Note that this is not a comprehensive list. There are things that don’t appear on this sheet that I will expect you to know (such as how sin, cos etc. are defined, or what the differential of $e^x$ is). You will also need to be able to understand and manipulate these expressions.

**Basic Trigonometry**

$$\cos^2 x + \sin^2 x = 1$$

$$\sin(x + y) = \sin x \cos y + \cos x \sin y$$

$$\cos(x + y) = \cos x \cos y - \sin x \sin y$$

**Cosine formula:**

$$a^2 = b^2 + c^2 - 2bc \cos A$$

**Sine formula:**

$$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$$

When $x \ll 1$:

$$\sin x \approx x \quad \text{and} \quad \cos x \approx 1 - \frac{x^2}{2}$$

**Basic Calculus**

$$\frac{df}{dx} = \lim_{h \to 0} \frac{f(x + h) - f(x)}{h}$$

**Product rule:**

$$\frac{d}{dx}(uv) = u \frac{dv}{dx} + v \frac{du}{dx}$$

**Integration by parts:**

$$\int uv \, dx = uv - \int v du$$

**Maclaurin series expansion:**

$$f(x) = f(0) + xf'(0) + \frac{x^2}{2!}f''(0) + \frac{x^3}{3!}f'''(0) + \cdots$$

Useful examples:

$$e^x = 1 + x + \frac{x^2}{2!} + \cdots \quad (1 + x)^n = 1 + nx + \cdots$$

**Vectors**

Vector $\mathbf{a} = [a_1, a_2, a_3]$

Unit vector:

$$\hat{a} = \frac{\mathbf{a}}{|\mathbf{a}|}, \quad |\mathbf{a}| = \sqrt{a_1^2 + a_2^2 + a_3^2}$$

Dot product:

$$\mathbf{a} \cdot \mathbf{b} = |\mathbf{a}||\mathbf{b}| \cos \theta = a_1 b_1 + a_2 b_2 + a_3 b_3$$
Cross product:
\[ \mathbf{a} \otimes \mathbf{b} = [a_2b_3 - a_3b_2, a_3b_1 - a_1b_3, a_1b_2 - a_2b_1] \]
\[ |\mathbf{a} \otimes \mathbf{b}| = |\mathbf{a}||\mathbf{b}| \sin \theta \]

**Lines and Planes**
Vector and algebraic equations of a line passing through \( r_0 = (x_0, y_0, z_0) \) parallel to \( \mathbf{v} = [a, b, c] \):

\[ \mathbf{r} = r_0 + t\mathbf{v} \quad \frac{x - x_0}{a} = \frac{y - y_0}{b} = \frac{z - z_0}{c} \]

Vector and algebraic equations of a plane passing through \( r_0 = (x_0, y_0, z_0) \) perpendicular to \( \mathbf{n} = [a, b, c] \):

\[ \mathbf{n} \cdot (\mathbf{r} - r_0) = 0 \quad a(x - x_0) + b(y - y_0) + c(z - z_0) = 0 \]

**Partial Differentials**
\[ \frac{\partial f}{\partial x} = \lim_{h \to 0} \frac{f(x + h, y) - f(x, y)}{h} \]

For \( z = f(x, y) \) the total differential \( dz \) is given by
\[ dz = \frac{\partial f}{\partial x} dx + \frac{\partial f}{\partial y} dy \]

The gradient of \( z = f(x, y) \) in two dimensions is given by
\[ \nabla f = \left[ \frac{\partial f}{\partial x}, \frac{\partial f}{\partial y} \right] \]

The directional derivative \( D_u f \) of \( f \) in the direction \( \hat{u} \) is given by
\[ D_u f = \hat{u} \cdot \nabla f \]