Warmup (NPC)

1a) Find the eigenvectors and eigenvalues of the following matrix, and hence sketch the resulting strain ellipse [5]:

\[
\begin{pmatrix}
5 & 4 \\
3 & 3 \\
4 & 3 \\
3 & 3
\end{pmatrix}
\]

b) Using Gauss-Jordan elimination, find \(B^{-1}\) where:

\[
B = \begin{pmatrix}
3 & 1 & 2 \\
0 & 1 & 1 \\
0 & 0 & 1
\end{pmatrix}
\]

[4]

c) Also write down \(|B|\). [2] [11 total]

2) a) Given two matrices \(A\) and \(D\), what does \((A D)^T\) equal in terms of \(A^T\) and \(D^T\)? [1]

b) Using your answer to a), what does \((A B C)^T\) equal in terms of \(A^T\), \(B^T\) and \(C^T\)? [2]

c) Here we are going to use two special kinds of matrices: a symmetrical matrix \(A\), where \(A = A^T\); and an orthogonal matrix \(B\), where \(B^T = B^{-1}\).

We’ll define a matrix \(D = B A B^{-1}\).

By taking the transpose of \(D\) and using your answer to b), show that \(D\) is also a symmetrical matrix. [4]

d) Also prove that the product of two orthogonal matrices is also orthogonal. [3]

[10 total]

3) Here we’ll consider the matrix

\[
C = \begin{pmatrix}
a & b \\
b & a
\end{pmatrix}
\]

a) Find an expression in terms of \(a\) and \(b\) for the two eigenvalues of this matrix [4].

b) Hence find the two eigenvectors of this matrix [3]

c) What is the angle between these two vectors? [1] [8 total]