1. Here we’re going to think about freezing ice shells.

a) During some period, a layer of water of thickness \( d_1 \) freezes to ice, of thickness \( d_2 \) (see Figure). If water and ice have densities of \( \rho + \Delta \rho \) and \( \rho \), respectively, write down an expression relating \( d_1, d_2, \rho \) and \( \Delta \rho \). [Hint: you can assume that \( d_1 \) and \( d_2 \) are very small compared to \( R \).] [3]

b) Because \( d_2 > d_1 \), the surface of the ice shell will expand outwards by a distance \( \Delta d = (d_2 - d_1) \). Write down an expression for \( \Delta d \) in terms of \( d_1, \Delta \rho \) and \( \rho \). [1]

c) The outwards expansion will cause the surface area of the body to increase. Write down an expression for the approximate change in surface area in terms of \( \Delta d \) and \( R \). [Hint: assume \( \Delta d \ll R \).] [2]

d) The surface strain \( \varepsilon \) is the change in area divided by the original area. Write down an expression for the strain in terms of \( \Delta d \) and \( R \). [1]

e) The surface strain on Charon is about 1%. How much has Charon’s shell thickened by if the radius of Charon is 606 km? [1]

f) The approximate timescale for a shell to thicken by \( \Delta d \) is \( 2d \Delta d / \kappa \), where \( d \) is the original shell thickness and \( \kappa \) is the thermal diffusivity. Taking \( d = 150 \) km and \( \kappa = 10^{-6} \) m\(^2\)s\(^{-1}\), calculate the thickening timescale and hence the strain rate for Charon [3]

g) Let’s assume that faults on Charon move when the stresses exceed 10 MPa. Use your answer to f) to calculate the average time interval between the faults moving. Take the Young’s modulus of ice to be 9 GPa. [2] [13 total]
2. Here we’re going to think about the tectonics caused by spin-down.

a) As a planet spins down, its shape will change and it will generate surface stresses. Do you expect the equator to be in compression or extension, and why? What about at the poles? [3]

For a thin-shell planet, the East-West and North-South stresses arising from despinsing can be described as follows:

\[ \sigma_{EW} = c (1 + 9 \cos 2\theta) \]
\[ \sigma_{NS} = c (-5 + 3 \cos 2\theta) \]

Here \( \theta \) is the latitude, compressional stresses are positive and \( c \) is a constant that depends on how much despinsing has happened.

b) Sketch how \( \sigma_{EW} \) and \( \sigma_{NS} \) vary with latitude from 0° to 90°. At what latitude do the EW stresses equal zero? [4]

c) This kind of despinsing will generate two different styles of faulting. What two styles are generated, and where will they be located? [2]

d) On somewhere like Mercury, contraction may have operated in addition to despinsing. Let’s assume that contraction generates an isotropic stress \( \sigma_{CO} = 4c \).

Make a new sketch of how \( \sigma_{EW} \) and \( \sigma_{NS} \) vary with latitude, taking into account the contraction. How many styles of faulting will be generated now? And where are the boundaries between the different styles of faulting located? [6] [15 total]
**Bonus/Grad Students**

3. Here we’re going to investigate the origin of novae on Venus.

   a) Using part a) of the figure above, write down an expression for the horizontal strain due to the formation of a graben with paired normal faults both dipping at an angle $\theta$ and with vertical offset $d$. [2]

   b) For typical novae, graben depths are about 0.4 km, the spacing between grabens is about 6 km, and the observed dip angle is about 45°. What is the implied strain? [1]

   c) Suggest one process which might cause us to underestimate the dip angle. What effect would this have on our estimate of the strain? [2]

   d) Let’s assume that novae form by an initially flat surface being pushed up (e.g. by an intrusion). Looking at part b) of the figure above, use Pythagoras to find a relationship between the uplift $h$, the width of the region $w$ and the radius of curvature $r$. [Hint: you can assume that $h<<r$]. [3]

   e) The strain is given by $T_e/2r$, where $T_e$ is the elastic thickness. For novae, we have $w=50$ km and $h=0.5$ km, and on Venus $T_e=30$ km. What is the predicted strain? [2]

   f) How does the strain predicted in part e) compare with the strain measured in part b)? [1]

   g) Suggest one possible explanation for the discrepancy [1] [12 total]