

We will describe the spin and orbit precession in terms of unit vectors aligned with the spin pole ($\hat{s}$), orbit pole ($\hat{n}$), and invariable pole ($\hat{k}$). In general, the spin and the orbit poles have complicated relative motion, resulting in the spin pole precessing, about the invariable pole. However, in a tidally damped spin configuration, or Cassini state, the spin pole adjusts its angular separation from the orbit pole such that it precesses at a rate which allows it to remain coplanar with the orbit pole and invariable pole.

These steady spin pole configurations can be found by two different, but completely equivalent, approaches. Below we will show how to find them directly from the differential equation for spin pole precession. An alternative, used by Colombo (1966) and Peale (1969), derives the same results via an extremum of the Hamiltonian.

We initially assume that the orbit pole $\hat{n}$ is precessing, at fixed inclination $i$ and uniform rate $\Omega$, about the invariable pole $\hat{k}$, such that

$$d\hat{n}/dt = \Omega (\hat{k} \times \hat{n})$$

and $\hat{n} \cdot \hat{k} = \cos i$

We also assume that the spin pole $\hat{s}$ is precessing about the instantaneous orbit pole $\hat{n}$, at a rate given by Bills (2005)

$$d\hat{s}/dt = (\Omega (\hat{n} \times \beta) + \beta) \times \hat{n}$$

where the rate parameters $x$ and $\beta$ are given by

$$x = 3n (J_{2} + C_{22})/2$$

$$\beta = 3n (4c(a - b) - \beta)/2$$

with $n$ denoting the orbital mean motion, or mean angular rate of motion along the orbit. The expression given here for $\beta$ is 1/2 the value cited in Bills (2005) and makes our result consistent with Peale (1969) and Ward (1975).

The differential equations for orbit precession (4) and spin precession (6) both describe motion as seen in an inertial reference frame. It is more convenient, in the present context, to examine the spin pole motion in a reference frame which is fixed in the precessing orbit plane. The resulting spin pole motion becomes

$$d\hat{s}/dt = (\Omega (\hat{n} \times \beta) + \beta) \times \hat{n} + \Omega (\hat{k} \times \hat{n})$$

For nominal Cassini state behavior, we seek an orientation of the spin pole $\hat{s}$ such that it will remain stationary, in this orbit fixed frame. We will use a Cartesian coordinate frame, hence the spin pole orientation in the Cassini is normal to the orbit plane, and the $z$-axis lies in the plane defined by the orbit pole $\hat{n}$ and invariable pole $\hat{k}$. To proceed, we substitute explicit values for the unit vectors; $\hat{s} = (x, y, z)$, $\hat{n} = (0, 1, 0)$, and $\hat{k} = (-\sin i, 0, \cos i)$. Steady precession motion thus corresponds to $dx/dt = \Omega dy/dt = \Omega dz/dt = 0$. Rewriting Eq. (8) component-wise, as three equations in terms of $x$, $y$, and $z$, it can be shown that $z = 0$ and that

$$(x + \beta + \eta \cos i) x - \eta \sin i = 0$$

The first of these constraints simply specifies that the spin pole $\hat{s}$ should lie in the plane defined by $\hat{n}$ and $\hat{k}$. If we now set $x = \sin i$, $y = 0$, and $z = \cos i$, where $i$ is the obliquity of the spin pole, and substitute into Eq. (9), we obtain

$$(x \cos i + \beta + \eta \sin i) = \eta \sin i$$

If the spin pole has a value of obliquity $i$ which is a solution of this equation, it will precess at a rate which will keep it fixed relative to both the orbit pole and the invariable pole, and it is said to be in a Cassini state.

The significance of Eq. (10) is that the orbit precession rate $\eta$ and inclination $i$ are known. Thus, if the obliquity $i$ can be measured, this places a constraint on the rate parameters $x$ and $\beta$. If, in addition, $J_{2}$ and $C_{22}$ are known, then Eqs. (10) and (7) can together be used to deduce the dimensionless moment of inertia $c$.

This approach to Titan, using the observed values of obliquity $i$ (Stiles et al., 2008), in combination with the observed gravity coefficients $J_{2}$ and $C_{22}$ (Liss et al., 2010), yields a dimensionless moment of inertia $c = 0.58$. This is in excess of the value for a homogeneous sphere $(2/5)$, and thus, if taken at face value, suggests that the rotation is controlled by a thin shell, partially decoupled from the deeper interior.

