1. a) What is the maximum gravity anomaly (in mGal), measured at the surface, created by an impact crater 2 km deep and 20 km wide in an ice shell of density 1000 kg m$^{-3}$? (1)

b) Would you expect to be able to detect this anomaly at a spacecraft altitude of 100 km or not, and why? (2)

c) Now we’ll assume that the crater is fully isostatically compensated by a varying ice shell thickness (see sketch). Let the ice and water densities be $\rho_i$ and $\rho_w$, respectively. Find an expression for the water uplift $w$ in terms of $d$, $\rho_i$ and $\rho_w$ (2).

d) Write down an expression for the gravity anomaly at the surface due to the crater itself, in terms of $\rho_i$, $d$ and $G$ (1)

e) Write down an expression for the gravity anomaly at the surface due to the water uplift in terms of $\rho_i$, $\rho_w$, $w$, $h$, $D$ and $G$. Make sure you include the effect of upwards attenuation. (2)

f) Using your results to c, d and e, write down an expression for the total gravity anomaly measured at the surface, as a function of $\rho_i$, $d$, $h$, $D$ and $G$. (4)

g) Sketch how the gravity anomaly will vary as a function of $D$, with everything else kept constant. Under what circumstance does an isostatically-compensated feature result in a negligible surface gravity anomaly? (3) (15 total)

2. Simple concepts of elasticity.

   a) Say we have an elastic band, of dimensions 10cm x 1cm x 0.1 cm. If we suspend a 1 kg mass from one end and observe that its length has increased by 1 cm, what is the Young’s modulus of the band? (4)
b) If the width of the band decreases by 0.1 cm, what is the Poisson’s ratio of the band? (2)

c) You add another 1 kg weight and the band increases in length by 1.3 cm. What do you conclude? (2) (8 total)

3. Here we are going to make use of a simple equation of state.

a) Saturn has a mass of $5.7 \times 10^{26}$ kg and an equatorial radius of 60,300 km. Calculate its mean density and the acceleration due to gravity at its equator (2)

b) For hydrogen at high pressures and temperatures, the approximate isothermal equation of state is given by

$$P = c \rho^{5/3}$$

where $P$ is pressure, $\rho$ is density, and $c$ is a constant ($=10^6$ in SI units).

Using this equation of state and the hydrostatic equation ($dP = \rho g dz$), derive an expression for how pressure varies as a function of depth $z$ within Saturn. You may assume that $g$ is constant and that the pressure at $z=0$ is zero. (5)

c) Using your expression, calculate the resulting pressure and density at the centre of Saturn. (2)

d) Name at least three factors/assumptions which could make your result inaccurate. For each factor, suggest whether it will result in an under- or over-estimate of the pressure and density at the centre. (6)

e) The true pressure and density at the centre of Saturn are about 40 Mbar and 13 g/cc. In the light of these numbers, what is probably the most important of the factors identified in part d) and why? (2) (17 total)

4. [Optional unless you are a grad student]

Here we are going to investigate the behaviour of a material with temperature-dependent viscosity. Viscosity $\eta$ varies as $A \exp(Q/RT)$, where $A$ is a constant, $Q$ is the activation energy, $R$ is the gas constant and $T$ is the absolute temperature. Note that $Q/RT$ is dimensionless.

a) Let the viscosity just below the melting temperature $T_m$ be $\eta_m$. Write down an expression for $A$ in terms of $T_m$, $\eta_m$ and the other variables (2).

b) Now express the viscosity $\eta$ at some different temperature $T$ in terms of $\eta_m$, $T_m$, $Q$ and $R$ (2)
c) By writing $T$ as $T_m - \Delta T$ and assuming that $\Delta T << T_m$, derive an expression for the viscosity in terms of $\Delta T, T_m, \eta_m, Q$ and $R$ (5)

d) What units does the quantity $RT_m^2/Q$ have? Suggest a physical meaning for this quantity. (3)

e) For silicates $T_m = 1350$ K, $Q = 300$ kJ/mol and $R = 8.34$ J mol$^{-1}$K$^{-1}$. What is the value of $RT_m^2/Q$? How much does the silicate viscosity change for a 100 K change in temperature? (2) (14 total)