1. Fig 4 of Cassen (1996) shows that the modelled surface density of condensed rocky materials reaches a maximum at intermediate semi-major axes, and is lower both closer to the Sun and further away from the Sun. Why?

2. Here we’ll construct a toy model of an evolving nebula.

a) Assume that the surface density $\sigma$ decays linearly with semi-major axis $r$, from $\sigma_0$ at $r=0$ to $\sigma=0$ at the edge of the disk ($r=R_d$). Find an expression for the total mass in the disk, $M_d$.

b) The disk evolves outwards, so that $M_d$ and $R_d$ are functions of time. We have seen that $M_dR_d^{1/2} \sim C$, a constant. Find an expression for how $\sigma_0$ varies with $R_d$.

c) Assume that the scale height $H=\beta r$, where $\beta$ is a constant, and that the density $\rho \sim \sigma/H$. Using the answer given in Week #1, problem 2c, show that the effective disk temperature $T_e(r,R_d)$ is given by

$$T_e \propto \frac{R_d}{R} \left(1 - \frac{1}{R_d} \right)^{2/5}$$

d) With an initial disk mass of 1% solar and initial $R_d=3$ AU, for $\beta=0.03$ I get

$$T_e \approx 1200K \left(\frac{1}{R} - \frac{1}{R_d} \right)^{2/5}$$

when $r$ and $R_d$ are expressed in AU.

At $r=1$AU, what is the temperature when $R_d=3$AU and $R_d=6$AU? Roughly what value of $R_d$ corresponds to the condensation of ice at 1AU ($T_e=200$K)? What is the gas surface density and sound speed at 1AU at this time?

e) (OPTIONAL) Given the surface density $\sigma$ (assumed locally constant) at a given distance $r$, use a Hill sphere approach to show that the isolation mass is $\sim \sigma^{3/2} r^{1/2} M^{1/2}$, where $M$ is the mass of the star. Hence find the radius of an ice planetesimal that forms at 1AU. Assume that ice represents 1% of the gas mass.

f) (OPTIONAL) Assuming that the viscosity $\nu \sim \alpha c^2/n$, where $c$ is the sound speed and $n$ is the mean motion, find the evolution timescale of the disk $r (dr/dt)$ at 1AU when it crosses the snow line. Take $\alpha \sim 10^{-2}$. 