EARTH164: PLANETARY ATMOSPHERES

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Next 2 Weeks – Dynamics

- Mostly focused on *large-scale*, *long-term* patterns of motion in the atmosphere
- What drives them? What do they tell us about conditions within the atmosphere?
- Three main topics:
  - Steady flows (winds)
  - Boundary layers and turbulence
  - Waves
- See Taylor chapter 8
- Wallace & Hobbs, 2006, chapter 7 also useful
- *Many* of my derivations are going to be simplified!

Key Concepts

- Hadley cell, zonal & meridional circulation
- Coriolis effect, Rossby number, deformation radius
- Thermal tides
- **Geostrophic** and cyclostrophic balance, gradient winds
- Thermal winds

$$Ro = \frac{u}{2L\Omega \sin\phi}$$

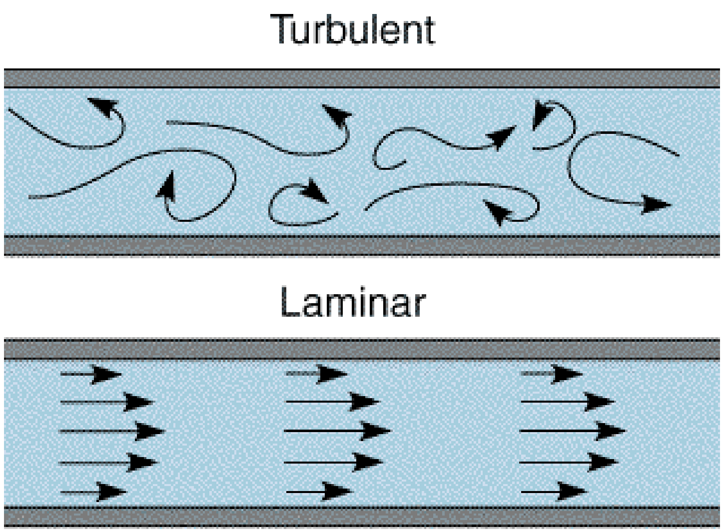
$$\frac{du}{dt} = -\frac{1}{\rho}\frac{\partial P}{\partial x} + 2\Omega \sin\phi v + F_x$$

$$\frac{\partial u}{\partial z} = -\frac{g}{fT}\frac{\partial T}{\partial y}$$

2. Turbulence

Turbulence

- What is it?
- Energy, velocity and lengthscale
- Boundary layers



Whether a flow is turbulent or not depends largely on the viscosity

Kinematic viscosity ν (m^2s^{-1})
Dynamic viscosity η (Pa s)

$$\nu=\eta/\rho$$

Gas dynamic viscosity ~10^{-5} Pa s
Independent of density, but it does depend a bit on *T*

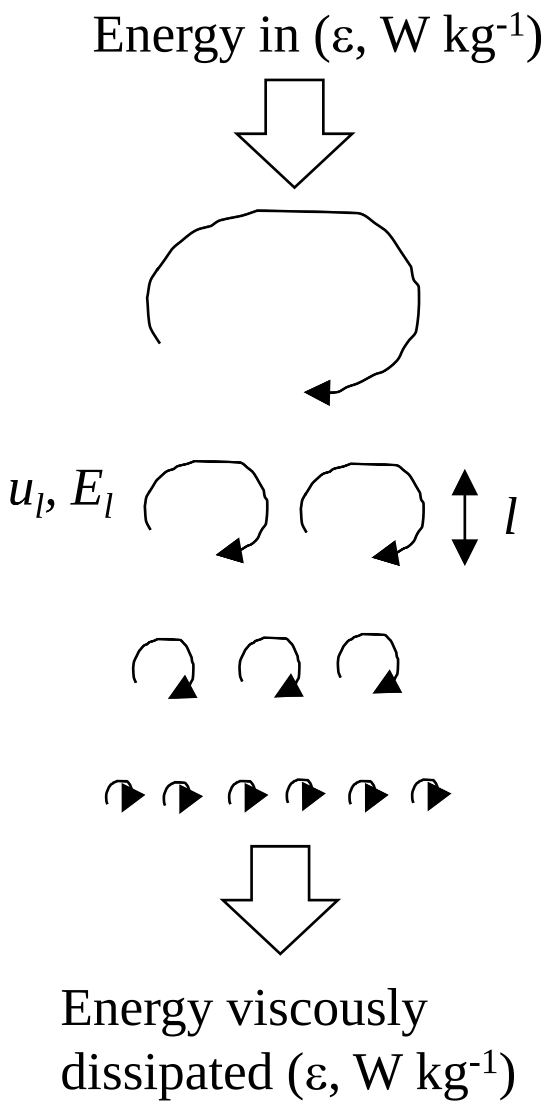
Reynolds number

- To determine whether a flow is turbulent, we calculate the dimensionless Reynolds number

$$\mathrm{Re} = \frac{uL}{\nu}$$

- Here u is a characteristic velocity, L is a characteristic length scale
- For Re in excess of about 10^3, flow is turbulent
- E.g. Earth atmosphere u~1 m/s, L~1 km (boundary layer), $\nu \sim 10^{-5}$ m^2/s so $Re \sim 10^8$ i.e. strongly turbulent