3.3. Non-uniform orbit precession

However, Titan does not quite satisfy the steady orbit precession criterion assumed above. The orbit precesses, with a period of 700 years and inclination of 0.28, about Saturn’s spin pole (Sinclair, 1977), but Saturn’s spin pole also precesses, with a period of 1.87 million years and inclination of 26.7, about its own orbit pole (Ward and Hamilton, 2004). There are also small solar perturbations to the orbit plane orientation on the Saturn orbit period time scale (Vienne and Duriez, 1995). Fortunately, the dynamical equivalence of a Cassini state configuration is easily extended to this multi-frequency situation.

The orientation of the orbit pole unit vector $\hat{n}$, relative to the invariable pole $\hat{k}$, can be represented as a complex scalar whose time evolution is given by a Poisson series

$$N(t) = \sum_{j} n_{j} \exp[\Omega(t + \phi_{j})]$$

where $n_{j}$, $\phi_{j}$, and $\Omega$ are the amplitude, rate, and phase for the $j$th mode of oscillation, and $i = \sqrt{-1}$. The linearized equation of motion for the complexified scalar spin pole $S$ is

$$dS/dt = f_{j}(N - S)$$

with rate parameter

$$\gamma = x + \beta = 3n (J_{2} + C_{22}) = 3n (8c(a - b) - (b - a))$$

The corresponding spin pole solution has a single free mode, and as many forced modes as there are modes in the orbit pole model. It has the form (Ward and deCampli, 1979; Bills, 2005)

$$S(t) = A \exp[-j\gamma t] \sum_{j} s_{j} \exp[j(\Omega t + \phi_{j})]$$

with coefficients obtained from the orbit pole via

$$s_{j} = \frac{1}{\gamma + jf_{j}}$$

If tidal energy dissipation is occurring, the free mode will eventually damp out, but the forced modes persist. This damped solution is the multi-frequency generalization of a Cassini state. As the orbit pole rate parameters $f_{j}$ are generally negative, and the spin pole rate parameter $\gamma$ is invariably positive, it is possible to have resonant amplification of forced modes for which $f_{j} \approx -\gamma$

In a multi-frequency version of the Cassini state, the spin and orbit poles are no longer aligned with the invariable pole, as has been observed for Titan (Stiles et al., 2008). This is not necessarily evidence of failure to be in a fully damped state, but may simply reflect the more complex orbit pole dynamics. However, if we use the moments of inertia inferred from the hydrostatic gravity (Liss et al., 2010), the nominal spin pole precession period is

$$2\pi/\gamma' \approx 235 \text{ years}$$

and thus only the main mode of orbit pole precession, with period 700 years, is significantly reflected in the spin pole.

The spin pole precession rate $\gamma'$ required for a fully damped solution to match the observed obliquity, including the effect of multi-frequency orbit pole precession, is smaller than this nominal value. The best-fitting value is roughly the nominal value, and implies a polar moment (via Eq. (13)) of $c = 0.45 \pm 0.02$. While
3.4. Application to Titan

The IAU spin pole model for Titan assumes zero obliquity, and is thus just a simple representation of the orbit pole motion. The right ascension $\alpha$ and declination $\delta$ are given by Seidelmann et al. (2007)

$$
\begin{align*}
\alpha(t) &= 36.41 - 0.0367t - 2.66 \sin S(t) \text{ deg} \\
\delta(t) &= 83.94 - 0.0047t - 0.30 \cos S(t) \text{ deg}
\end{align*}
$$

where

$$
S(t) = (29.80 - 52.17) \text{ deg}
$$

and $t$ is time in Julian centuries from J2000. The constant and linear terms describe the motion of Titan's Laplace pole. That is the pole about which the orbit would precess, at constant inclination, if the precession were steady (Dobrovolskis, 1993; Jacobson and Owen, 2004; Tremaine et al., 2009). For satellites close to Saturn, the Laplace pole is essentially coincident with Saturn's spin pole, while for distant satellites it approaches Saturn's orbit pole. For comparison, the IAU spin pole model for Saturn (Seidelmann et al., 2007) is

$$
\begin{align*}
\alpha_0 &= 40.589 - 0.0367 \text{ deg} \\
\delta_0 &= 83.537 - 0.0047 \text{ deg}
\end{align*}
$$

