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**Question 1.** Here we’re going to investigate two possible explanations for proposed cryovolcanic features on Ariel.

a) The thickness of the proposed flow features is $h$ (see Figure). What is the approximate conductive cooling timescale of these features (in years)? Take $\kappa=10^{-6}$ m$^2$s$^{-1}$. [1]

b) Use your answer to a) and the equation for an axisymmetric flat flow in your notes to deduce the viscosity of the flow feature. Take $\rho=1000$ kgm$^{-3}$ and $g=0.27$ ms$^{-2}$. [3]

c) How does this viscosity compare to that for solid ice near its melting point? [1]

d) One possible way of explaining the inferred viscosity is that the fluid is a low-temperature slush of solid ice + liquid. Taking $\phi=0.5$, $a=1$mm, $\eta=10^{-2}$ Pa s and $\Delta\rho=100$ kg m$^{-3}$, what is the compaction timescale? How does this compare to the cooling timescale? What do you conclude about this potential explanation for the inferred viscosity? [4]

e) Rather than assuming a Newtonian rheology as in part b), one can instead assume a Bingham rheology. Taking the levee width to be $w/2$ (see Figure), what is the implied Bingham yield stress? [2]

f) How does this yield stress compare to that for silicates? [1] [12 total]

**Question 2.**

a) The Enceladus plume reaches about 200 km into space. If the surface gravity of Enceladus is about 0.1 ms$^{-2}$, what is the surface velocity of the plume material? (You can assume $g$ is constant). [2]
b) Water vapour has a specific heat capacity of about 1400 J/kg K. Based on your answer to a), what is the implied temperature of the erupting material? Does this seem reasonable? [2]

c) One way of reducing the exit velocity of vapour is to mix in some solids. The *effective* specific heat capacity is reduced by a factor of (1+s), where s is the ratio (mass of solids:mass of vapour). If the initial vapour temperature is 300 K, what value of s is required to give the observed exit velocity? Is the plume mostly gas or mostly solid? [2]

d) The total (solid) mass flux in the plume is about 50 kg/s. Assume that these solids started out as liquid water at 300 K and cooled and solidified to 80 K. Taking $L=300 \text{ kJ/kg}$ and $C_p=2000 \text{ J/kgK}$, what is the power (in W) advected by this mass flux? [3]

e) The measured heat flow from the South Pole of Enceladus is about 5 GW. How does this compare with your answer to d), and what do you conclude? [2]

f) Let’s assume that the solid mass flow is being supplied by upwards flow of water through about 10 vents (cracks). We’ll assume that each vent is about as wide as it is long. *Using this assumption*, write down an expression for the volume flow (in m³/s) through one vent in terms of its width w, viscosity η, g and density contrast $\Delta \rho$. [2]

g) The mass flow through one vent is about 5 kg/s, the viscosity of water is $10^{-3} \text{ Pa s}$, and we’ll assume there is a positive density contrast driving upwards flow of 100 kg/m³. Taking the density of water to be 1000 kg/m³, what is implied crack width? [2]

h) Using your answer to g), what is the maximum upwards flow velocity in the crack? [1]

i) Using this upwards flow velocity, the width of the crack, and the thermal diffusivity of ice ($10^{-6} \text{ m}^2/\text{s}$), calculate how deep the flow could start from without the material in the crack freezing before it reaches the surface. Does this depth seem reasonable? [3] [19 total]

**Question 3 (Bonus/grad students).** Here we’re going to investigate fragmentation during magma ascent in a bit more detail.

We’re going to make two extra assumptions. First, the density contrast between magma and solid is given by $\Delta \rho = \rho \phi$, where φ is the bubble volume fraction. Second, φ depends on height above some initial point $h$: $\phi = c h$, where c is a constant with units m⁻¹.

a) Starting with the equation given in your notes, show that these assumptions yield a first order differential equation: $\frac{dh}{dt} = \frac{g \rho c h w^2}{8 \eta}$ where w is the width of the conduit.[2]

b) Solve this equation subject to the initial conditions that $\phi = \phi_0$, $h = h_0$ at $t=0$. [2]

c) Your solution should be an exponential with a time constant. Write down an expression for the time constant. [1]
d) Differentiate your solution to get an expression for the velocity $u$ as a function of time. [1]

e) Fragmentation happens when $u = \sigma \frac{w}{\phi \eta}$. Find an expression for the time it takes to reach fragmentation in terms of $\sigma$, $\phi_0$, $g$, $\rho$, $w$ and $c$. [3]

f) Explain whether the time gets longer or shorter as $\sigma$, $\eta$, $g$ and $w$ increase, and explain why each result makes sense [4] [13 total]