

PROBLEM SET #2

Here we'll consider what happens to an icy satellite as its surface ice shell refreezes. To be definite, we'll consider Pluto, with $g=0.6 \text{ ms}^{-2}$, $T_s=40 \text{ K}$ and a total H_2O thickness of 300 km (you can assume Cartesian geometry).

- a) Let's assume that ice viscosity is 10^{14} Pa s at the melting temperature (270 K). How thick would the ice shell have to be before convection initiates? You can take $\alpha=10^{-4} \text{ K}^{-1}$, $Q=50 \text{ kJ/mol}$, $\rho=900 \text{ kgm}^{-3}$ and $\kappa=10^{-6} \text{ m}^2\text{s}^{-1}$.
- b) Roughly how long would it take a conductively-cooling ice shell to reach this critical thickness?
- c) Once the ice shell is convecting, what is the characteristic heat flux? You can take $k=4 \text{ Wm}^{-1}\text{K}^{-1}$.
- d) Roughly how long will it take for the convecting ice shell to freeze completely? You can take $L=300 \text{ kJ/kg}$.
- e) If the long-term supply of heat from the rocky interior is 3 mW/m^2 , what is the long-term equilibrium temperature at the base of the solid ice shell?
- f) Make a sketch of how the shell thickness and heat flux evolve with time.