Show all your working for full credit

1. Here we’re going to consider a circular viscous flow spreading radially (like a pancake dome). In the notes we saw that the flow speed is given by

\[ u = \frac{\rho gh^3}{\eta r} \]

where \( r \) and \( h \) are the radius and central thickness of the flow at a particular time and \( \eta \) is the flow viscosity.

a) Assuming that volume is conserved, write an expression relating \( h, r \) and the drop volume \( V \). You can assume \( h \) doesn’t vary significantly with radius and that the flow is axisymmetric. [1]

b) Hence write an expression for \( u \) in terms of \( \rho, g, V, \eta \) and \( r \). [1]

c) Because the flow is spreading, \( r \) is increasing, so we have \( u = \frac{dr}{dt} \). Make use of this relationship and your answer to b) to find an expression for \( r \) as a function of \( t \). You can assume the boundary condition that the initial radius at \( t=0 \) is \( r=r_0 \). [4]

d) After a long time, we can assume that \( r \gg r_0 \). This allows you to write down a simpler version of your answer to c) – what is it? [1]

e) Use your answers to a) and e) to find an expression for how \( h \) varies with time. (Hint: you should find it goes as \( V^{1/4} t^{-1/4} \)). [2]

f) Show that your answer to e) has the correct units. [1]

g) Now we’ll assume that the flow continues to spread until it cools conductively. Write down the approximate timescale for the flow to cool conductively, taking the thermal diffusivity to be \( \kappa \). [1]

h) Using your answers to f) and g), find an expression for the final flow thickness \( h \) in terms of \( V, \eta, \rho, g \) and \( \kappa \). [3]

i) What happens to the final flow thickness if the viscosity increases? Does this make sense? [2]

j) A typical pancake dome on Venus might have \( h=1 \) km, \( V=700 \) km\(^3\), \( \kappa=10^{-6} \) m\(^2\)s\(^{-1}\), \( \rho=3000 \) kg/m\(^3\) and \( g=9 \) ms\(^{-2}\). What magma viscosity is implied? [2]

k) What do you conclude about the composition of these magmas on Venus? [1] [19 total]
2. Here we’re going to consider melt transport on Io assuming porous flow.

a) We can use the compaction timescale from your notes to derive an ascent velocity $u$ for magma in a partially-molten Io. Write down an expression for $u$ in terms of viscosity, grain size, melt fraction $\phi$, $g$ and density contrast $\Delta \rho$. [2]

b) The rate at which the melt buries the surface $u_s = \phi u$. What is the reason for the additional factor of $\phi$? [1]

c) Use your notes to write down an expression for the surface heat flux due to melt transport, in terms of $u_s$, density, specific heat capacity, latent heat and temperature change $\Delta T$. [1]

d) Let’s assume that the surface heat flux $F$ is known. Use your answers above to find an expression for the melt fraction $\phi$ that produces the specified heat flux $F$. [3]

e) If the melt viscosity increases, what happens to $\phi$? Does this make sense? [2]

f) On Io, we can take $F=1$ W$m^{-2}$, $(C_p \Delta T + L)=1.6$ MJ/kg, $\rho=3$ g/cc, $\Delta \rho=0.1$ g/cc, $a=1$ mm, $g=1.7$ ms$^{-2}$, $C=100$ and $\eta=100$ Pa s. What is the melt fraction required? [2]

g) Does this melt fraction imply that the mantle of Io is behaving like a liquid or a solid? [1] [12 total]

**Bonus/Grad. Students**

3a) A convective adiabat is defined by

$$T(z) = T_0 \exp \left( \frac{\alpha g z}{C_p} \right)$$

where $T_0$ is a constant, $g$ is gravity, $z$ is depth, $\alpha$ is thermal expansivity and $C_p$ is specific heat capacity. Using a Taylor series expansion (or otherwise), convert this equation to a linear relationship between $T$ and $z$. (Hint: you can take $\alpha g z / C_p$ to be small.) [2]

b) The solidus temperature $T_m$ increases at a rate depending on the Clapeyron slope $\gamma = dT_m/dP$ where $P$ is pressure. We can write an expression for the solidus

$$T_m(z) = T_{m0} + cz$$

where $T_{m0}$ and $c$ are constants. Write down an expression for $c$ in terms of $\gamma, \rho$ and $g$. [2]

c) Use your answers to a) and b) to determine an expression for the depth at which melting starts in terms of $T_0, \alpha, C_p, T_{m0}, \gamma, g$, and $\rho$. [2]

d) For Earth, we have $T_0=1300^\circ$C, $T_{m0}=1100^\circ$C, $C_p=1200$ J/kgK, $\alpha=3 \times 10^{-5}$ K$^{-1}$, $g=10$ ms$^{-2}$, $\rho=3000$ kgm$^{-3}$ and $\gamma=100$ K/GPa. At what depth does melting start? [2]
e) Do you expect to get melting beneath normal continental lithosphere? [1]

f) The Earth produces about 30 km$^3$ of melt each year. Using the same values as for melts on Io in Question 2, what heat flux does this represent? Take the radius of the Earth to be 6400 km. [3]

g) Using your answer to 2d, what average melt fraction is implied by the heat flux you calculated in f) above (assuming porous flow)? You can take all the other parameters to be the same as for Io, except that for Earth $g=10$ ms$^{-2}$. [2] [13 total]