1 a) $A = \begin{pmatrix} 1 & 2 \\ -1 & 3 \end{pmatrix}$  $A^T = \begin{pmatrix} 1 & -1 \\ 2 & 3 \end{pmatrix}$  $AA = \begin{pmatrix} 1 & 2 \\ -1 & 3 \end{pmatrix} \begin{pmatrix} 1 & 2 \\ -1 & 3 \end{pmatrix} = \begin{pmatrix} -1 & 8 \\ 4 & -7 \end{pmatrix}$

$|A| = 5$  $A^{-1} = \frac{1}{5} \begin{pmatrix} 3 & -2 \\ 1 & 1 \end{pmatrix}$

b) $B = \begin{pmatrix} 4 & 1 & 1 \\ 0 & 1 & 2 \\ 0 & 0 & 1 \end{pmatrix}$  $(4 \ 1 \ 1) \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix} = \begin{pmatrix} 4 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix}$

$B^{-1}$

$\begin{pmatrix} 8 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix} = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix}$

$\frac{1}{8} R_1$  $\begin{pmatrix} 1 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 1 & 0 \end{pmatrix}$  $B^{-1}$  $\begin{pmatrix} 1 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 1 & 0 \end{pmatrix}$

2 a) $(AD)^T = (D^T A^T)$

b) Let $D = B C$  then $(ABC)^T = (AD)^T = D^T A^T$

but $D = B C$  and $(BC)^T = C^T B^T$  so $(ABC)^T = C^T B^T A^T$

c) $D = B A B^{-1}$  $D^T = (B^{-1})^T A^T B^T$  (from b),

$B^{-1} = B^T$  so $(B^{-1})^T = (B^T)^T = B$

$\Rightarrow D^T = B A^T B^T$  but $A^T A = I$  so $D^T = B A B^{-1}$

so $D^T = D$  Q.E.D.

d) $(AB)^T = B^T A^T = B^{-1} A^{-1}$ if $A$ and $B$ are orthogonal.

$(AB)^T A (B^{-1} A^{-1}) = A^{-1}$  so $(AB)^T B^{-1} A^{-1} = I$

so $(AB)^{-1} (AB) B^{-1} A^{-1} = (AB)^{-1}$  $\Rightarrow B^{-1} A^{-1} = (AB)^{-1}$

so $(AB)^T = (AB)^{-1}$  Q.E.D.

3 c) $y_1 = m x_1 + c$  $y_2 = m x_2 + c$  $y_3 = m x_3 + c$

$\Rightarrow A \begin{pmatrix} x_1 \\ x_2 \\ x_3 \end{pmatrix} = \begin{pmatrix} c \\ 0 \\ 0 \end{pmatrix}$  $x = \begin{pmatrix} c \\ 0 \\ 0 \end{pmatrix}$  $b = \begin{pmatrix} y_1 \\ y_2 \\ y_3 \end{pmatrix}$
b) \[ A^T (A^T x - b) = 0 \Rightarrow A^T A x = A^T b \]
\[ d = A^T b \quad C = A^T A \]

c) \[
A = \begin{pmatrix} 1 & 1 & 1 \\ 1 & 3 & 1 \\ 3 & 1 & 1 \\ 2 & 1 & 3 \\ 3 & 1 & 1 \end{pmatrix} \quad A^T = \begin{pmatrix} 1 & 1 & 1 & 2 & 3 \\ 1 & 3 & 1 & 1 & 1 \\ 3 & 1 & 1 & 2 & 3 \\ 2 & 1 & 3 & 1 & 1 \\ 3 & 1 & 1 & 2 & 3 \end{pmatrix} \quad C = \begin{pmatrix} 1 & 1 \\ 1 & 3 \\ 1 & 3 \\ 3 & 1 \\ 6 & 4 \end{pmatrix} \]
\[ d = \begin{pmatrix} 1 & 1 & 1 & 2 & 3 \\ 1 & 3 & 1 & 1 & 1 \\ 3 & 1 & 1 & 2 & 3 \\ 2 & 1 & 3 & 1 & 1 \\ 3 & 1 & 1 & 2 & 3 \end{pmatrix} \begin{pmatrix} 0.8 \\ 4.2 \\ 8.7 \\ 13.7 \end{pmatrix} = \begin{pmatrix} 5.8 \\ 13.7 \end{pmatrix} \]

d) \[ 3c + 6m = 5.8 \quad 6c + 12m = 11.6 \quad 6c + 12m = 13.7 \]
\[ 6c + 12m = 11.6 \quad 6c + 12m = 13.7 \quad 2m = 2.1 \quad m = 1.05 \]
\[ 3c + 6.3 = 5.8 \quad 3c = -0.5 \quad c = -0.167 \]

e) By eye, points are close to \( y = xc \) so \( m \) should be \( 1 \) and \( c \) should be \( \approx 0 \).