EART164 Homework #6 Due Wednesday 22nd May 2013

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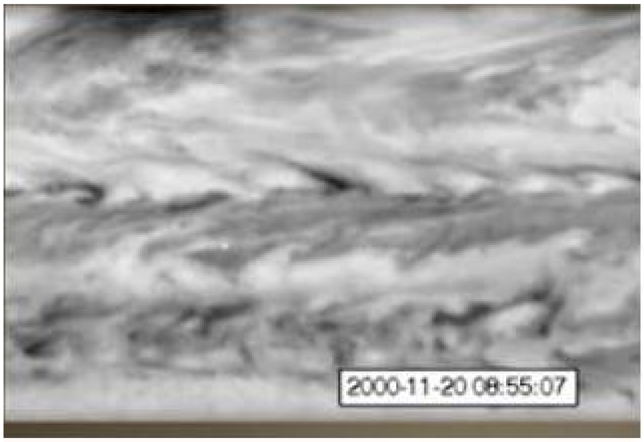

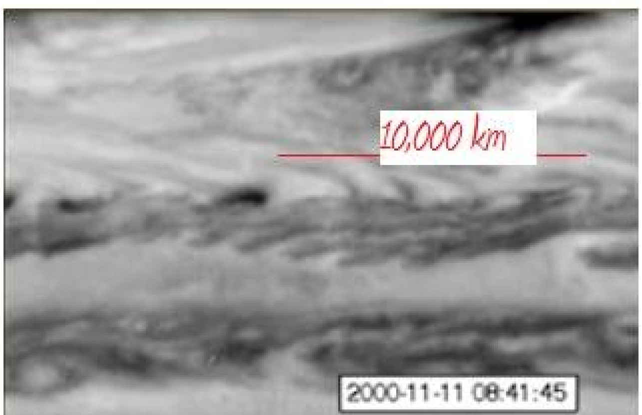

1. The picture above left shows two snapshots of Rossby waves imaged on Jupiter (the video from which these stills are taken is more convincing). These waves have wavelengths of about 10,000km.

a) Jupiter has a rotation period of about 10h and a radius of 72,000 km. What is the wind speed associated with these Rossby waves? [2]

b) The wave period is roughly the separation between the two snapshot (9 days). What does this imply about the propagation velocity of the *waves*? Is it faster or slower than the wind velocity? [2]

c) The sound speed in a gas is given by $(P/\rho)^{1/2}$, where P is pressure and ρ is density. Write down an expression for the sound speed in terms of the scale height H and the acceleration due to gravity g. [2]

d) What is sound speed on Jupiter, taking H=25 km and g=25 ms^{-2}, and how does it compare with the inferred wind velocity? [2]

e) Explain in words why Rossby waves only propagate in an eastwards direction [3] [11 total]

2. The picture above right shows a Martian south polar crater 35 km in diameter in which fog has formed. Winds blowing from the bottom right have caused wave patterns in the fog. Assuming that these waves are gravity waves, we are going to deduce atmospheric structure.

a) Estimate the wavelength λ of the waves [1]

b) We estimate the wind speeds to be 4 m/s. Using the expression from your notes, determine the Brunt-Vaisala frequency for the Martian atmosphere, and hence the local lapse rate (in K/km). You may take g=3.7 ms^{-2}, T=200 K and C_p=800 J/kg K. [3]

c) Suggest two reasons why this lapse rate might be different from the adiabatic one. [2]

d) For the same wind velocity and Brunt-Vaisala frequency, how high a mountain could these winds blow over? [2] [8 total]

3. Here we're going to consider turbulence.

a) The turbulent boundary layer on the Earth is about 1 km high. If atmospheric density (assumed constant) is 1.3 kgm^{-3} and the solar constant is 1300 Wm^{-2}, what is the mean dissipation rate ε (in W/kg) in this boundary layer? (2)

b) Hence estimate the turbulent velocity in this boundary layer. Does your result seem reasonable? (2)

c) Using this characteristic velocity, what is the thickness of the viscous boundary layer? Take the viscosity of air to be 10^{-5} m^{2}s^{-1}. (1)

d) On Mars, the solar constant is smaller (600 Wm^{-2}) and so is the air density (0.01 kg m^{-3}). Assuming the boundary layer height is the same as that on Earth, what is the turbulent velocity on Mars? (2)

e) The velocity required to loft dust is given by (approximately) $v=(\rho_s\, g\, D/\rho)^{1/2}$, where ρ_s and ρ are the density of the dust and air, respectively, g is gravity and D is the dust diameter. Taking g=3.7 ms^{-2} and ρ_s=2 g/cc, what velocity is required to loft 100 micron dust particles on Mars? (1)

f) Comment on your answer to e) in light of your answer to d). (2) [10 total]

4. Here's how we can apply mixing length theory to heat transfer via convection in gas giants.

a) Using the expressions in your notes, show that you can write down an expression for the steady state temperature gradient dT/dz as follows:

$$\frac{dT}{dz} = \frac{g}{C_p} - c^{2/3}\frac{g}{T^{1/3}}$$

where g is gravity, C_p is specific heat capacity and c contains a bunch of other variables. Here I'm taking z positive downwards. [4]

b) How does the temperature gradient change as you go deeper into a convecting atmosphere? [1]

This is a non-linear differential equation, so it's hard to solve analytically. But we can use it to calculate how the actual temperature gradient compares with the adiabatic gradient. For Jupiter, let's take g=25 ms^{-2}, F=5.4 Wm^{-2} (heat generated in the interior), C_p=10^4 J kg^{-1} K^{-1}, T=300 K, μ=0.002 kg mol^{-1} and P=10 bar.

c) What is the adiabatic temperature gradient (in K/km)? [1]

d) How much does the actual temperature gradient (dT/dz) differ from the adiabatic gradient? What does this imply about the efficiency of heat transfer in convecting atmospheres? [4] [10 total]