PS#3

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a) assume $\sigma = \sigma_0 e^{i\omega t}$ $\quad \dot{\varepsilon} = \dot{\varepsilon}_0 e^{i\omega t}$

Maxwell $\quad \dot{\varepsilon} = \dfrac{\sigma}{\eta} + \dfrac{\dot{\sigma}}{\mu} = \dfrac{\sigma_0 e^{i\omega t}}{\eta} + \dfrac{i\omega\sigma_0 e^{i\omega t}}{\mu}$

$$\dot{\varepsilon}_0 = \sigma_0 \left\{ \frac{1}{\eta} + \frac{i\omega}{\mu} \right\}$$

$$\eta^* = \frac{\sigma_0}{\dot{\varepsilon}_0} = \left\{ \frac{1}{\eta} + \frac{i\omega}{\mu} \right\}^{-1} = \left(\frac{1}{\eta}\right)^{-1} \left\{ 1 + \frac{i\omega\eta}{\mu} \right\}^{-1}$$

$$\eta^* = \frac{\eta}{1 + \frac{i\omega\eta}{\mu}}$$

when $\omega \to 0$ $\eta^* \to \eta$ (ok)

when $\omega \to \infty$ $\eta^* \to -i\dfrac{\mu}{\omega}$

b) $A = \cos\varepsilon \quad \tan\varepsilon = q \quad 1 + \tan^2\varepsilon = 1 + q^2 = \sec^2\varepsilon = \dfrac{1}{\cos^2\varepsilon} = \dfrac{1}{A^2}$

$$\therefore A^2 = \frac{1}{1+q^2}$$

c) low viscosity $q \to 0 \Rightarrow A \to 1$ so if the viscosity is low enough the response saturates (makes sense)

high viscosity $A \approx \dfrac{1}{q} \approx \dfrac{2\rho g R}{19\eta\omega}$ if viscosity is high, $A \to 0$ material is effectively rigid.

d) high frequency limit $q \gg 1$ $A \approx \dfrac{1}{q}$

$$A \approx \frac{2\rho g R}{19\omega\left(-\frac{i\mu}{\omega}\right)} = \frac{2\rho g R}{19\mu} i$$

compare with standard elastic response* $\dfrac{1}{1 + \frac{19\mu}{2\rho g R}} \approx \dfrac{2\rho g R}{19\mu}$ when $\mu \gg \rho g R$.

* here I'm neglecting the 3/2 or 5/2 constant

so in the high-frequency limit a viscous fluid behaves in the same way as an elastic body - it doesn't have time to flow.

The response is complex ($\pi/2$ phase lag) because for an elastic body the stress and strain rate are $\pi/2$ out of phase.