EART164 Homework #1 Due Wednesday 10^{th} April 2013

Show all your working for full credit

1. Here we'll consider a simple model of the solid components of the protoplanetary nebula.

a) Let the surface density of the nebula (in units of mass/surface area) be σ, where σ is a function of radial distance r. Write down an expression for the increment in mass dm associated with an increment in radial distance dr. [1]

b) A general expression for σ is

$$\sigma(r)=f\,c\,r^{-3/2}$$

where c is a constant and f is a constant that has one value inside the snowline and another beyond it (this is to represent the availability of solid water ice beyond the snowline).

Assuming that f=1 for r<5 AU and f=10 for r>5 AU, find an expression for the total mass contained within the nebula, if the outer edge of the nebula is at 40 AU. [5]

c) Using your answer to b), if the total mass contained within the nebular was 65 M_E (Earth masses), what is the value of c? (You can use AU as your unit of distance and M_E as your unit of mass). [2]

d) Jupiter's solid core has a mass of about 10 M_E. How wide a region was this solid core accreted from, if the inner edge of this region was at the snowline (5AU)? [3]

e) How does the width of this region compare to the distance to Saturn? [1] [12 total]

2. We can use the nebular composition handout table from your notes to assess whether present-day atmospheres are primordial. Below we'll use Titan as an example, assuming that the volatiles are initially accreted as solids.

a) The four volatile species that might be condensed are H, C, N and Ar. Assume that H combines with C to form CH_4 and that any excess H escapes. Write down the percentages (by volume) of CH_4, N_2 and Ar in the resulting atmosphere. [4]

b) The actual percentages in Titan's stratosphere are 98.4% N_2, 1.4% CH_4 and 7ppm Ar. One possible explanation for the discrepancy is that CH_4 is destroyed by photodissociation. If the original ratio of N_2:CH_4 was given by your prediction in a) and no N_2 has been lost, how many times the present CH_4 abundance has been lost? [1]

c) At present-day rates, the current CH_4 in the atmosphere will be destroyed in 50 Myr. Is this consistent with the result you obtained in part b)? If not, can you suggest one possible explanation? [3]

d) Is there more or less Ar in Titan's atmosphere than predicted in part a)? Suggest one possible reason for this discrepancy (Hint: the surface temperature of Titan is about 90 K). [2] [10 total]

3. When deriving the pressure scale height, we normally assume that g is constant. But that may not be a good assumption for bodies with very extended atmospheres.

a) For a body of mass M, write down how g varies with radial distance r [1]

b) The hydrostatic equation may be written $\frac{dP}{dr} = -\rho(r)\, g(r)$. Use this expression together with the ideal gas equation to show that the pressure in an isothermal atmosphere is given by

$$P = P_0 \exp\left(\frac{GM\mu}{RT} \left[\frac{1}{r} - \frac{1}{r_0} \right] \right) \tag{1}$$

where G is the gravitational constant, μ is the molar mass, R is the gas constant, T is the temperature and P_0 is a reference pressure at radial distance r_0. (Here we are neglecting the mass of the atmosphere itself when calculating g.) [6]

c) We can write altitude z as $z=r-r_0$. Show that equation (1) reduces to the usual pressure scale height expression in the limit when $z \ll r_0$. [3]

d) Sketch how pressure varies with altitude according to equation (1), and also sketch the pressure variation using the standard scale height expression (from your notes). What is the physical reason for the divergence of the two curves? [3]

e) What is the most important assumption that makes equation (1) inaccurate for real planetary atmospheres? [1] [14 total]