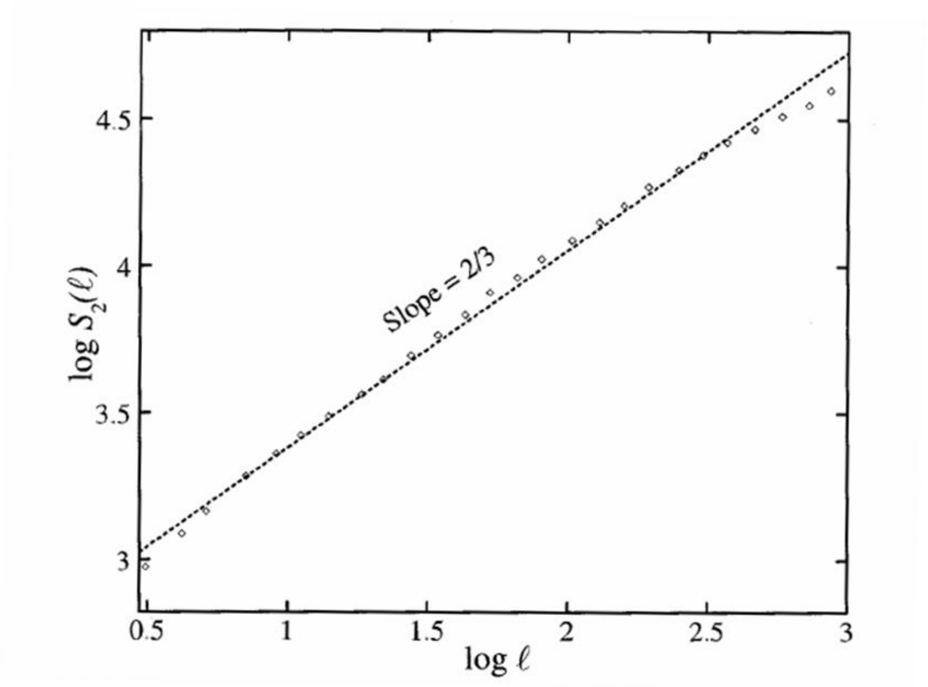
Energy cascade (Kolmogorov)



- *Approximate* analysis (~)
- In steady state, ε is constant
- Turbulent kinetic energy (per kg): $E_l \sim u_l^2$
- Turnover time: $t_l \sim l / u_l$
- Dissipation rate $\varepsilon \sim E_l / t_l$
- So $u_l \sim (\varepsilon\, l)^{1/3}$ (very useful!)
- At what length does viscous dissipation start to matter?

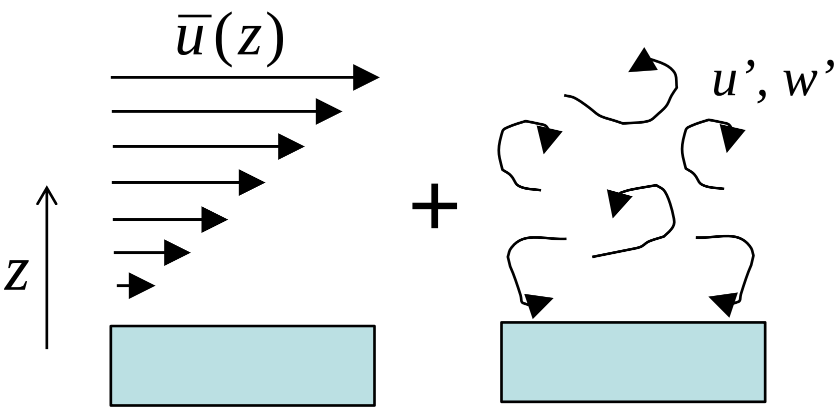
Kinetic energy and lengthscale

- We can rewrite the expression on the previous page to derive $E_l \sim \varepsilon^{2/3} l^{2/3}$
- This prediction agrees with experiments:



Turbulent boundary layer

- We can think of flow near a boundary as consisting of a steady part and a turbulent part superimposed
- Turbulence causes velocity fluctuations *u'~ w'*



•Vertical gradient in steady horizontal velocity is due to vertical momentum transfer

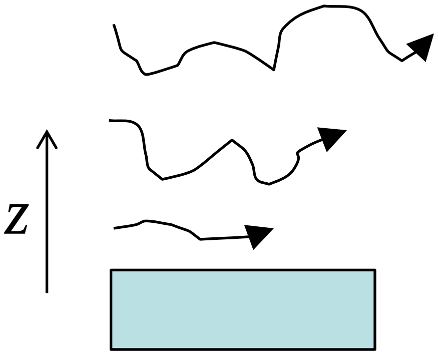
•This momentum transfer is due to some combination of viscous shear and turbulence

•In steady state, the vertical momentum flux is *constant* (on average)

•Away from the boundary, the vertical momentum flux is controlled by *w'*.

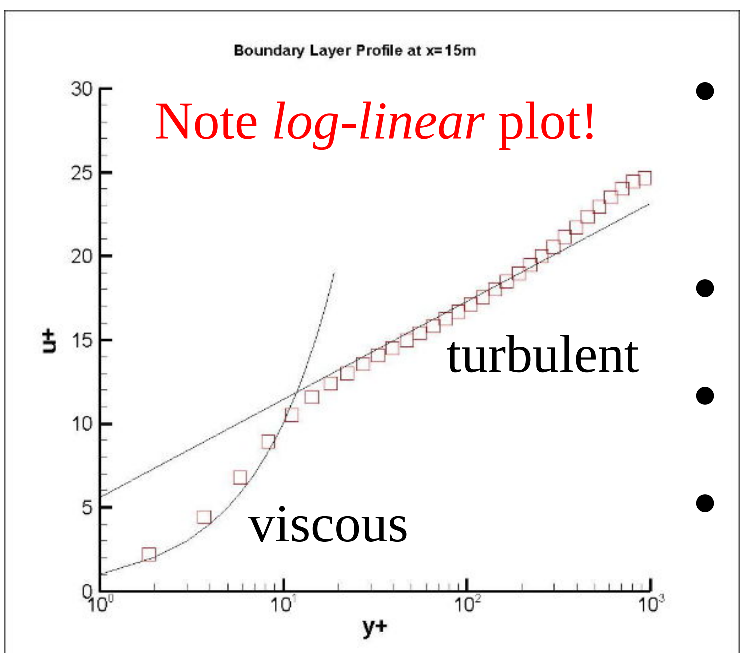
•So *w'* is ~ constant.

