PS#4

1 a) $\frac{da}{dt} = 3\frac{k_2}{Q}\frac{m}{M}\left(\frac{R}{a}\right)^5 \frac{(GM)^{1/2}}{a^{3/2}} a. \quad \therefore \int a^{11/2}\,da = \int 3\frac{k_2}{Q}\frac{m}{M} R^5 (GM)^{1/2}\,dt$

$\therefore \int a^{11/2}\,da = \int c\,dt \quad c = 3\frac{k_2}{Q}\frac{m}{M} R^5 (GM)^{1/2}$

$\therefore \frac{2}{13}a^{13/2} = ct + d \quad t=0 \quad a=a_0 \quad \therefore \frac{2}{13}a^{13/2} = ct + \frac{2}{13}a_0^{13/2}$

$\therefore a^{13/2} - a_0^{13/2} = \frac{13}{2}ct \qquad a_0^{13/2}\left(\left(\frac{a}{a_0}\right)^{13/2} - 1\right) = \frac{13}{2}ct$

b) $c = 3\frac{k_2}{Q}\frac{m}{M} R^5 (GM)^{1/2} = 4.5\times10^{36}. \quad a^{13/2} - a_0^{13/2} = ~~1.12\times10^{42}~~ \quad 5.26\times10^{53}$

$\Rightarrow t = 570$ Myr $\ll$ age of solar system

c) $L = mna^2\sqrt{1-e^2} = m(GM)^{1/2}a^{1/2}\sqrt{1-e^2}$

in absence of external torques $\frac{dL}{dt} = 0 \Rightarrow m(GM)^{1/2}\frac{1}{2}a^{-1/2}(1-e^2)^{1/2}\frac{da}{dt}$

$= -m(GM)^{1/2}a^{1/2}\frac{1}{2}(1-e^2)^{-1/2}\cdot 2e\frac{de}{dt}$

$\Rightarrow \frac{L}{2a}\frac{da}{dt} + Le\frac{de}{dt} = 0. \quad \frac{1}{2a}\frac{da}{dt} = e\frac{de}{dt}.$

$E = -\frac{GMm}{2a} \quad \frac{dE}{dt} = \frac{GMm}{2a^2}\frac{da}{dt}$

torques due to primary $\frac{da}{dt} = 3\frac{k_2}{Q}\frac{m}{M}\frac{R^5}{a^5}na. \qquad n = 7.7\times10^{-5}\,s^{-1}$

" " " satellite $\frac{da}{dt} = \frac{2a^2}{GMm}\dot{E}$

$\Rightarrow 3\frac{k_2}{Q}\frac{m}{M}\frac{R^5}{a^5}na = \frac{2a^2}{GMm}\dot{E} \Rightarrow \dot{E} = 3\frac{k_2}{Q}\frac{m}{M}\frac{R^5}{a^5}na\frac{GMm}{2a^2}$

$\dot{E} = \frac{3}{2}\frac{k_2}{Q}\frac{m^2R^5}{a^6}Gn = 3.1\times10^{10}$ W

$\dot{E} = \frac{21}{2}\frac{n^5R_s^5}{G}\frac{k_2}{Q}e^2 \Rightarrow \frac{k_2}{Q} = 6\times10^{-4}$ doesn't seem totally unreasonable...