

Show all your working for full credit

This set of problems will span several of the concepts you have used in the course. Treat them as revision questions.

1. a) Write down an expression for the gravity $g(r)$ inside a planet, where r is the radial distance from the centre of the planet and density ρ is uniform. (1)
- b) Using this answer and the hydrostatic assumption, write down a general expression for the pressure $P(r)$ inside the planet (3)
- c) Using the boundary condition that $P=0$ at the surface ($r=R$), write down an expression for the pressure at the centre of the planet in terms of ρ, G and R (2)
- d) If geological materials start to flow at a pressure of roughly 10 MPa, how big can a planet of density 4000 kgm^{-3} get without assuming a spherical shape? (1)
- e) Would you expect most asteroids to be spherical or not? (1) (8 total)

2. a) Write down an expression for the surface acceleration due to gravity, g_s , of a planet with mass M and radius R (1)
- b) Write down an expression for the acceleration $g(z)$ at a height z above the surface, in terms of g_s, R and z . (2)
- c) By taking out a factor of GM/R^2 and using the binomial expansion, show that your expression simplifies to $g(z)=g_s (1 - [2z/R])$ when $z \ll R$ (2)
- d) Now use the hydrostatic assumption and the ideal gas equation ($P\mu=\rho rT$ - note that here r is the gas constant!) to find the pressure as a function of z when g varies as given in c) above. Express your answer in terms of z, μ, g_s, R, T, r and P_0 , the pressure at the surface (4) (9 total)

3. Short-lived isotopes like ^{26}Al decay according to the following scheme:

$$H(t)=H_0 \exp(-\lambda t) \quad (1)$$

where $H(t)$ is the heat production rate i.e. the power (in W kg^{-1}) at time t , H_0 is the initial heat production rate (at solar system formation, $t=0$) and λ is a decay constant (s^{-1}).

- a) Using equation (1), write down an expression including H_0 and λ for the energy per kg released by a radioactive isotope between $t=0$ and some later time τ (Hint: think about how energy and power are related) (1)
- b) Assuming that conduction is negligible, write down an expression for the corresponding change in temperature between $t=0$ and $t=\tau$ if the specific heat capacity of the material is C_p (1)
- c) For an asteroid of radius R , write down an expression for the approximate time it takes for heat to diffuse out to the surface (and thus for the asteroid to reach conductive equilibrium). (1)
- d) Before conductive equilibrium is reached, the temperature will increase. By assuming that τ is given by your answer to c) and also that $\tau \ll \lambda$, write down an expression for the expected temperature increase in terms of H_0, R, C_p and the thermal diffusivity κ (Hint: remember that $e^x \approx 1 + x$ when x is small) (3)

- e) Explain what happens to your answer to d) as H_0, R, κ and C_p vary, and why these results make sense. Why does λ not appear in the equation? (3)
- f) The initial heat production rate H_0 is about $10^{-7} \text{ W kg}^{-1}$. If melting begins for a temperature increase of 1000 K, how big can an asteroid be before it starts to melt? Assume $\kappa=10^{-6} \text{ m}^2\text{s}^{-1}$ and $C_p=1000 \text{ J kg}^{-1} \text{ K}^{-1}$ (1)
- g) What happens to your answer to f) if the asteroid forms at a time later than $t=0$? (1)
- h) Use your Week 8 notes to write down an expression for the gravitational potential energy per unit mass released during the accretion of an asteroid in terms of G, R and its density ρ . (2)
- i) Using your answers to d) and h), write down the ratio of the temperature change due to radioactive decay to the temperature change due to gravitational potential energy. Assuming $\rho=3000 \text{ kg m}^{-3}$, which cause of temperature change is more important? What is one surprising implication of your answer? (3) (16 total)