1. Radar altimeters have a couple of disadvantages compared to laser altimeters.

(a) The horizontal resolution of radar instruments is limited by the wavelength $\lambda$ that they use. Panel a) above shows that the horizontal resolution $d$ depends on $\lambda$ and the range $r$. Use Pythagoras to find an expression for $d$ in terms of $\lambda$ and $r$. [Hint: you can ignore any terms in $\lambda^2$ because they are negligible.] [3]

(b) For a radar instrument operating at 10 cm wavelength with a range of 100 km, what is the horizontal resolution? [1]

c) Another problem, shown in panel b), is that mountains not directly below the transmitter can interfere with the direct echo received if they are tall enough (this is called “layover”). Use Pythagoras again to find an expression for the height $h$ of a mountain that will just interfere with the direct echo if it is at a distance $x$ and the range to the transmitter is $r$. [2]

d) A similar problem will arise if the ground is sloping. Using your answer to c), write an expression for the critical slope $s$ in terms of $r$ and $x$. [Hint: use radians, and assume that $h$ is much smaller than $x$.] [2]

e) Real slopes tend to obey an expression $s = cx^H$ where $c$ and $H$ are constants. Using your answer to d), solve to find the critical distance $x$ beyond which layover should cease to be a problem. [3]

f) A rough surface might be described by $s = 0.5x^{0.6}$, where $x$ is in metres. Find the critical distance in this case for the same transmitter as in b). [1] [12 total]
2. Here we’ll consider a spherical, rotating Earth (see Figure below).

![Diagram of Earth with labels](image)

a) At the equator and at the pole, a pendulum will point straight down (i.e. towards the centre of the Earth). At intermediate latitudes, the pendulum will be slightly deflected. Why? And which way (equatorwards or polewards) will it be deflected? [2]

b) In class we saw that the potential at the surface of a spherical rotating body is given by

\[ V = -\frac{GM}{r} - \frac{1}{2} r^2 \Omega^2 \sin^2 \theta \]

Where \( M \) is the mass of the Earth, \( G \) is the gravitational constant, \( r \) is the radius, \( \Omega \) is the angular rotation frequency and \( \theta \) is the colatitude (see figure). The radial (inwards) acceleration \( a_r \) is given by differentiating \( V \) with respect to \( r \). Write down expressions for the radial acceleration at the pole (\( \theta = 0 \)) and the equator (\( \theta = 90^\circ \)). Which is larger? [4]

c) The tangential acceleration \( a_\theta \) is given by differentiating \( V \) with respect to \( \theta \), and then dividing by \( r \). Write down an expression for \( a_\theta \), and state the value of \( a_\theta \) at the equator. [2]

d) The angular deflection of the pendulum (in radians) is given by \( a_\theta / a_r \). Write down an expression for this deflection at \( \theta = 45^\circ \). [2]

e) For the Earth, \( g = 9.8 \text{ m s}^{-2} \), \( r = 6400 \text{ km} \) and \( \Omega = 7.3 \times 10^{-5} \text{ s}^{-1} \). What is the angular deflection in degrees at \( \theta = 45^\circ \)? [2] [12 total]

3a) The hypsograms in your lecture notes show an asymmetry: there is a “long tail” towards high elevations, but not towards low elevations. Why do you think this might be? [2]
b) Titan has very little topographic relief compared to other bodies. Why might this be? [3] [5 total]

**Question 4 (Bonus/Grad students)** A more precise way of predicting the flattening $f$ of a fluid planet is to use $f = \frac{\Omega^2 a^2}{2g} h_2$ where $h_2$ is a dimensionless number called the Love number. Given the observed flattening, this expression can be used to deduce the Love number.

a) What value of $h_2$ have we assumed in the class notes? [1]

b) For a fluid planet, the normalized moment of inertia $c$ depends on $h_2$ as follows:

$$c = \frac{2}{3} \left[ 1 - \frac{2}{5} \left( \frac{5}{h_2} - 1 \right)^{1/2} \right]$$

For a uniform body, the moment of inertia is 0.4. What is the corresponding value of $h_2$? [2]

c) Using the Table in the class notes, deduce the $h_2$ of the Earth given its observed flattening [2]

d) What is the normalized moment of inertia of the Earth, and what is this telling you about its internal structure? [2]

e) If you repeat the same exercise for Mars, what values of $h_2$ and $c$ do you derive? How might you explain your answers? [4] [11 total]