1. a) attenuation factor = \((\frac{c}{a})^n\) \((\text{sphere}) = e^{-kz} (\text{Cartesian})\) \(1\)

\[ z = a - r \quad \left(\frac{c}{a}\right)^n = \left(\frac{a-x}{a}\right)^n = 1 - \frac{x}{a}, \ldots \quad e^{-kz} = 1 - kz + \ldots \]

so \(n = ak\) or \(n = \frac{2\pi}{k}\). \(n\) is how many wavelengths fit onto the circumference of the sphere \(3\)

2. a) \(\int \frac{d\theta}{dt} = K(\theta) \int r^2 \frac{dz}{dt} \quad \text{so} \quad 1 \frac{d\theta}{dt} = \frac{1}{K(\theta)} \int r^2 \frac{dz}{dt} = c.\)

\[ \frac{d\theta}{dt} - K(\theta) = 0 \quad \text{and} \quad \frac{1}{R^2} \int r^2 \frac{dz}{dt} - cR = 0 \] \(2\)

b) \(x = \frac{2\pi}{k} \frac{dz}{dt} = \frac{2\pi}{k} \frac{dz}{dt} \frac{dr}{dx} \quad \frac{dr}{dx} = \frac{\pi}{k} \left(\frac{dr}{dx}\right) \frac{d^2r}{dx^2} = \frac{\pi}{k} \left(\frac{d^2r}{dx^2}\right) \quad \frac{d}{dx} \left(\frac{dr}{dx}\right) \frac{d^2r}{dx^2} = \frac{c^2}{k} \frac{d^2z}{dx^2} \]

\[ \frac{d}{dx} \left(\frac{dr}{dx}\right) \frac{d^2r}{dx^2} = \frac{c^2}{k} \frac{d^2z}{dx^2} \quad \text{so} \quad \frac{d}{dx} \left(\frac{dr}{dx}\right) \frac{d^2r}{dx^2} - cR \frac{dr}{dx} = 0 \]

Let’s assume \(R = b \sin \frac{nx}{2c} \quad \frac{dr}{dx} = b \cos \frac{nx}{2c} \quad \frac{d^2r}{dx^2} = -b \sin \frac{nx}{2c} \)

\[ \frac{d}{dx} \left(\frac{dr}{dx}\right) \frac{d^2r}{dx^2} = b \cos \frac{nx}{2c} - b \sin \frac{nx}{2c} \quad \frac{d}{dx} \left(\frac{dr}{dx}\right) \frac{d^2r}{dx^2} = \frac{c^2}{k} \frac{d^2z}{dx^2} \quad \text{so we need} \quad \frac{c^2}{k} \frac{d^2z}{dx^2} = -c^2 \quad \text{so} \quad c = \frac{n^2 \pi^2}{a^2} \] \(5\)

c) \(\frac{d\theta}{dt} = Ke^{\theta} \quad \Rightarrow \quad \int \frac{d\theta}{e^\theta} = K \int dt \quad \text{so} \quad \theta = \theta_0 e^{\left(\frac{Ke}{\theta_0}\right)} = \theta_0 e^{\left(\frac{K\pi^2}{\theta_0} - \frac{t}{\pi^2}\right)}

we can rewrite this as \(e^{\left(-\frac{t}{\tau}\right)}\) where \(\tau = \frac{a^2}{n^2 \pi^2 K}\)

Here \(\tau\) is the diffusion timescale \(\left(\frac{\sigma^2}{\theta_0}\right)/K\) where \(\left(\frac{\sigma^2}{\theta_0}\right)\) is the effective wavelength of the surface attenuation and \(K\) is diffusion. As expected, short wavelength features attenuate more rapidly with depth.

d) Short wavelength features attenuate more rapidly. So the final tsunami profile will consist of the form \(\sin \frac{t}{\pi c}\), plus a possible \(\sin \frac{t}{\pi c}\) component that doesn’t depend on \(\theta\). Fields will become smoother with time \(12\)