<u>EART164: Equations You Should Know</u> (in addition to high school physics)

Basics

Ideal gas law $P\mu = \rho R_g T$

Hydrostatic equbm. $dP = -\rho g\, dz$

Isothermal scale height $H = R_g T / g\mu$

Mean free path $\lambda = \frac{m_{mol}}{\pi\rho r_{mol}^2}$

Energy and temperature

Equilibrium temp. $T_{eq} = \left(\frac{S(1-A)}{4\sigma\varepsilon}\right)^{1/4}$

Moist adiabat $\frac{dT}{dz} = -\frac{g}{C_p + L_H \frac{dx}{dT}}$

Adiabat $P = c\rho^{\gamma}$ $[\gamma = C_p/C_v]$

Clouds and Dust

Clausius-Clapeyron $\frac{dP_s}{dT} = \frac{L_H P_s}{R_g T^2}$

Dust sinking time $t \approx \frac{\eta H}{g r^2 \Delta\rho}$

Radiative Transfer

Optical depth $\frac{\partial\tau}{\partial z} = \alpha\rho$

Radiative diffusion $F = -\frac{16}{3}\frac{\sigma T^3}{\alpha\rho}\frac{\partial T}{\partial z}$

Stratosphere $T_0 = \frac{1}{2^{1/4}} T_{eq}$

Greenhouse effect $T = T_0\left(1 + \frac{3}{2}\tau\right)^{1/4}$

Time constant $t_{rad} = \frac{P}{g}\frac{C_p}{\sigma T^3}$

Dust optical depth $\tau = \frac{3h\rho}{4r\rho_s}$

Dynamics

Governing eqn. $\frac{\partial v}{\partial t} = -\frac{1}{\rho}\frac{\partial P}{\partial y} - 2\Omega\sin\phi u + F_y$

Rossby number $Ro = \frac{u}{2L\Omega\sin\phi}$

Thermal winds $\frac{\partial u}{\partial z} = -\frac{g}{2\Omega\sin\phi T}\frac{\partial T}{\partial y}$

Reynolds number $\mathrm{Re} = \frac{uL}{\nu}$

Turbulence $u_l \sim (\varepsilon\, l)^{1/3}$

Brunt-Vaisala frequency $\omega_{NB}^2 = \frac{g}{T}\left(\left(\frac{dT}{dz}\right) + \frac{g}{C_p}\right)$

Rossby deformation radius $\lambda \approx \left(\frac{uR}{\Omega}\right)^{1/2}$ Convective heat transport $F \sim \rho C_p \left(\left.\frac{dT}{dz}\right|_{ad} - \frac{dT}{dz}\right)^{3/2} \left(\frac{g}{T}\right)^{1/2} H^2$

Climate Evolution

Urey cycle $MgSiO_3 + CO_2 \rightarrow MgCO_3 + SiO_2$

Jeans escape $\phi = n v_{th} (1+\lambda) e^{-\lambda}$ [$\lambda = \left(v_{esc} / v_{th}\right)^2$] [$v_{th} = \sqrt{3 R_g T / \mu}$]

Impact blowoff $\Delta m = \frac{v_i^2}{v_{esc}^2} \pi R_i^2 \rho_0 H$ Energy-limited blowoff $\frac{dm}{dt} = \varepsilon \frac{\pi R_{ext}^2 F_{ext}}{GM / R}$