Boundary Layer (cont'd)



- A common assumption for turbulence (Prandtl) is that

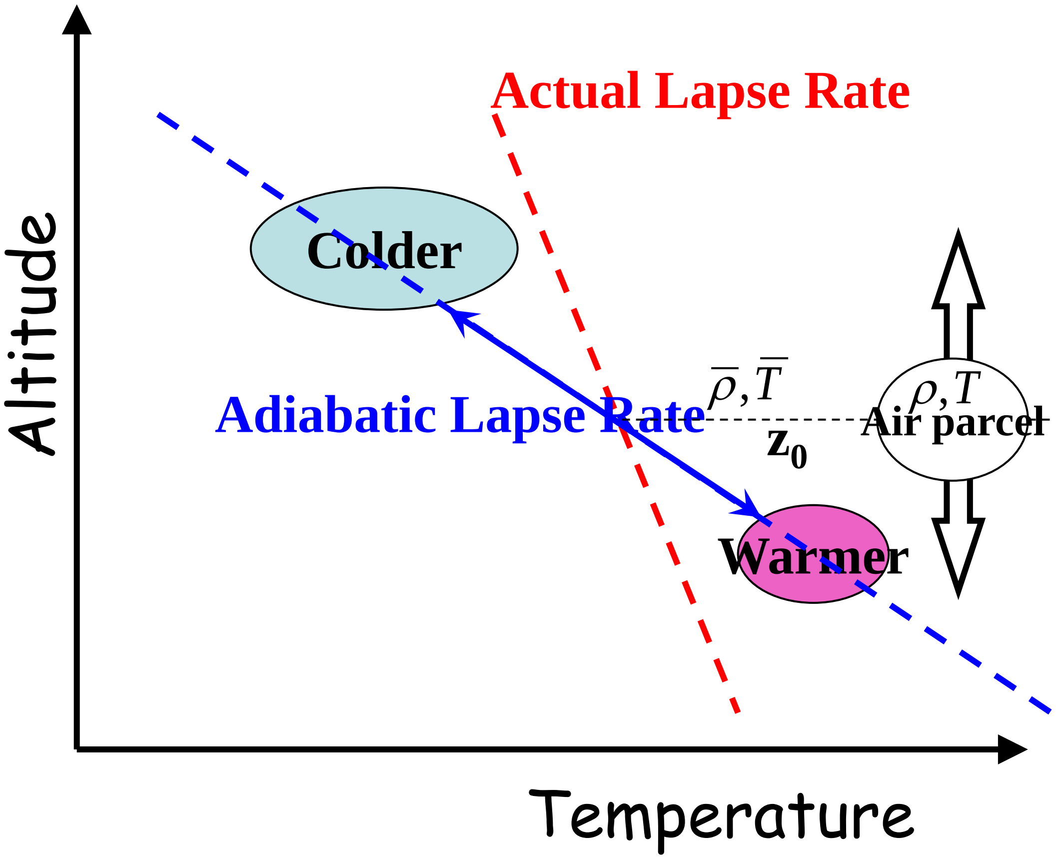
$$w' \sim z \frac{d\bar{u}}{dz}$$



- But we just argued that *w'* was constant (indep. of *z*)
- So we end up with $u \sim \ln z$
- This is observed experimentally
- Note that there are really *two* boundary layers

3. Waves

Atmospheric Oscillations



$$\rho \frac{d^2 z}{dt^2} = -g(\rho - \bar{\rho})$$

$$\frac{\rho - \bar{\rho}}{\rho} \approx \frac{T - \bar{T}}{T}$$

$$\frac{d^2 z}{dt^2} = \frac{g}{T}\left(\left(\frac{dT}{dz}\right) - \left(\frac{dT}{dz}\right)_a\right) z$$

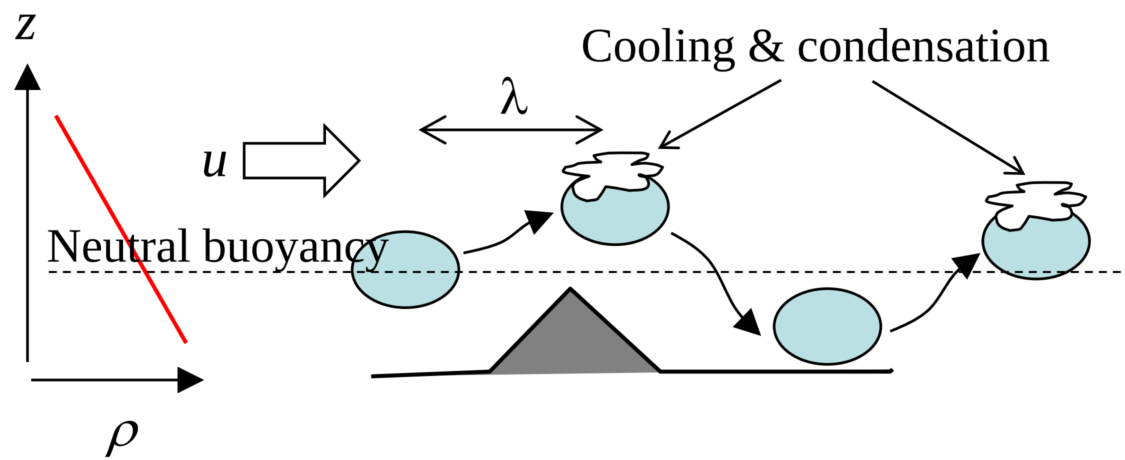
$$\frac{d^2 z}{dt^2} = -\omega_{NB}^2 z$$

$$\omega_{NB}^2 = \frac{g}{T}\left(\left(\frac{dT}{dz}\right) + \frac{g}{C_p}\right)$$

