1. Here we are going to do a case study on Mercury, the terrestrial planet about which least is known. Most of our knowledge currently comes from three distant fly-bys by Mariner 10 in 1974-5, but the Messenger spacecraft just completed three more fly-bys and goes into orbit soon.

Many of the equations you need are given in this week’s lecture. Make sure you state what assumptions you are making.

a) Mercury’s mass and radius are $3.3 \times 10^{23}$ kg and 2440 km, respectively. What are its density, and the surface acceleration due to gravity? (2)

b) If Mercury is made of a core of density 7000 kg m$^{-3}$ and a mantle of density 3400 kg m$^{-3}$, how thick is the mantle? (4)

c) Use your answer to b) to predict the normalized moment of inertia of Mercury ($C/MR^2$). (3)

d) We don’t yet know the moment of inertia of Mercury. Why not? (1)

e) The Caloris impact basin on Mercury is about 3 km deep and 1300 km in diameter. If the impact basin were uncompensated, how big a gravity anomaly would you see at the surface (assume a crustal density of 3 g/cc)? (2)

f) How big a gravity anomaly would you measure at the spacecraft altitude of 200 km? (2)

g) If the spacecraft actually measured a gravity anomaly of 70 mGal, what is the degree of compensation of the impact basin? (2)

h) What does this degree of compensation imply about the elastic thickness of the lithosphere on Mercury? Assume Poisson’s ratio = 0.25, Young’s modulus = 100 GPa, and a crust-mantle density contrast of 500 kg m$^{-3}$. (4)

i) The surface temperature is 400 K. Assuming that rock stops being elastic at about 1000 K, what is the heat flux implied by the elastic thickness you have just derived? Assume the thermal conductivity of rock is 3 W m$^{-1}$ K$^{-1}$. (2)

j) If the only source of heat is radioactive decay within the mantle, what is the heating rate (in W kg$^{-1}$) in the mantle responsible for the heat flux you have estimated? (3)
k) One possible extra source of heating in the mantle is tidal heating. Calculate the amplitude of the equilibrium fluid tidal bulge for Mercury due to tides raised by the Sun, and hence state whether you think tidal heating is important. The mass of the Sun is $2 \times 10^{30}$ kg and its distance from Mercury is 24,000 Mercury radii. (3)

l) The core of Mercury is thought to be molten, which suggests a temperature at the base of the mantle of about 2000 K. Estimate the Rayleigh number of Mercury’s mantle, stating what parameter values you are assuming. Is Mercury’s mantle convecting? (4)

m) Mercury may have cooled and contracted over time. If its mantle has cooled by 100 K, how much strain will have developed? (Assume a thermal expansivity of $3 \times 10^{-5}$ K$^{-1}$) (2)

n) What would the corresponding stress be? (1)

o) At what depth in Mercury’s crust would this stress equal the lithostatic pressure? (2)

p) Mercury has surface features called lobate scarps which look like thrust faults and are tens of km wide. Based on your answer to o), do you think the lobate scarps could have been formed by thermal contraction, and why? (2) (39 total)