In class we used conservation of angular momentum to find the outwards motion of a satellite due to dissipation in the primary:

\[ \frac{da}{dt} = 3 \frac{k_{2p}}{Q_p} \frac{m_s}{m_p} \left( \frac{R_p}{a} \right)^5 na \]

where subscripts p and s refer to the primary and satellite, respectively.

a) Integrate this expression to find \( a(t) \) given that \( a = a_0 \) at \( t = t_0 \).

b) For Mimas we have \( a = 3.08 R_p \) at the present day, \( m_s = 3.7 \times 10^{19} \) kg, \( m_p = 5.7 \times 10^{26} \) kg, \( R_p = 60,300 \) km. Astrometric observations give a present-day \( k_2/Q \) of about \( 1.5 \times 10^{-4} \). How long ago was Saturn at the Roche limit, roughly \( 2 R_p \)?

c) One way of slowing down Mimas’s outwards evolution is to appeal to dissipation inside Mimas. What would the present-day dissipation rate inside Mimas need to be to result in a net \( da/dt = 0 \)? What \( k_2/Q \) for Mimas would this imply? Is this reasonable? Mimas’s present eccentricity is 0.02 and radius is 198 km.