- E.g. Earth $(dT/dz)_a$=-10 K/km, dT/dz=-6K/km (say), T=300 K, ω_{NB}=0.01s^{-1} so period ~10 mins

ω_{NB} is the *Brunt-Vaisala* frequency

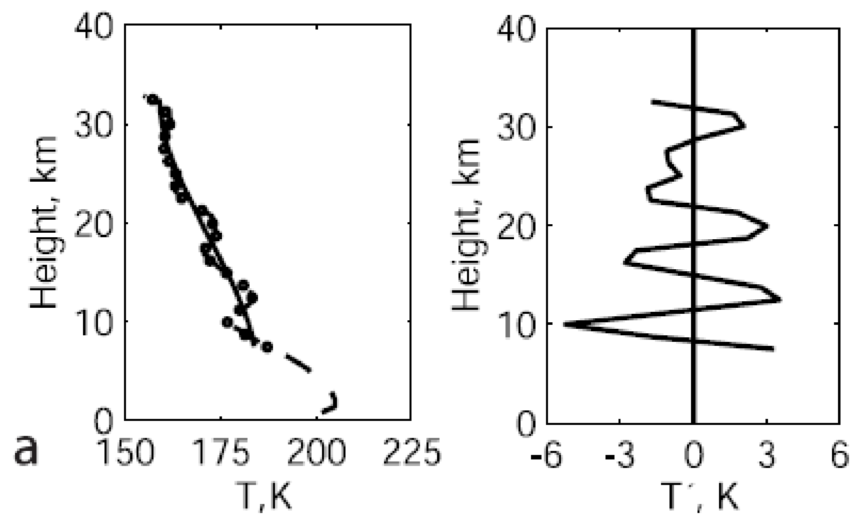
Gravity Waves



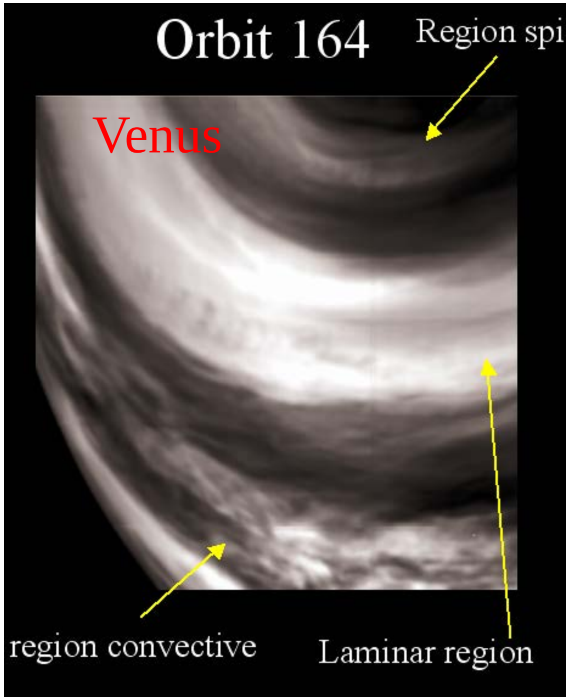
- Common where there’s topography
- Assume that the wavelength is set by the topography
- So the velocity

