Dissipation at tidal and seismic frequencies in a melt-free, anhydrous Mars

F. Nimmo¹ and U. H. Faul²

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[1] The measured inward motion of Phobos provides a constraint on the tidal dissipation factor, \( Q \), within Mars. We model viscoelastic dissipation inside a convective Mars using a modified Burgers model based on laboratory experiments on anhydrous, melt-free olivine. The model tidal \( Q \) is highly sensitive to the mantle potential temperature and grain size assumed but relatively insensitive to the bulk density and rigidity structure. \( Q \) thus provides a tight constraint on the Martian interior temperature. By fitting the observed tidal \( Q \) and tidal Love number (\( k_2 \)) values and requiring present-day melt generation, we estimate that for a grain size of 1 cm the current mantle potential temperature is 1625 ± 75 K, similar to that of the Earth. This estimate is consistent with recent petrologically derived determinations of mantle potential temperature but lower than estimates in some thermal evolution models. The presence of water in the Martian mantle would reduce our estimated temperature. Our preferred mantle grain size of \( \approx 1 \) cm is somewhat larger than that of the Earth’s upper mantle. The predicted mantle seismic \( Q \) is about 130 and is almost independent of depth. The Martian lithosphere represents a high seismic velocity lid, which should be readily detectable with future seismological observations.


1. Introduction

[2] The rate at which tidal energy is dissipated within solid bodies can be measured and provides constraints on the interior properties of those bodies. Dissipation is often quantified by a frequency-dependent tidal quality factor \( Q \), where large \( Q \) means small dissipation. The \( Q \) of the solid Earth is 280 at a period of 12.5 h [Ray et al., 2001]; the value of \( Q \) for Mars is significantly smaller (see below), indicating more dissipation. On the face of it, this is a surprising result: tides on Mars have shorter periods compared to Earth tides, which should lead to higher, not lower, \( Q \) values. Furthermore, simple (Maxwell) viscoelastic dissipation models applied to Mars [Bills et al., 2005] require mantle viscosities that are several orders of magnitude smaller than expected values and are likely inappropriate based on experimental observations. For solid silicate bodies, the tidal \( Q \) is highly sensitive to the temperature structure and grain size adopted but relatively insensitive to the density and rigidity structure (see section 5.1 below). \( Q \) thus provides a convenient probe of mantle temperature, itself an important indicator of a planet’s thermal and volcanic evolution.

[3] In this paper, we apply a dissipation model [Nimmo et al., 2012] based on laboratory experiments [Jackson and Faul, 2010] to the case of an anhydrous, melt-free Mars. We show that the measured Martian \( Q \) can be generated for reasonable interior structure models. Our preferred model yields a Martian mantle potential temperature similar to that of the Earth’s upper mantle. The inferred Martian mantle grain size is larger than that of the Earth’s upper mantle, suggesting a higher mantle viscosity for Mars.

1.1. Previous Work

[4] Many previous works have modeled the density structure of Mars, using the bulk density and moment of inertia as observational constraints [e.g., Bertka and Fei, 1998; Sohl et al., 2005; Verhoeven et al., 2005]. Some have also considered the rigidity (and thus seismic velocity) structure, constrained by the inferred tidal response (Love number) [e.g., Sohl and Spohn, 1997; Van Hoolst et al., 2003; Gudkova and Zharkov, 2004; Khan and Connolly, 2008; Rivoldini et al., 2011]. Unlike many of these works, we do not provide a detailed, multimineral model of Martian density and rigidity, because that is not the aim of this paper. Our primary focus is on the tidal dissipation factor \( Q \) which (as we show below) is relatively insensitive to the assumed density and rigidity structures. While we do make use of the Love number and moment of inertia constraints,
we concentrate on $Q$ because it is particularly well suited to
determining the mantle temperature structure.

[5] Although the tidal $Q$ of Mars is well known, there have been relatively few attempts to relate it to models of the interior. The most similar approach to ours is briefly described in Castillo-Rogez and Banerdt [2013], where the authors adopt an Andrade rheological model and derive a mantle temperature $\approx 1700$ K, assuming dry olivine. Sohl and Spohn [1997] adopted a Burgers rheological model but prescribed the ratio (rigidity:unrelaxed rigidity) as a fixed constant rather than calculating it as we do below. Bills et al. [2005] showed that a homogeneous, Maxwellian Mars would have to have a viscosity $\approx 10^{15}$ Pa s to match the inferred $Q$, compared with typical terrestrial mantle viscosities of $10^{20}$–$10^{22}$ Pa s. Efroimsky [2012] discusses the formalism for applying a hybrid Andrade-Maxwell model to silicate bodies.

[6] Lognonne and Mosser [1993] and Lognonne and Johnson [2007] briefly discuss the $Q$ of Mars and use an absorption-band model to argue that, other things being equal, a dissipative Mars at tidal frequencies can be reconciled with the known seismic $Q$ of the Earth if the Martian mantle is about 100–150 K colder than that of Earth. As will be seen below, we obtain a similar result if the Martian mantle grain size is comparable to that of the Earth's upper mantle ($\sim$1 mm). However, we favor an alternative in which Earth and Martian mantle temperatures are similar but the Martian mantle has a coarser grain size.

[7] Zharkov and Gudkova [1997] outlined how to calculate the tidal $Q$ of a layered Mars and discussed its frequency dependence; they did not, however, use a rheological model to link $Q$ to quantities like temperature or grain size, as we do here. Khan and Connolly [2008] included calculations of the $Q$ of Mars, but specific details were not included in the paper.