We now present some figures which illustrate various aspects of the precessional motion of Titan's spin and orbit poles. Fig. 1a illustrates the current and past locations of relevant poles. It shows the motion, for the past 1000 years, of the orbit and invariant poles of Titan, and the spin pole of Saturn, as represented by the IAU models listed above. Also shown is an estimate (Stiles et al., 2008) of the location of Titan's spin pole at epoch 1 August 2006. The small ellipse surrounding the estimate represents seven times the standard deviation. It is enlarged in this figure simply to make it visible. The dashed segments joining the Laplace pole to the orbit pole and spin pole illustrate the point that the spin pole is not quite coplanar with the other two.

Fig. 1b shows the motion, also for the past 1000 years, of two different damped spin pole solutions, one with the expected precession rate parameter ($\gamma = \gamma'$), as given by Eq. (17), and another with slower spin precession ($\gamma = 0.526\gamma'$). While the latter value gives a better fit to the observed spin pole, it is still well outside the observational errors.

Fig. 2 shows how the misfit, or angular distance between the observed spin pole and the current location of a damped spin pole, changes with the precession rate parameter $\gamma$. The trajectories shown in Fig. 2 are the nominal case ($\gamma = \gamma'$) and the best fitting ($\gamma = 0.526\gamma'$) case. This misfit distance, for a damped spin pole model, is just the amplitude required for the free mode, in a forced model which exactly fits the observed pole.

4. Future prospects

As was noted earlier, the present spin pole orientation of Titan can be reproduced by any member of a one-parameter family of models which include both free and forced modes of precession. When additional Cassini radar data are processed, to further constrain the precessional motion, this should allow a resolution of the present ambiguity. We will then face the task of understanding the implications of the inferred precession rate and free mode amplitude.

Fig. 3 illustrates the spin pole precession trajectories for two representative models. Both are constrained to pass through the observed (Stiles et al., 2008) location, at the observation epoch, which is 1 August 2006, but they have different precession rates and correspondingly different free mode amplitudes. The motions are shown for a 15-year time span, roughly corresponding to the anticipated duration of the Cassini mission observations of Titan. By construction, the two models agree at the nominal epoch, and they initially move in very similar directions, but at different rates. Thus future observations of the spin pole position will be able to quantify the respective contributions of the free and forced precession.

If a significant free precession mode exists, the challenge will be to properly understand its origin. While it is possible to have impulsive excitation, such as would result from a recent impact (Peale, 1976), it seems more likely to represent a quasi-steady balance between excitation and damping. The most obvious source of excitation is the atmosphere (Tokano, 2010). If the strength of that excitation can be quantified, then the equilibrium amplitude will put important constraints on the rate of dissipation, which are likely diagnostic of internal structure (Sohl et al., 1995; Tohie et al., 2005).

If the resolved model has a spin precession rate different from that inferred from the hydrostatic gravity data (17), it will strongly suggest that Titan does not precess as a rigid body, but has a surface shell which is partially decoupled (e.g. by an ocean) from the deeper interior. However, the precessional dynamics of a thin shell, overlayers a global subsurface ocean, are not yet fully understood (Noir et al., 2009).

The problem of precessional coupling between Earth's fluid core and solid mantle has long been studied experimentally (Stewart and Roberts, 1963; Noir et al., 2001), analytically (Noir et al., 2003; Schmitt and Jault, 2004), and observationally (Charlot et al., 1995; Lambert and Dehant, 2007). More recently, the decoupling of Mercury's mantle from its core has received considerable attention (e.g. Peale et al., 2002; Peale, 2005; Margot et al., 2007). Like Mercury, but in contrast to the Earth, the moment of inertia and rotational dynamics of Titan are likely dominated by the portion beneath the fluid layer. Analysis of the situation at Titan is still in its

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Fig. 1. (a) Spin and orbit poles for Titan. Curves show motion for the past 1000 years, while dots indicate current locations. (b) Trajectories (dashed lines) for two damped spin poles. See text for discussion.
infancy, although Van Hoolst et al. (2009) and Karatekin et al. (2008) have considered gravitational coupling between layers, while Noyelles (2008) investigated the possibility of a resonantly-forced wobble. Further studies of this kind are likely to yield additional insight into the dynamics both of Titan’s solid shell, and potentially its liquid interior.

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References