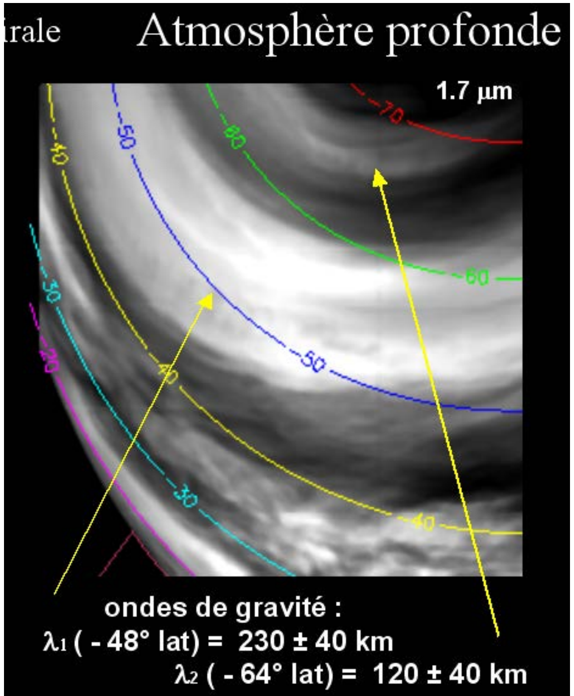
$$u = \frac{\omega_{NB}}{2\pi}\lambda$$

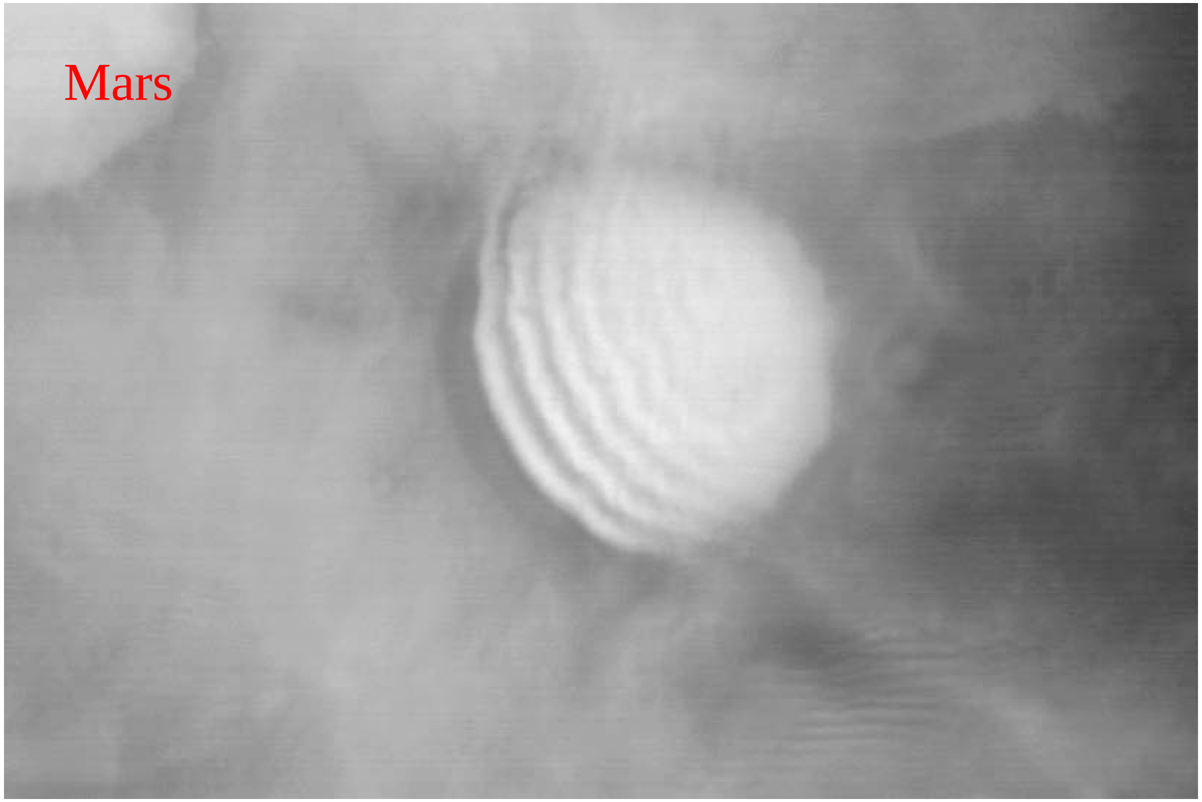
- You also get gravity waves propagating *upwards:*



Gravity Waves



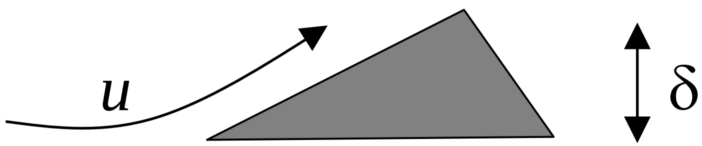


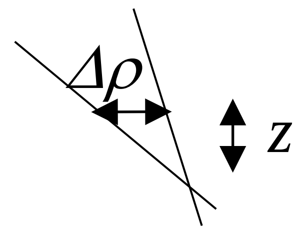


- What is happening here?

Overcoming topography

- What flow speed is needed to propagate over a mountain?





$$\Delta PE = \Delta\rho g\ dz = \frac{\partial\rho}{\partial z} g z\ dz$$

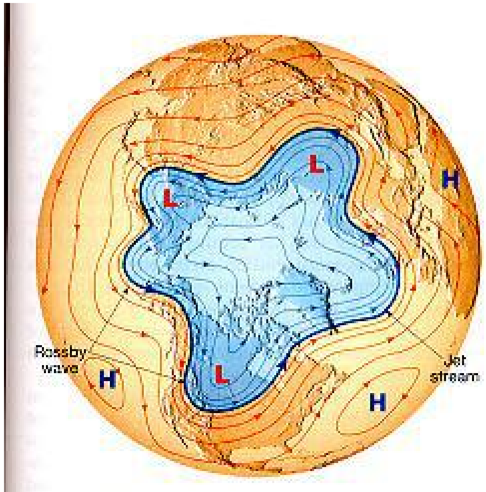
$$KE = \frac{1}{2}\rho u^2$$

$$PE = \frac{1}{2}\delta^2 g \frac{\partial\rho}{\partial z}$$

$$\frac{1}{\rho} d\rho \approx \frac{1}{T} dT$$ (from before)

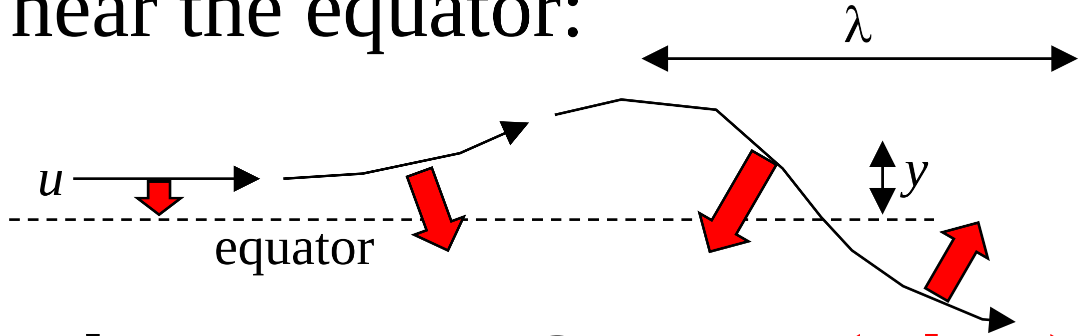
- So we end up with: $u^2 \geq \omega_{NB}^2 \delta^2$
- The Sierras are 5 km high, ω_{NB}~0.01s^{-1}, so wind speeds need to exceed 50 ms^{-1} (110 mph!)