2. Observations

[8] The response of a planetary body to periodic forcing can be described using two parameters: the degree-2 tidal Love number $k_2$, which quantifies the amplitude of the response, and the dissipation factor $Q$, which quantifies the phase lag between forcing and response. The Love number depends on the density and rigidity structure of the body, while $Q$ also depends on whether the body undergoes creep. As a result, $Q$ is much more sensitive to temperature and grain size than is $k_2$. Both $Q$ and $k_2$ are expected to be frequency dependent. In the case of Mars, measurements of both $Q$ and $k_2$ exist at different frequencies and will form our primary observational constraints.

[9] Phobos orbits Mars inside the synchronous point. Dissipation due to tides raised by Phobos on Mars causes Phobos to drift inward at an astrometrically measurable rate [e.g., Bills et al., 2005]. This rate depends on the Martian value of $k_2/Q$ at the synodic period of Phobos (11.106 h). Mars also experiences tides raised by the Sun. Because of Mars' rotation, the solar tides give rise to a periodic term in the gravity field which can be measured [Yoder et al., 2003].

The magnitude of the response depends on the value of $k_2$ at the synodic period of the Sun (24.66 h, roughly Mars' rotation period). Thus, in principle, separate estimates of $k_2$ and $Q$ can be derived. However, there are two complications. First, the measured value of $k_2$ at the solar synodic period will be slightly larger than that at Phobos' period. More seriously, Phobos is sufficiently close to Mars that higher-degree terms ($k_3$, $k_4$, ...) affect the rate of orbital evolution [Bills et al., 2005].

[10] Yoder et al. [2003] analyzed the response of Mars to solar tides to derive $k_2$. Correcting for the small effects of the fluid core and atmospheric tides, $k_2$ for Mars at 24.66 h period is 0.149 ± 0.017. Using our baseline model presented below, $k_2$ is found to be about 0.6% smaller at Phobos' synodic period than at the solar synodic period. Thus, at Phobos' period we estimate $k_2 = 0.148 ± 0.017$ (0.131–0.165).

[11] Bills et al. [2005] included the effects of higher-degree terms in estimating $k_2/Q$ of Mars. However, when inferring $Q$ from $k_2/Q$, they used a value of $k_2 = 0.0745$ appropriate for a homogeneous elastic sphere, which is too low. As a result, their estimate of $Q = 85.58 ± 0.37$ needs correcting. Lainey et al. [2007] used a more realistic value of $k_2 = 0.152$ but explicitly neglected the higher-degree terms. As a result, their estimate of $Q = 79.91 ± 0.69$ also requires correcting.

[12] To investigate the effect of the higher-degree terms, we will follow Bills et al. [2005]. Their equations (13) and (44) can be written

$$
\frac{1}{n} \frac{dn}{dt} = \frac{k_2}{Q} 9n \mu \left( \frac{R}{a} \right)^3 \left[ 1 + \frac{k_3}{k_2} \left( \frac{R}{a} \right)^2 + \frac{10 k_4}{3 k_2} \left( \frac{R}{a} \right)^4 \right]
$$

[13] Here $n$ is the mean motion of Phobos, $\frac{1}{n} \frac{dn}{dt}$ is its measured secular acceleration, $a$ is the semimajor axis, $R$ is the radius of Mars, and $\mu$ is the Phobos/Mars mass ratio. This expression assumes that the tidal lag angle $\gamma$ is small and frequency independent; the term in square brackets is the correction factor $f$ taking into account higher-degree terms ($k_3$ and $k_4$). If these terms are neglected ($f = 1$), equation (1) shows that the measured secular acceleration can be used to determine $Q$ if $k_2$ is known.

[14] Unfortunately, $k_3$ and $k_4$ are unknown. For a homogeneous body, the correction factor $f = 1.13$ [Bills et al., 2005]. For an inhomogeneous body, this factor (which depends mainly on $k_3/k_2$) will be different. Van Hoolst et al. [2003] find Martian model Love numbers $h_2$, $h_3$, and $h_4$ in the range 0.13–0.23, 0.09, and 0.06, respectively. Assuming that the Love numbers behave similarly, the Van Hoolst et al. results imply that the ratios $k_3/k_2$ and $k_4/k_2$ are in the range 0.39–0.69 and 0.26–0.46, compared with the homogeneous values of 0.435 and 0.252. Taking the upper limits on these ratios, the maximum correction factor $f = 1.20$.

[15] Given estimates of $k_2$ and $f$, equation (1) can then be used to determine $Q$ for Mars. Lainey et al. [2007] take $f = 1$ and $k_2 = 0.152$ to derive their value of $Q$; using our preferred values of $f = 1.13$ ± 0.07 and $k_2 = 0.148 ± 0.017$, we obtain $Q = 88 ± 16$. This is very similar to the preferred value of 92 given by Yoder et al. [2003]. The values in Bills et al. [2005] give $f = 1.13$ and $\frac{k_3}{k_2} = 9.83 \times 10^{-4}$, implying $Q = 170 ± 20$ for our preferred $k_2$. The source of the discrepancy between these two approaches is unclear but may in part be due to the different time span of observations analyzed in the two studies. For the bulk of this MS we will adopt the Lainey et al. [2007] corrected value of $Q = 88 ± 16$. However, where appropriate we will also discuss the implications of the Bills et al. [2005] corrected estimate.
[16] Secondary observational constraints include the moment of inertia \(C/MR^2 = 0.3650 \pm 0.0012\), inferred from the observed precession of the Martian rotation pole [Yoder et al., 2003] and the inferred elastic thickness \(T_e\) of the Martian lithosphere. This latter quantity may exceed 300 km at the north pole at present [Phillips et al., 2008] and is estimated at >70 km, 2-80 km, <100 km, and >20 km for four Amazonian-aged features [McGovern et al., 2004]. A compilation by Grott et al. [2013] concludes that present-day \(T_e\) values exceed 150 km. There is probably spatial variability in \(T_e\) as a result of variations in lid thickness induced by convection [Kiefer and Li, 2009; Grott and Breuer, 2010]. Note that only part of the stagnant, nonconvecting lid will respond in an elastic manner over geological timescales; thus, the stagnant lid thickness is generally greater than the effective elastic thickness.

