

WEEK #6 – LIBRATIONS & CASSINI STATES

Topics/Concepts

Torque, moments of inertia (A, B, C)

Precession, inclination, Cassini state, obliquity, inclination damping

Free vs. forced librations, optical vs. physical librations

Equations

Longitudinal torque on static (fossil) bulge $T = 3 \frac{GM}{a^3} (B - A) \cos\varphi \sin\varphi$

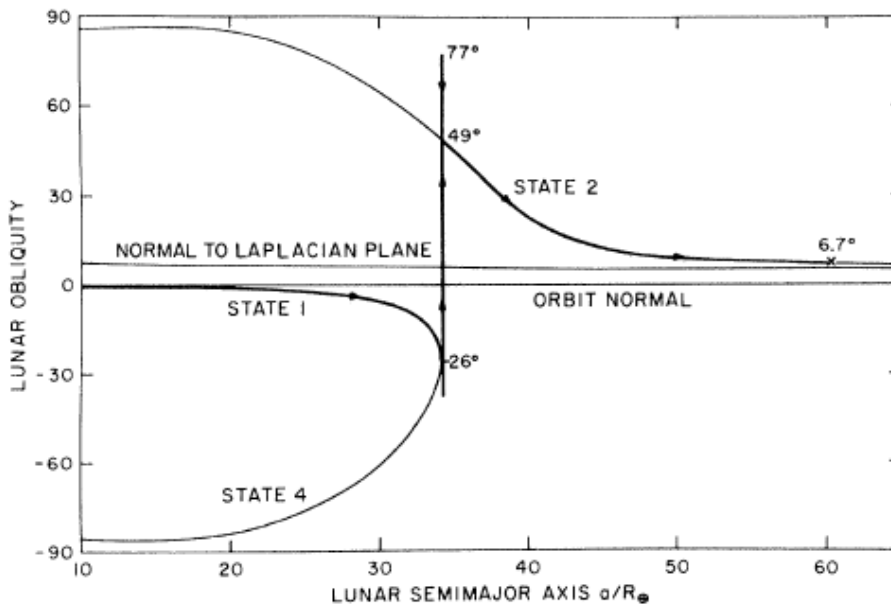
Free libration period $\omega_0 = n \sqrt{\frac{3(B-A)}{C}}$

Vertical component of angular momentum $L_z = m(GM)^{1/2} a^{1/2} \cos i$

Damped Cassini state $(v + (u - v)\cos\theta)\sin\theta = \sin(i - \theta)$

Tidal dissipation (synchronous) $\dot{E} = \frac{3 R^5 \eta^5 k_2}{2 G Q} [7e^2 + \sin^2\theta]$

Inclination damping $2 \tan i \frac{di}{dt} = \frac{2a\dot{E}}{GMm}$



References

Chen & Nimmo, Tidal dissipation in the lunar magma ocean and its effect on the early evolution of the Earth-Moon system, *Icarus* 275, 132-142, 2016.

Bills & Nimmo, Forced obliquity and moments of inertia of Titan, *Icarus* 196, 293-297, 2008.

Tiscareno, Thomas, Burns, The rotation of Janus and Epimetheus, *Icarus* 204, 254-261, 2009.

Ward, Past orientation of the lunar spin axis, *Science* 198, 377-379, 1975.