EART290Q 2020

PROBLEM SET #5 – due Friday 6th March

One of the proposed *Discovery* missions is to Triton. Triton has a bulk density of 2061 kg/m^3 and a radius of 1353 km. It orbits synchronously with a 5.88 day period and an effectively zero eccentricity.

a) We might model Triton as a two-layer body having an inner silicate region of density 3.5 g/cc and a radius 1026 km, and an outer ice shell of density 0.95 g/cc. What is the dimensionless moment of inertia $c' = C/MR^2$?

b) Assuming that Triton is hydrostatic, determine the *secular* Love numbers k_{2s} and h_{2s}.

c) Also determine J_2, C_{22} and the equatorial flattening (in km).

d) Triton is retrograde, with an inclination of i=156.87^{o} and a ratio of mean motion to orbital precession rate p=-4.27x10^4. The Cassini state constraint can be written

$$3[(J_2+C_{22})\cos\theta_0 + C_{22}]\, p \sin\theta_0 = 2\, c' \sin(i-\theta_0)$$

where θ_0 is the obliquity. What value does θ_0 take to satisfy this equation? (You may have to solve it numerically, and it will have a *negative* value).

e) Let's say we measure an obliquity of -0.6^{o}. What value of c' would you deduce based on your assumed J_2 and C_{22}, and what would you conclude?

f) Calculate the expected period of free libration. You will need to know that $(B-A)/MR^2 = 4\, C_{22}$.

g) In principle we could use an observation of forced libration amplitude to help determine $(B-A)$. Why is this technique unlikely to be useful at Triton?