3. Model

[17] In this work we are most interested in dissipation within Mars and the factors which control it (primarily temperature and grain size). We will therefore adopt a relatively elaborate model of the dissipation process (see section 3.2 below). Conversely, our model will in other ways be significantly simplified, e.g., in its treatment of phase changes and its calculation of pressure and density (section 3.1). These simplifications are adopted for three reasons: first, our current knowledge of Mars’ internal structure and composition is highly uncertain; second, the approximations involved can be shown a posteriori not to affect our main conclusions; and third, a simplified approach makes the sensitivity of our results to uncertainties more transparent.

[18] Even the dissipation model has its simplifications. In particular, it assumes that the dissipative behavior (but not the density/rigidity structure) of the Martian mantle can be adequately represented by the experimentally characterized response of dry, melt-free Fo90 olivine [Jackson and Faul, 2010]. In detail, this assumption is unlikely to be correct. The bulk of the Martian mantle is most likely olivine but of a more iron-rich kind than typical terrestrial values [Dreibus and Wanke, 1985]. The water content and extent of melting within the Martian mantle are not well known, but both can have significant effects on dissipation. The lower Martian mantle hosts higher-pressure phases (wadsleyite, ringwoodite, and possibly perovskite) [Longhi et al., 1992] which are treated in a simplified manner below.

[19] At one level, our decision to model the dissipative component of the Martian mantle using results from Fo90 is a purely pragmatic decision: other minerals and compositions have not been sufficiently well characterized to allow their rheological behavior to be described. However, it also represents a reasonable initial approach: dissipation is likely to be dominated by the most abundant mineral present, and a dry, melt-free mantle represents a useful end-member. We discuss the potential effects of melt and water further below (section 5.1). The simplified treatment of phase changes can be justified a posteriori as our best fit models result in most dissipation happening in the upper (low-pressure) part of the mantle (see Figure 1). The role of iron in controlling olivine dissipation is currently poorly understood but may be quite dramatic [Zhao et al., 2009]; thus, the approach adopted here probably underestimates the dissipation for a given set of conditions.

3.1. Bulk Structure

[20] For our simplified Martian structure, we will assume a five-layer model consisting of core, a mantle split into olivine (\(\text{ol}\)), wadsleyite (\(\text{wd}\)), and ringwoodite (\(\text{ri}\)) phase assemblages, and a crust. For the purposes of calculating the gravity and pressure within Mars, we will assume that the mantle and core have constant densities \(\rho_m\) and \(\rho_c\), respectively. For a specified value of \(\rho_c\) we calculate the core radius and mantle density which satisfy the measured bulk density and moment of inertia of Mars, taking the crust into account (see Table 3). This approach is evidently not self-consistent, but the errors introduced are much smaller than other uncertainties (see below).

[21] Taking the core radius to be \(R_c\), the gravitational acceleration within the mantle is given by

\[
g(r) = \frac{4}{3} \pi G_g \left[ (\rho_c - \rho_m) \frac{R_c^3}{r^2} + r \rho_m \right]
\]

where \(G_g\) is the gravitational constant, and here we are neglecting the small density contrast between crust and mantle. In the mantle the pressure increment \(dP\) is given by

\[
dP = -\rho_m g dr
\]

which yields

\[
P(r) = \frac{4}{3} \pi G_g \rho_m \left[ R_c^2 \Delta \rho \left( \frac{1}{r^2} - \frac{1}{R^2} \right) + \frac{\rho_m}{2} \left( R^2 - r^2 \right) \right]
\]

where \(R\) is the radius of Mars and \(\Delta \rho = \rho_c - \rho_m\).

[22] In reality, the mantle density \(\rho\) will vary with depth because of compressibility, thermal expansion, and phase changes. For a particular phase assemblage \(x\), we calculate \(\rho_x\) as follows:

\[
\rho_x(r) = \rho_{0x} \left( 1 + \frac{P(r)}{K_x(r)} \right) (1 - \alpha_x [T(r) - T_0])
\]

where \(K_x\) is the bulk modulus, \(T\) is the temperature, \(T_0\) is a reference temperature, and \(\alpha_x\) is the thermal expansivity (assumed constant). The quantity \(\rho_{0x}\) is a reference density. In what follows, we vary the reference density \(\rho_0\) of the olivine phase so that the bulk density of the mantle equals \(\rho_m\). The other two reference densities are kept fixed. The bulk modulus is calculated as described in section 3.2.

[23] While this approach is not fully self-consistent, the errors introduced are small. The difference between the analytical values for \(g(r)\) (equation (2)) and those obtained by numerically integrating the calculated density profile (equation (4)) never exceeds 3%. Similarly, the difference between the analytical value for \(P(r)\) (equation (3)) and that obtained by numerical integration never exceeds 4%. Given other major uncertainties (notably the location of the core-mantle boundary), such errors are acceptable.

