

Show all your working for full credit

1. Here we are going to consider the orbital evolution of Io.

- a) If dissipation in the *satellite* dominates, the satellite's orbit circularizes and (to conserve angular momentum) the satellite moves *inwards*. Does moving inwards require energy or give off energy? (1)
- b) The satellite's gravitational potential energy E is given by $-GMm/2a$, where m and M are the mass of the planet and satellite, respectively, and a is the semi-major axis. By using the chain rule, find an expression for the rate of change of energy dE/dt in terms of the rate of change in semi-major axis da/dt . (2)
- c) Using Kepler's laws, find the power dE/dt in terms of M , a , da/dt and the satellite's orbital frequency ω (2)
- d) Let's assume that this gravitational power is what is ultimately driving Io's volcanism. If Io's power output is 5×10^{13} W, $M=9 \times 10^{22}$ kg, it has a period of 1.77 days and a semi-major axis of 4.2×10^5 km, how fast is Io approaching Jupiter in km/Myr? (3)
- e) Using Kepler's laws again, how much will Io's period change in 1 Myr as it moves inwards towards Jupiter? (2) (10 total)

2. Here we're going to use an approximate way of calculating tidal heating on Enceladus.

- a) The magnitude of the diurnal tide is $3e$ times the equilibrium tide, where e is the eccentricity. Using your notes, write down an expression for the magnitude of the satellite diurnal tide in terms of e , the satellite radius R , satellite mass M , primary mass m and semi-major axis a . (1)
- b) The tidal strain is simply the diurnal tide divided by R . Write down an expression for the tidal strain. (1)
- c) The strain rate is the tidal strain divided by the orbital period. Using Kepler's laws, determine the period and thus write down an expression for the strain rate in terms of e, m, M, R, a and the gravitational constant G . (4)
- d) Assuming that the satellite is a viscous fluid, the heating rate per unit volume (Wm^{-3}) is given by (viscosity \times strain rate²). Viscosity has units of Pa s. Show that the expression for this heating rate is dimensionally correct. (2)
- e) Using c) and d), write down the heating rate per unit volume in terms of e, m, M, R, a, G and the viscosity η (1)
- f) Hence find the *total* heating rate in the satellite. Write down your answer in terms of e, m, M, R, a, G and η . (1)
- g) What happens to the heating rate if e decreases or a increases? Explain why this behaviour makes physical sense (2)
- h) It turns out that the total heating should be multiplied by another factor f , which is related to the lag of the tidal bulge and means that only some of the energy gets turned into heat. Using $f=0.01$, calculate how much tidal heat (in W) Enceladus should generate, using the following parameters: $M=1 \times 10^{20}$ kg, $m=5.7 \times 10^{26}$ kg, $R=250$ km, $a=2.4 \times 10^5$ km, $\eta=10^{13}$ Pa s (this is the value for warm ice), $e=0.0045$ and $G=6.67 \times 10^{-11}$ in SI units. (3)

- i) Spacecraft observations indicate that the South pole of Enceladus has a surface heat flux of roughly 0.2 Wm^{-2} . Comment on this value in view of your answer to h). (3)
- j) Mimas has a similar mass and radius to Enceladus, but has an eccentricity $e=0.02$ and has a semi-major axis $a=1.8 \times 10^5 \text{ km}$. How many times more heat should Mimas be producing than Enceladus? (2)
- k) In fact, as far as we can tell, Mimas is geologically dead and produces no measurable heat. Give one possible explanation for why this might be so. (2) (24 total)