Rossby (Planetary) Waves



Idealized air flow of the westerlies at the 500-millibar level. The five long-wavelength undulations, called *Rossby waves*, compose this flow. The jet stream is the fast core of this wavy flow.

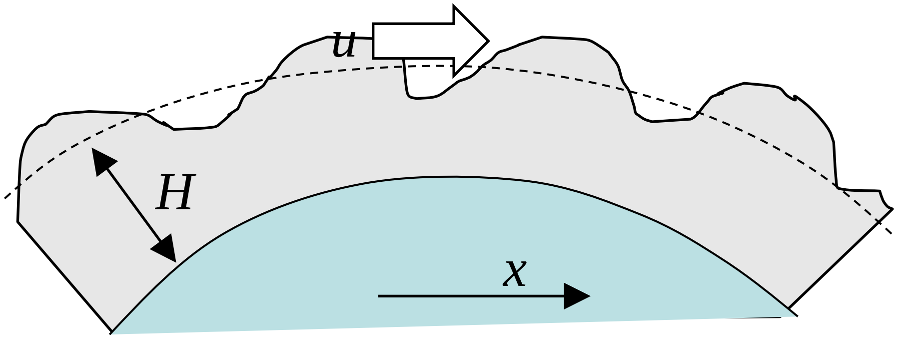
- A result of the Coriolis acceleration $2\underline{\Omega} \times \underline{u}$
- Easiest to see how they work near the equator:

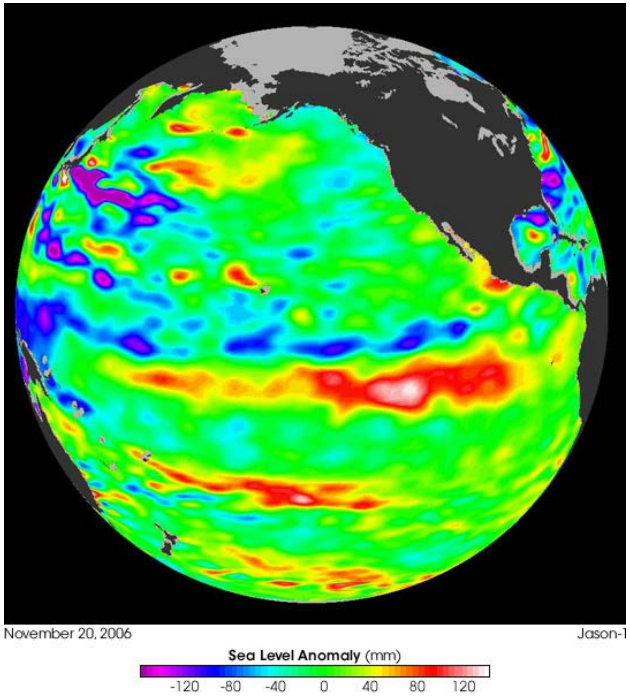


- Magnitude of acceleration ~ $-2\Omega u\, y/R$ (why?)
- So acceleration α – displacement (so what?)
- This implies wavelength $\lambda \sim (uR/\Omega)^{1/2}$
- What happens if the velocity is westwards?