[24] The temperature structure of the Martian mantle likely includes a conductive stagnant lid overlying a convecting adiabatic mantle [e.g., Ogawa and Yanagisawa, 2011]. A simple parameterization approximating this behavior may be obtained as follows. The horizontally averaged potential temperature \(T_p\) is given by

\[
T_p(z) = T_s + (T_m - T_s) \tanh(z/L)
\]
Figure 1. (a) Temperature as a function of depth. Bold red line is our preferred model with $L = 125$ km and $T_m = 1600$ K (equation (5)). Thin black lines are present-day temperature profiles from different convection runs of Ogawa and Yanagisawa [2011]; note that their computational domain is only 1000 km deep. Thick green line is the peridotite solidus of Hirschmann [2000]. Dotted line is the petrological geotherm from Bertka and Fei [1997]. (b) Density as a function of pressure. Bold red line is our preferred model (equation (4)). Circles are replotted from Khan and Connolly [2008, Figure 2] and show calculated mantle densities based on experiments [Bertka and Fei, 1998]. (c) Model unrelaxed rigidity modulus $G_U$ (equation (9)) and $S$ wave velocity ($V_s = \sqrt{G/\rho}$) at 1 Hz as a function of depth. Thin black lines are two velocity models from Mocquet and Menvielle [2000]. (d) Model $Q$ as a function of depth, evaluated at 25$\mu$Hz (Phobos’ synodic period) and at 1 Hz.
[25] Here $T_s$ is the surface temperature, $T_m$ is the potential temperature of the convecting interior, and depth $z = R - r$. The near-surface heat flux in this model is $k(T_m - T_s)/L$, where $k$ is the thermal conductivity. The quantity $L$ is an indication of how thick the conductive lid is; roughly 90% of the total temperature drop occurs across a thickness of $1.5L$, so we may take the stagnant lid thickness to be $1.5L$. Because most materials do not undergo significant creep below about 70% of their melting point [Frost and Ashby, 1982], the effective elastic thickness of the lithosphere is approximated by $L$.

[26] To convert from potential temperature to actual temperature $T(z)$, we multiply $T_s(z)$ by an adiabatic factor $f_{ad}$. The adiabatic temperature gradient $dT/dz = \alpha g T/C_p$, where $\alpha$ and $C_p$ are thermal expansivity and specific heat capacity, respectively. Taking these two quantities to be constant and using equation (2) for $f_{ad}$, we find

$$\ln f_{ad} = \frac{4}{3} \frac{G_m \alpha}{C_p} \left[ \frac{R^2}{R} \Delta \rho - \frac{z R}{1 - (z/R)} + \rho_m \left( Rz - \frac{z^3}{2} \right) \right].$$

[27] Equations (5) and (6) can thus be combined to derive the real temperature profile.

[28] Our simplified model does not treat the crust in detail. This is because $Q$, our main focus, will not be significantly affected by a cold, thin surficial layer. However, because the crust may be enhanced in radiogenic elements, it will affect the temperature structure; in particular, the conductive heat flux through the lid derived from equation (5) may be less than the actual surface heat flux.

### 3.2. Dissipation Model

[29] Local dissipation in the Martian mantle is calculated by using a model based on laboratory experiments to determine the complex rigidity as a function of depth. We generally adopt the same approach and parameters as in Nimmo et al. [2012]; only a summary is given below.

[30] Laboratory experiments on melt-free, polycrystalline olivine show that the dissipation factor $Q$ goes as $\omega^2$, where $\omega$ is the forcing angular frequency and $\alpha$ is a constant $\approx 0.3$ [e.g., Gribb and Cooper, 1998; Jackson et al., 2004]. One advantage of investigating Mars rather than the Moon is that the tidal frequencies are higher ($\sim 10^4$ s$^{-1}$ rather than $\sim 10^6$ s$^{-1}$). This means that, unlike the Moon, essentially no extrapolation in frequency is required when applying the laboratory-derived parameters to Mars.

[31] Qualitatively, dissipation appears to arise from two different mechanisms: a background effect, inferred to be due to a diffuse process and modeled as a distribution of relaxation times, and a superimposed peak corresponding to elastically accommodated grain boundary sliding. Both these mechanisms are included. The result is that $Q$ and the rigidity modulus $G$ are dependent on temperature, pressure, and forcing period. The sensitivity of $Q$ to temperature and frequency is much larger than that of $G$; for Mars-sized objects the pressure sensitivity is minor but not negligible. The results are also sensitive to the grain size $d$ adopted; we examine the effects of varying this parameter below.

[32] In detail, we treat the shear modulus as a complex quantity $G^* = 1/J^*$, where $J^* = J_e + iJ_i$ is the complex compliance and $i = \sqrt{-1}$. At a particular frequency the dissipation factor $Q = J_e/J_i$. For the extended Burgers model, the real and imaginary components of the complex compliance are as follows:

$$J_e(\omega) = \frac{1}{G_U} \left[ 1 + \Delta \int_{\tau_L}^{\tau_U} D(\tau) \frac{d\tau}{1 + \omega^2 \tau^2} \right]$$

and

$$J_i(\omega) = \frac{1}{G_U} \left[ \omega \Delta \int_{\tau_L}^{\tau_U} \tau D(\tau) \frac{d\tau}{1 + \omega^2 \tau^2} + \frac{1}{\omega \tau_M} \right]$$

[33] Here $G_U = 1/J_0$ is the unrelaxed (infinite-frequency) shear modulus calculated using equation (9) below, $\tau$ is a dummy variable, $\tau_M = \eta/G_U$ is the Maxwell time for a material of steady state viscosity $\eta$, $\Delta$ describes the strength of the relevant relaxation mechanism, $\omega$ is the angular frequency, and $\tau_L$ and $\tau_U$ are the integration limits corresponding to short and long periods, respectively. In the low-frequency limit, equation (8) reduces to Maxwellian behavior, in which $J_i = (\omega \eta)^{-1}$.

