

PROBLEM SET #1 – due Friday 31st Jan

1. I want you to write a simple 1D, spherical thermal conduction code including exponentially-decaying heat production and make two plots. The first is temperature vs radial position inside an asteroid, at different times: 0.5, 1, 1.5, 2.5, 5 and 10 Myr. The second is to plot the central temperature as a function of time. This second plot should be compared with the analytical solution from Carslaw & Jaeger, Section 9.8:

$$T(t) = \frac{\kappa H_0}{k\lambda} e^{-\lambda t} \left[\frac{a(\frac{\lambda}{\kappa})^{1/2}}{\sin[a(\frac{\lambda}{\kappa})^{1/2}]} - 1 \right] + \frac{2a^2 H_0}{\pi^2 k} \sum_{n=1}^{\infty} \frac{(-1)^n}{(n^2 - \frac{\lambda a^2}{\pi^2 \kappa})} e^{-\frac{\kappa n^2 \pi^2}{a^2} t}$$

The relevant parameters are: $a=30$ km, $\kappa=10^{-6}$ m²s⁻¹, $C_p=10^3$ J kg⁻¹ K⁻¹. The heat production rate is $H_0 e^{-\lambda t}$ where $H_0=6.28 \times 10^{-8}$ Wkg⁻¹ and $\lambda=3.14 \times 10^{-14}$ s⁻¹. Density is 3000 kgm⁻³ and you can take the initial/surface temperature to be 200 K.