PS#2

a) $\Delta T_{rh} = \gamma^{-1} = \frac{RT_0^2}{Q} = \frac{8\times(270)^2}{50\times10^3} = 12\ \mathrm{K}$

$Ra_{cr} = 20(\gamma\Delta T)^4 \qquad \Delta T = 230\ \mathrm{K}$

$= 2.7\times10^6$

$Ra = \frac{\rho g\alpha\Delta T d^3}{\kappa\eta_b} \quad \therefore \quad d_{crit} = \left(\frac{Ra_{cr}\,\kappa\eta_b}{\rho g\alpha\Delta T}\right)^{\frac{1}{3}} = \underline{30\ \mathrm{km}}$

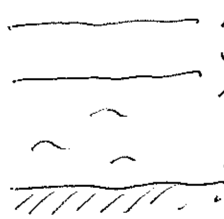
b) $t \sim \frac{d^2}{\kappa}$ (Stefan correction will introduce a factor of order unity).

$= \underline{30\ \mathrm{Myr}}$

c) $F = \frac{k\Delta T}{S_0} \qquad S_0 \approx 2d(Ra^{-1/3})(\gamma\Delta T)^{4/3}$

$F = \frac{k}{2}\left(\frac{\rho g\alpha}{\kappa\eta_b}\right)^{1/3}\gamma^{-4/3} \approx \underline{45\ \mathrm{mW\,m^{-2}}}$

d)



$\Delta t = \frac{\rho L\Delta d}{F} = \underline{52}\ \mathrm{Myr}$

e) $F_b = \frac{k}{2}\left(\frac{\rho g\alpha}{\kappa\eta_b}\right)^{1/3}\gamma^{-4/3} \Rightarrow \eta_b^{1/3} = \frac{k}{2F_b}\left(\frac{\rho g\alpha}{\kappa}\right)^{\frac{1}{3}}\gamma^{-4/3}$

$\Rightarrow \eta_b = \underline{3.3\times10^{17}\ \mathrm{Pa\,s}}$

$\eta = \eta_b \exp\left(\frac{Q}{RT_s} - \frac{Q}{RT_0}\right) \qquad \ln\left(\frac{\eta_b}{\eta_0}\right) = \frac{Q}{RT_b} - \frac{Q}{RT_0} \qquad T_b = \underline{198\ \mathrm{K}}$

(linearized approx. would give 173K).

<u>But</u> $Ra = \frac{\rho g\alpha\Delta T d^3}{\kappa\eta_b} = 7\times10^5.$

$Ra_{cr} = 20\left(\Delta T\cdot\frac{Q}{RT_b^2}\right)^4 \qquad \Delta T = 158\ \mathrm{K} \quad T_b = 198\ \mathrm{K} \quad Ra_{cr} = 7\times10^6$

So this is <u>not</u> convectively unstable.

Alternative is conduction?

$$F = \frac{k\Delta T}{d} \Rightarrow \Delta T = \frac{Fd}{k} = \frac{3\times10^{-3}\times3\times10^{5}}{4} = 225\,K$$

$$\therefore T_b = \underline{265\,K}$$

But at $T_b = 265\,K$ the shell will definitely be convecting.
So we have a paradox!

Most likely resolution is cyclical behaviour: flipping between conductive (and melting) and convective.

f) At 30km thickness $F_{cond} = \frac{k\Delta T}{d} = 31\,mWm^{-2}$

$F_{conv} = 45\,mWm^{-2}$.