[34] As noted above, laboratory experiments suggest that there are two different relaxation mechanisms operating, with different distributions of relaxation times. We therefore include both a high-temperature background with strength $\Delta_B$ and an additional peak with strength $\Delta_P$. Both distributions are a strong function of grain size and temperature. Further details may be found in Jackson and Faul [2010] and Nimmo et al. [2012]. Unless noted otherwise below, all rheological parameters used are identical to those given in Nimmo et al. [2012, Table 1]; one exception is the reference grain size (13.4 $\mu$m) which was incorrectly stated to be 3.1 $\mu$m in that paper.

[35] The elastic (unrelaxed) rigidity modulus $G_U$ depends on pressure and temperature and is calculated as in Nimmo et al. [2012]:

$$G_U(T, P) = G_U(T_R, P_R) + (T - T_R) \frac{\partial G}{\partial T} + (P - P_R) \frac{\partial G}{\partial P}$$

where $P_R$ and $T_R$ are the reference temperature and pressure, and $\partial G/\partial T$ and $\partial G/\partial P$ are experimentally measured quantities (see below). Note that here we have neglected higher-order pressure derivatives, owing to the relatively modest pressures on Mars. We adopt different values of the reference rigidity $G_U(T_R, P_R)$ and the partial derivatives for the three different phases (see Table 1).

[36] We calculate the (unrelaxed) bulk modulus $K$ in an analogous fashion:

$$K(T, P) = K(T_R, P_R) + (T - T_R) \frac{\partial K}{\partial T} + (P - P_R) \frac{\partial K}{\partial P}$$

The bulk modulus is assumed to not vary significantly with period, but is important in determining the density structure (equation (4)).

[37] The calculations detailed in Nimmo et al. [2012] yield the complex shear modulus $G$ at a specified frequency, where the ratio of the real to the imaginary parts of the modulus contains information on the local value of $Q$. At high frequencies, the mantle response is essentially elastic and $G \approx G_U$. At lower frequencies, dissipation becomes more important and $G < G_U$. To calculate the global, frequency-dependent quantities $Q$ and $k_2$, the method of Roberts and Nimmo [2008] is used. The mantle is discretized into layers 50 km thick, while the core is assumed to be fully
In this section, we discuss the parameterization of the mantle properties. We adopt the same parameters for the olivine-wadsleyite transition as for the olivine assemblage. We note that these values are not identical to those given in Nimmo et al. [2012, Table 1] except the reference grain size (13.4 $\mu$m). “Eqn.” stands for equation.

### 3.3. Parameters Adopted

[39] Except as noted below, the material properties of the mantle are taken from Jackson and Faul [2010, Table 2]. These authors were using F$_{og9}$ olivine; more iron-rich olivine (likely appropriate for Mars) will have slightly different elastic properties [Bass, 1995]. For all three phases we obtain $G_U(T_R,P_R)$ and $K(T_R,P_R)$ and their derivatives from Stixrude and Lithgow-Bertelloni [2005, 2011] for an Mg$_9$O$_7$S$_2$; equations (9) and (10) are then used to determine $G_U$ and $K$ at the conditions of interest. With this approach we are assuming that the higher iron content only affects the elastic moduli but not the grain boundary viscosity or diffusivity of olivine. The latter assumption can be justified by the observation that silicon is the slowest-diffusing species, controlling strain rates for both grain boundary and volume diffusion [e.g., Hirth and Kohlstedt, 2003; Dohmen and Milke, 2010]. On a more pragmatic level, there are currently no experimental data on dissipative behavior in iron-rich olivines.

[40] The three reference densities $\rho_0$ were chosen to approximately satisfy the petrologically derived density curve of Bertka and Fei [1998] (see Figure 1b) while yielding the observed moment of inertia and bulk density. Varying these values has no significant effect on the resulting $Q$ (see section 5.1). The thermal expansivities were likewise chosen to approximately match the more sophisticated calculations of Sohl and Spohn [1997, Figure 4]; likely variations have only minor effect on our results (section 5.1).

[41] Our baseline core parameters were chosen to be consistent with the observed bulk density and moment of inertia. We examine the effect of varying these parameters below (section 5.1) and conclude that, although they do affect the model $k_2$, they have almost no effect on the model $Q$ value. We emphasize again that $Q$ is sensitive to temperature and grain size but not rigidity or density. As a result, more sophisticated and self-consistent models of rigidity and density will not produce appreciably different dissipation results compared with the simple models we have adopted here.

### 4. Results

[42] Figure 1 plots the result of our nominal model, for which $L = 125$ km, $T_m = 1600$ K, and $d = 1$ cm. Figure 1a

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**Table 1. Parameter Values for the Nominal Model**

