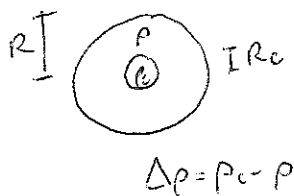


PS#5



$$a) \frac{C}{M R^2} = \frac{0.4 \left(1 + \frac{\Delta p}{\rho} \left(\frac{R}{r}\right)^5\right)}{\left(1 + \frac{\Delta p}{\rho} \left(\frac{R}{r}\right)^3\right)} = \underline{0.308}$$

$$\Delta p = 2.55 \rho = 0.95 \quad \frac{R_c}{r} = 0.758$$

$$b) \text{ Darwin-Radau } \frac{C}{M R^2} = \frac{2}{3} \left[1 - \frac{2}{5} \left(\frac{5}{h_{rs}} - 1\right)^2\right] \quad \therefore \frac{5}{h_{rs}} - 1 = \left(\frac{5}{2}\right)^2 \left[1 - \frac{3}{2} \frac{C}{M R^2}\right]^2$$

$$\therefore h_{rs} = \underline{1.78} \quad h_{rs} = h_{rs} - 1 = \underline{0.78}$$

$$c) J_2 = \frac{5}{6} q h_2 \text{ (synchronous)} \quad q = \frac{R^3 \omega^2}{G m_s} = \frac{3 \omega^2}{G 4 \pi \rho h} = 2.66 \times 10^{-4}$$

$$\omega = 1.237 \times 10^{-5} \text{ s}^{-1}$$

$$\therefore J_2 = \underline{1.73 \times 10^{-4}} \quad C_{22} = \frac{3}{10} J_2 = \underline{5.18 \times 10^{-5}}$$

$$a-c = 2 q h_2 R = \underline{1.28 \text{ km}}$$

$$d) 3[(J_2 + C_{22}) \omega \sin \theta_0 + C_{22}] \rho \sin \theta_0 = 2 c' \sin(\frac{1}{2} - \theta_0) \quad \begin{matrix} p = -4.27 \times 10^9 \\ c' = 156.87^\circ \end{matrix}$$

$$\text{say } \theta_0 = \underline{-0.39^\circ} \quad \left. \begin{array}{l} \text{LHS} = 0.241 \quad \text{RHS} = 0.238 \\ \theta_0 = \underline{-0.38^\circ} \quad \text{LHS} = 0.235 \quad \text{RHS} = 0.238 \end{array} \right\} \text{ roughly } \theta_0 = \underline{-0.385^\circ}$$

$$e) \text{ if } \theta_0 = -0.60^\circ, \text{ infer } c' = 0.484$$

This is larger than expected for a uniform body (!) and suggests that the obliquity is large because the shell is decoupled from the interior e.g. by an ocean.

$$f) \text{ free vibrations } \omega_0 = n \sqrt{\frac{3(B-A)}{C}} = n \sqrt{\frac{3(B-A)}{M R^2} \frac{M R^2}{C}} = n \sqrt{\frac{12 C_{22}}{C'}}$$

$$\Rightarrow \text{ free vibration period} = \underline{131 \text{ days}}$$

g) Because the eccentricity is so small there are almost no torques acting and so the forced vibration amplitude will be very small