Warmup (NPC)

1 a) Write down the equation of a line that passes through the point (2,3,0) and is perpendicular to the vectors [1,2,1] and [0,1,1]. [3]

b) Find the equation of the plane that is everywhere equidistant from the points (2,1,2) and (-2,1,0). [3]

c) Given that \( f(x, y) = y \cos xy \), write down \( \frac{\partial f}{\partial x} \), \( \frac{\partial f}{\partial y} \) and \( \frac{\partial^2 f}{\partial x \partial y} \). [3] [9 total]

Longer Questions

2a) You are given the equations of two lines which intersect each other:

\[
(x - 4) = 3y = 3(z - 1)
\]

and

\[
2(x - 1) = y + 1 = z
\]

For each line, write down a vector parallel to the line. [2]

b) Also write down the coordinates of the point where the two lines intersect (Hint: solve two simultaneous equations for \( x \) and \( y \) first) [3].

c) For each line, calculate the \( y \) and \( z \) coordinates when \( x = 0 \) [2].

d) What is the angle between the two lines? [2]

e) Using your answers to a) and b), find the equation of a plane (in terms of \( x, y \) and \( z \)) which contains both of the lines. [4]

[13 total]

3a) The density of a sphere \( \rho \) is given by \( 3M/4\pi r^3 \), where \( M \) is its mass and \( r \) is its radius. Write down an expression for the fractional uncertainty in density \( \Delta \rho/\rho \) in terms of the fractional uncertainties in radius \( \Delta r/r \) and in mass \( \Delta M/M \). [3]

b) The radius of Pluto is uncertain by 0.13%. If its mass was uncertain by 0.2%, what would be the fractional uncertainty in Pluto’s density? Is the radius or the mass contributing more to the overall uncertainty? [2] [5 total]
4a) Temperature can vary as a function of position and time. The 1D temperature evolution equation is given by

\[ \frac{\partial T}{\partial t} = C \frac{\partial^2 T}{\partial x^2} \]  \hspace{1cm} (1)

where \( T(x, t) \) is temperature, \( x \) is distance, \( t \) is time and \( C \) is a constant (with units of \( \text{m}^2 \text{s}^{-1} \)).

You may assume that a solution to this equation is given by

\[ T(x, t) = T_0 e^{-t/\tau} \cos \left( \frac{2\pi x}{\lambda} \right) \]  \hspace{1cm} (2)

where \( T_0, \lambda \) and \( \tau \) are constants (with units of temperature, length (m) and time (s), respectively).

Substitute equation (2) into equation (1) and hence find an expression for \( \lambda \) in terms of \( C \) and \( \tau \). Are the units correct? [6]

b) Assume that \( \lambda \) gives the lengthscale of the system and \( \tau \) gives the approximate cooling time. If an igneous body with a lengthscale of 1 km is emplaced at depth, how long will it take to cool? Take \( C = 10^{-6} \text{ m}^2 \text{s}^{-1} \). [2] [8 total]

[35 total]