<table>
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<th>Quantity</th>
<th>Value</th>
<th>Units</th>
<th>Eqn.</th>
<th>Quantity</th>
<th>Value</th>
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<td>-18</td>
<td>M Pa K$^{-1}$</td>
<td>10</td>
</tr>
<tr>
<td>$\rho_w$</td>
<td>3580</td>
<td>kg m$^{-3}$</td>
<td>4</td>
<td>$\rho_s$</td>
<td>3770</td>
<td>kg m$^{-3}$</td>
<td>4</td>
</tr>
<tr>
<td>$G_i(T_0,P_0)$</td>
<td>88.8</td>
<td>GPa</td>
<td>9</td>
<td>$G_i(T_0,P_0)$</td>
<td>102</td>
<td>GPa</td>
<td>9</td>
</tr>
<tr>
<td>$\partial G/\partial T$</td>
<td>-15</td>
<td>M Pa K$^{-1}$</td>
<td>9</td>
<td>$\partial G/\partial P$</td>
<td>1.4</td>
<td>-</td>
<td>9</td>
</tr>
<tr>
<td>$\partial G/\partial P$</td>
<td>1.4</td>
<td>-</td>
<td>9</td>
<td>$\partial G/\partial P$</td>
<td>1.4</td>
<td>-</td>
<td>9</td>
</tr>
<tr>
<td>$\alpha$</td>
<td>$2 \times 10^{-5}$</td>
<td>K$^{-1}$</td>
<td>6</td>
<td>$\alpha$</td>
<td>$2 \times 10^{-5}$</td>
<td>K$^{-1}$</td>
<td>6</td>
</tr>
<tr>
<td>$K(T_R,P_R)$</td>
<td>159</td>
<td>GPa</td>
<td>10</td>
<td>$K(T_R,P_R)$</td>
<td>167</td>
<td>GPa</td>
<td>10</td>
</tr>
<tr>
<td>$\partial K/\partial T$</td>
<td>-12</td>
<td>M Pa K$^{-1}$</td>
<td>10</td>
<td>$\partial K/\partial P$</td>
<td>-29</td>
<td>M Pa K$^{-1}$</td>
<td>10</td>
</tr>
<tr>
<td>$\partial K/\partial P$</td>
<td>4.3</td>
<td>-</td>
<td>10</td>
<td>$\partial K/\partial P$</td>
<td>4.1</td>
<td>-</td>
<td>10</td>
</tr>
</tbody>
</table>

*Other rheological parameters are identical to those given in Nimmo et al. [2012, Table 1] except the reference grain size (13.4 $\mu$m). “Eqn.” stands for equation.
Table 2. Model Results and Comparison With Observations

<table>
<thead>
<tr>
<th>$T_m$</th>
<th>$L$</th>
<th>$d$</th>
<th>$Q$ (Phobos)</th>
<th>$k_2$ (Phobos)</th>
<th>$k_2$ (1 Hz)</th>
<th>$\Delta T_{sol}$ (K)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Models</td>
<td>1600</td>
<td>125</td>
<td>10</td>
<td>89</td>
<td>0.141</td>
<td>0.136</td>
</tr>
<tr>
<td></td>
<td>1600</td>
<td>150</td>
<td>10</td>
<td>89</td>
<td>0.141</td>
<td>0.136</td>
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<td></td>
<td>1590</td>
<td>100</td>
<td>10</td>
<td>89</td>
<td>0.141</td>
<td>0.136</td>
</tr>
<tr>
<td></td>
<td>1610</td>
<td>200</td>
<td>10</td>
<td>96</td>
<td>0.140</td>
<td>0.136</td>
</tr>
<tr>
<td></td>
<td>1450</td>
<td>150</td>
<td>1</td>
<td>98</td>
<td>0.138</td>
<td>0.132</td>
</tr>
<tr>
<td>Observed</td>
<td>-</td>
<td>-</td>
<td>-</td>
<td>88 \pm 16</td>
<td>0.148 \pm 0.017</td>
<td>-</td>
</tr>
</tbody>
</table>

*Because $Q$ and $k_2$ are frequency dependent, we specify the period at which they are evaluated. $\Delta T_{sol}$ gives the smallest difference between the mantle temperature and the solidus temperature and quantifies how easy it is to generate melt. The final row gives the observational constraints, as described in section 2.*
dissipation (decreases $Q$), as expected, while a thicker lid reduces dissipation. Similarly, in Figure 2b a thicker lid reduces $k_2$ (because an elastic lid reduces the amplitude of tidal deformation), while increasing $T_m$ increases $k_2$ (because a warmer planet is more deformable).

The red lines in Figure 2a show the estimated range of $Q$ for Mars, and the white star denotes our nominal model (Figure 1). There is evidently a trade-off between $L$ and $T_m$: a thicker lid requires a hotter mantle to match the observed value (see Table 2). However, the sensitivity to lid thickness is not very great: for a constant $Q$, increasing $L$ by 100 km requires an increase in $T_m$ by only 40 K.

The red line in Figure 2b shows the lower bound on $k_2$. Compared to $Q$, $k_2$ varies rather little with temperature and thus does not provide a strong constraint on $T_m$. The cross-hatched region in Figure 2b shows the parameter space in which $Q$ is satisfied. For $L > 100$ km, the mantle potential temperature $T_m$ has to exceed 1550 K. To further constrain $L$ and $T_m$ requires other arguments, presented in the following section.

Figure 3 is identical to Figure 2 but shows what happens for a smaller grain size ($d = 1$ mm). Smaller grains result in lower $Q$ and lower rigidities; as a result, matching the observations requires correspondingly lower mantle temperatures. As before, the cross-hatched region in Figure 3b shows the parameter space in which $Q$ is satisfied, while $k_2$ does not provide a tight constraint. Depending on the value of $L$ adopted, temperatures could be anywhere in the range 1420–1650 K. We will argue below that values of $L$ in the range 100–200 km are more likely, in which case temperatures cannot exceed 1525 K. This result is reminiscent of the suggestion made by Lognonne and Mosser [1993] that the bulk $Q$ of Mars can be explained if its mantle is colder than that of the Earth.
4.2. Degeneracies

Figures 2 and 3 demonstrate that different combinations of \( T_m, L, \) and \( d \) can satisfy the observational constraints. Here we discuss how these degeneracies can be reduced.