Kelvin Waves

- Gravity waves in zonal direction



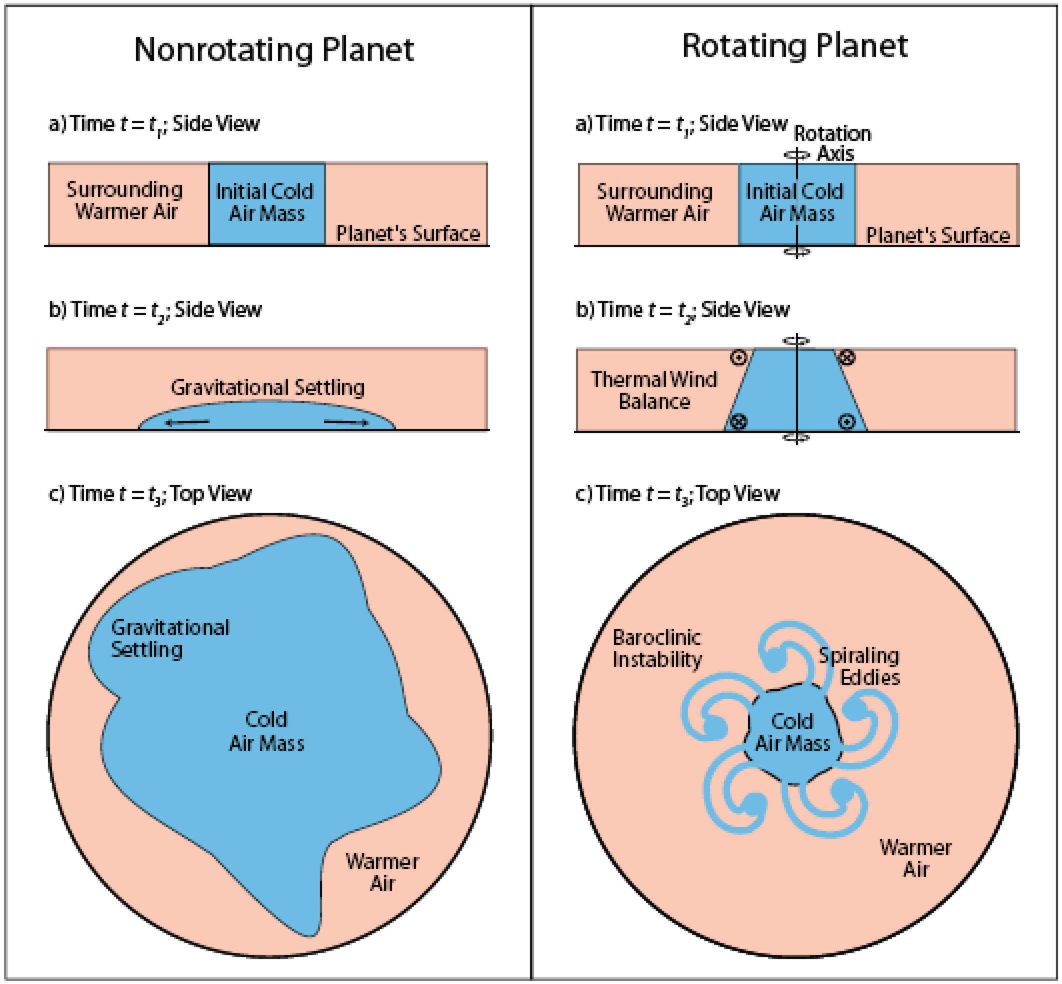


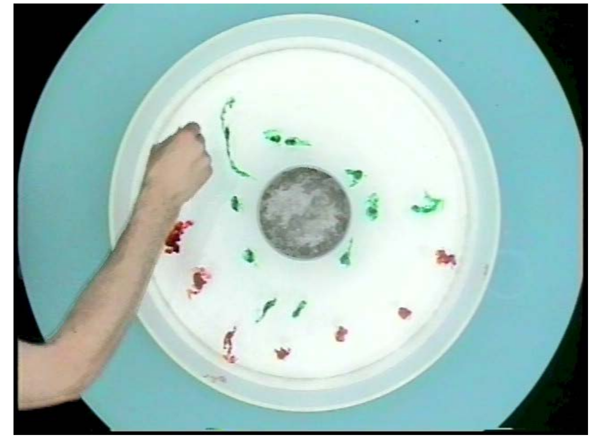
- Let's assume that disturbance propagates a distance L polewards until polewards pressure gradient balances Coriolis acceleration (simpler than Taylor's approach)
- Assuming the relevant velocity is that of the wave, we get

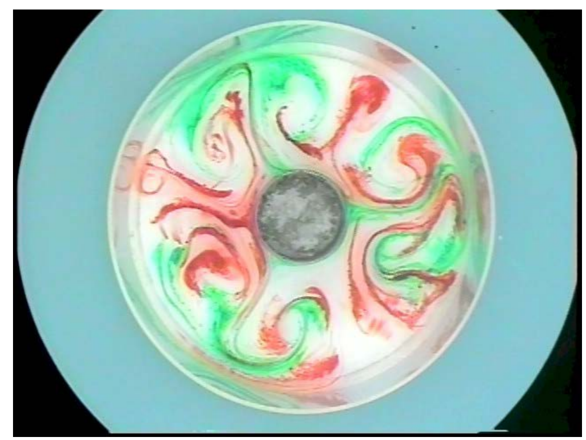
$$L^2 \sim \frac{R}{\Omega}\sqrt{gH} = \frac{R}{\Omega}u$$

(Same as for Rossby λ!)

Baroclinic Eddies



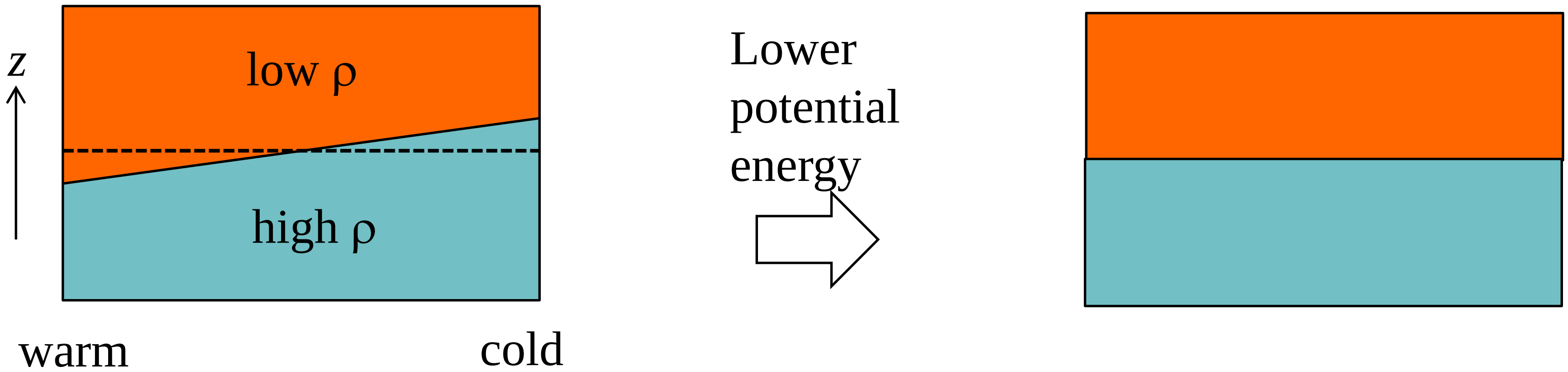




- Important at mid- to high latitudes

Nadiga & Aurnou 2008

Baroclinic Instability



- Horizontal temperature gradients have potential energy associated with them
- The baroclinic instability converts this PE to kinetic energy associated with baroclinic eddies
- The instability occurs for wavelengths $\lambda > \lambda_{crit}$:

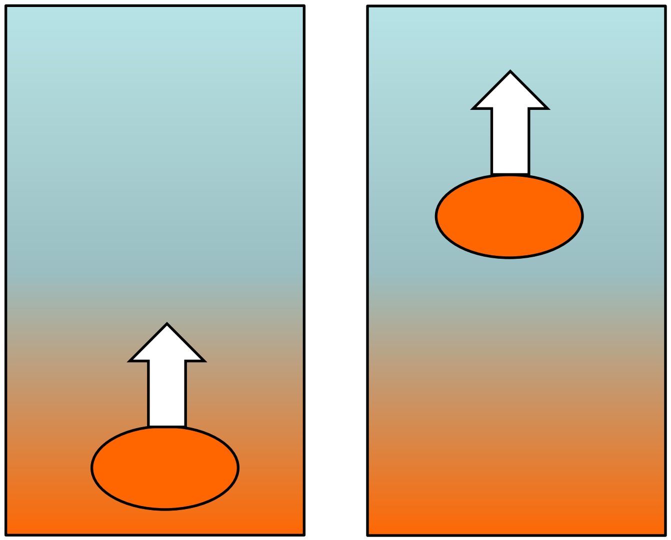
$$\lambda_{crit}^2 \Omega^2 \approx gH \frac{\Delta\rho}{\rho}$$

Where does this come from?
Does it make any sense?

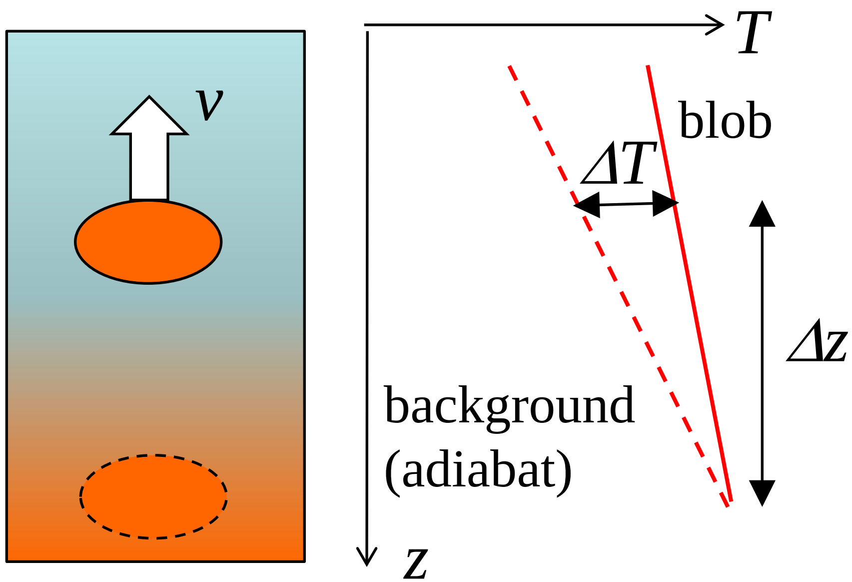
Not obvious why it is omega and not wave frequency

Mixing Length Theory

- We previously calculated the *radiative* heat flux through atmospheres
- It would be nice to calculate the *convective* heat flux
- Doing so properly is difficult, but an approximate theory (called mixing length theory) works OK
- We start by considering a rising packet of gas:



- If the gas doesn't cool as fast as its surroundings, it will continue to rise
- This leads to convection



- So for convection to occur, the temperature gradient must be (*very slightly*) "super-adiabatic"
- Note that this means a less negative gradient!

- The amount of heat per unit volume carried by the blob is given by

$$\Delta E = \rho C_p \Delta T = \rho C_p \left(\left.\frac{dT}{dz}\right|_{ad} - \left|\frac{dT}{dz}\right| \right) \Delta z$$

- Note the similarity to the Brunt-Vaisala formula
- The heat flux is then given by

$$F = \rho C_p \Delta T v = \rho C_p \left(\left.\frac{dT}{dz}\right|_{ad} - \left|\frac{dT}{dz}\right| \right) v \Delta z$$

$$F = \rho C_p \Delta T v = \rho C_p \left(\left. \frac{dT}{dz} \right|_{ad} - \left. \frac{dT}{dz} \right| \right) v \Delta z$$

- So we need the velocity v and length-scale Δz
- Mixing-length theory gives approximate answers:
  - The length-scale $\Delta z \sim H$, with H the scale height
  - The velocity is roughly $v \sim H\omega$, ω is the B-V frequency
- So we end up with:

$$F \sim \rho C_p \left(\left. \frac{dT}{dz} \right|_{ad} - \left. \frac{dT}{dz} \right| \right) H^2 \omega \sim \rho C_p \left(\left. \frac{dT}{dz} \right|_{ad} - \left. \frac{dT}{dz} \right| \right)^{3/2} \left(\frac{g}{T} \right)^{1/2} H^2$$

- Does this equation make sense?
- So we can calculate the convective temperature structure given a heat flux (or vice versa)

Key Concepts

- Reynolds number, turbulent vs. laminar flow
- Velocity fluctuations, Kolmogorov cascade
- Brunt-Vaisala frequency, gravity waves
- Rossby waves, Kelvin waves, baroclinic instability
- Mixing-length theory, convective heat transport

$$\mathrm{Re} = \frac{uL}{\nu} \qquad u_l \sim (\varepsilon\, l)^{1/3} \qquad \omega_{NB}{}^2 = \frac{g}{T}\left(\left(\frac{dT}{dz}\right) + \frac{g}{C_p}\right)$$

$$\lambda \sim (uR/\Omega)^{1/2}$$

$$F \sim \rho C_p \left(\left.\frac{dT}{dz}\right|_{ad} - \left|\frac{dT}{dz}\right|\right)^{3/2} \left(\frac{g}{T}\right)^{1/2} H^2$$