The main additional constraint is to consider the importance of melting. Hartmann et al. [1999] used crater counts to deduce volcanism happening within the last 1–2% of Mars’ history, presumably as a result of plume activity (see also Vaucher et al. [2009]). For such volcanism to occur, the mantle temperature must exceed the solidus, at least locally. As shown in Figure 1, this is most likely to occur close to the base of the lid. Figures 2a and 3a contour the smallest temperature difference between the mean mantle temperature and the solidus temperature, \( \Delta T_{\text{sol}} \). For instance, for our nominal model the smallest difference is \( \Delta T_{\text{sol}} = 199 \text{ K} \). Evidently, smaller lid thicknesses make it easier to simultaneously satisfy the requirements of \( Q, k_2, \) and melt generation. Furthermore, comparison of Figures 2a and 3a show that it is harder to satisfy all three requirements with a small grain size, because the lower mantle temperatures make it difficult to generate melt.

In a convecting mantle, upwellings will be hotter than the average mantle temperature and the local stagnant lid thickness will be reduced [e.g., Solomatov and Moresi, 1996]. Full calculations of convective melt production are beyond the scope of this work, but we can provide some rough constraints as follows. Kiefer [2003] states that a lid thickness of about 270 km (\( L \approx 180 \text{ km} \) in our terms) produces no melt in a model with a potential temperature of 1804 K. Although much hotter than our models, this produces no melt in a model with a potential temperature of 1804 K. Although much hotter than our models, this result suggests that \( \Delta T_{\text{sol}} \approx 150 – 200 \text{ K} \) is enough to stop melting. With a mantle potential temperature \( T_m = 1600 \text{ K}, \Delta T_{\text{sol}} = 199 \text{ K} \) for \( L = 125 \text{ km} \) and 140 K for \( L = 100 \text{ km} \) (Figures 2,3), suggesting that localized melt production should be possible in these cases. Furthermore, inspection of Figure 1a shows the close similarity between our nominal temperature profile (\( L = 125 \text{ km} \)) and the profile from model CM7H of Ogawa and Yanagisawa [2011], for which melting occurs at the present day.

Evidently, melting depends sensitively on the value of \( L \) adopted. McGovern et al. [2004] estimated heat flux upper bounds of 24 and 28 mW m\(^{-2}\) for two Amazonian-age volcanic features. Figure 2b shows that such values imply \( L \gtrsim 150 \text{ km} \), in rough agreement with the bounds discussed above. Conversely, the Phillips et al. [2008] value of \( L = 300 \text{ km} \) estimated at the north pole of Mars would result in no melting at all and is presumably reflective of local rather than global conditions [Kiefer and Li, 2009; Grott and Breuer, 2010].

Overall, values of \( L \) in the range 100–200 km seem most likely. Under this assumption, the \( d = 1 \text{ cm} \) cases (Figure 2) can simultaneously satisfy the requirements of \( k_2, Q, \) and melt production. For \( L \) in the range 100–200 km, the range of \( T_m \) within which \( Q \) is satisfied is roughly 1550–1700 K. Conversely, for \( d = 1 \text{ mm} \) the temperatures required to satisfy the tidal (\( Q, k_2 \)) constraints are so low that simultaneously generating melt is very problematic.

In summary, the tidal constraints (\( k_2 \) and \( Q \)) permit some pairs of \( T_m \) and \( L \) values. The requirement for local melt generation permits other pairs of \( T_m \) and \( L \). Given a rough constraint on the lithospheric structure (\( L \approx 100–200 \text{ km} \)), no \( d = 1 \text{ mm} \) grain size case is capable of satisfying both the tidal and melt-generation constraints (the mantle is too cold). With a \( d = 1 \text{ cm} \) grain size and \( L = 100–200 \text{ km} \), we estimate that \( T_m = 1625 \pm 75 \text{ K} \).

5. Discussion

5.1. Robustness

Our model involved both simplifications and the choice of poorly known parameter values. It is therefore important to explore the extent to which these factors are likely to affect our results.

One important uncertainty is in the bulk structure of Mars adopted. In Table 3 we list a range of different structures, all of which satisfy the moment of inertia and bulk density constraints, but with different core radii and core and mantle mean densities. Calculations of pressure, rigidity, etc. are carried out exactly as in section 3, with similar thermal parameters as our nominal model (\( T_m = 1600 \text{ K}, L = 150 \text{ km}, \text{ and } d = 1 \text{ cm} \)). As expected, the Love number shows systematic changes as the bulk density structure is changed. However, of much more interest is the fact that \( Q \) is almost unchanged. This is because \( Q \) is mainly controlled by what is happening immediately beneath the lithosphere (Figure 1d), and thus the bulk structure adopted does not greatly affect it. This result is important, because it means that uncertainties in the bulk structure of Mars will not greatly affect our conclusions. In other words, \( Q \) depends a lot on \( T_m \) and \( d \) but rather little on the rigidity and density structure (which are the primary controls on \( k_2 \)). Thus, to determine the temperature structure of Mars, \( Q \) is the most important factor to model.

Although our results are insensitive to the bulk structure, they are sensitive to factors which affect temperature, such as thermal expansivity. For instance, increasing \( \alpha \) of olivine by 50% causes a change in the near-surface temperature structure (equation (6)) and results in a 10% reduction in \( Q \).

As discussed above, the fact that tidal \( Q \) is lowest just beneath the lid means that our simplified treatment of phase changes at depth is unlikely to affect our conclusions. However, relaxing the assumption of anhydrous, melt-free conditions may have more significant consequences. The main effect of both melt and water is to significantly reduce the effective viscosity and thus to enhance dissipation. The mantle of Mars can still apparently generate melt, but the sparse and localized nature of recent volcanic activity makes it somewhat unlikely that the global \( Q \) value will be significantly affected.

Table 3. Sensitivity of Results to Bulk Structure Assumed

<table>
<thead>
<tr>
<th>( \rho_m )</th>
<th>6700</th>
<th>7000</th>
<th>7300</th>
<th>7600</th>
<th>( \text{kg m}^{-3} )</th>
</tr>
</thead>
<tbody>
<tr>
<td>( R_c )</td>
<td>1766</td>
<td>1700</td>
<td>1643</td>
<td>1593</td>
<td>( \text{km} )</td>
</tr>
<tr>
<td>( \rho_o )</td>
<td>3311</td>
<td>3300</td>
<td>3290</td>
<td>3281</td>
<td>( \text{kg m}^{-3} )</td>
</tr>
<tr>
<td>( k_z )</td>
<td>0.153</td>
<td>0.141</td>
<td>0.131</td>
<td>0.123</td>
<td>-</td>
</tr>
<tr>
<td>( Q )</td>
<td>90</td>
<td>92</td>
<td>93</td>
<td>95</td>
<td>-</td>
</tr>
</tbody>
</table>

*The different combinations of mean mantle and core density (\( \rho_m, \rho_o \)) and core radius \( R_c \) yield the observed bulk density and moment of inertia, assuming a 50 km thick crust with a density of 2900 kg m\(^{-3}\). These different structures yield different \( k_2 \) Love numbers, evaluated at Phobos’ synodic frequency. However, for a given set of thermal parameters (here we take \( L = 150 \text{ km}, T_m = 1600 \text{ K}, \text{ and } d = 1 \text{ cm} \)), \( Q \) scarcely changes.

20.153 0.141 0.131 0.123 -
1766 1700 1643 1593 km
3311 3300 3290 3281 kg m\(^{-3}\)
0.153 0.141 0.131 0.123 -
90 92 93 95 -
implies a present-day mantle potential temperature of 1613 K of 1613 K, implying viscosities of about 10^{19} Pa s [Kiefer, 2011] has a present-day temperature of about 1800 K, implying a viscosity of about 10^{19} Pa s. Similarly, the parameterized evolution models EPT21 and STL21 of Breuer and Spohn [2003] have temperatures in the range 1900–1950 K, implying viscosities of about 2–3 \times 10^{19} Pa s. Kiefer [2003] assumed a core-mantle boundary temperature equivalent to a mantle potential temperature of 1804 K, somewhat above our inferred range. These low viscosities and high temperatures arise in thermal evolution models because stagnant lid convection is relatively inefficient at getting rid of heat, and thus the rate of cooling within Mars has been quite modest. In contrast, the lower temperatures in our models might have arisen from early rapid cooling of the Martian mantle, perhaps due to plate-tectonic-like behavior [Nimmo and Stevenson, 2000] or advection of melt. It will obviously be of future interest to reconcile these different approaches to estimating Martian interior properties.

6. Conclusions and Future Work

The main conclusion of this paper is simple: the observed tidal Q of Mars can be understood using laboratory-derived material parameters. An anhydrous, melt-free Martian mantle with a potential temperature similar to (but a grain size somewhat larger than) that of the Earth’s upper mantle and a moderate-thickness lithosphere is consistent with the available tidal and melt-generation constraints. A wetter mantle might permit lower mantle potential temperatures without violating the tidal or melt-generation constraints.

One obvious shortcoming in our approach is that dissipation models based on Fo_{90} olivine may not be appropriate to the Martian mantle. It will therefore be of interest to carry out experimental studies into how iron content affects dissipation. Likewise, as noted above, this study and petrological studies have deduced lower mantle potential temperatures than some thermal evolution models. An obvious next step would be to take the output from some of

5.2. Implications

We have argued above that the tidal Q of Mars implies a present-day mantle potential temperature of 1625 ± 75 K for a grain size of 1 cm. This potential temperature is almost identical to the estimate by Herzberg et al. [2007] of 1613 ± 60 K for the Earth’s upper mantle. Strikingly, two recent petrological analyses, based respectively on Fe, Si, and Th concentrations [Baratoux et al., 2011] and pyroxene concentrations [Baratoux et al., 2013], have reached exactly the same conclusion. Furthermore, these latter studies imply source depths for recent volcanism of ≈150 km, consistent with our estimates of L discussed above. There are thus two independent lines of evidence supporting an Earth-like mantle potential temperature.

On Earth, mantle grain size is affected by two competing effects [Karato, 1988]. Passage through the spinel-perovskite phase transition results in recrystallization and reduction in grain size, while grain growth is a continuous process with a rate which strongly increases as temperature increases. As a result, the equilibrium grain size is expected to depend on the frequency with which grains are cycled through the phase transition and the temperatures they experience. On Mars, grain growth will proceed as on Earth, but the (likely) absence of a spinel-perovskite phase transition means that this grain size reduction mechanism is unlikely to occur [Demouchy et al., 2011]. Because Martian and terrestrial mantle temperatures appear to be so similar, one would therefore expect mantle grain sizes to be larger on Mars than on Earth. The grain size of the Earth’s upper mantle is probably a few millimeters [e.g., Karato, 1984], less than our inferred Mars value and thus in line with expectations.
these evolution models are consistent with those observed. Ultimately, it is to be hoped that the InSight mission will lay many of the remaining discrepancies to rest.

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References


